Spectrum Trading with Interference Rights

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Abstract—**Spectrum, which supports the transmission of sound, data, and video, is critical to the implementation of mobile connected society. Federal Communications Commission (FCC) has been seeking policies that offer more spectrum access opportunities, such as secondary spectrum market and TV white space. In this paper, we develop the idea of trading spectrum in Interference Right. It means to alleviate the spectrum scarcity by cooperative spectrum usage and making the spectrum market as liquid as possible. We use some plausible case studies to illustrate the characteristics and features. The paper therefore includes a detailed description of trading procedures along with some first order quantitative modeling of the cases coupled with qualitative analysis.**

I. Introduction

Spectrum rights have been viewed as a solution for spectrum allocation since Coase pointed out that the most efficient way to assign spectrum is to define a property- like right and give it to users who value it the most. Since then, several researchers tried to define spectrum rights via exclusive usage rights, which use various dimensions to define spectrum exclusivity. This includes DeVany's three dimensional model [1] and Matheson's seven dimensional model [2] of electrospace. The exclusive usage rights are implemented through transmission power caps and guard bands, which are determined by regulators. Without these two aspects, it is difficult to prevent spatial, out-of-band and in-band interference.

In contrast to exclusive usage rights, defining rights based on interference restriction regulates how much interference could be tolerated in a certain region. This value is impacted by the terrain, current spectrum usage, noise level, and the receiver's performance. Interference restriction is not the opposite of transmitter power limits. In essence, interference restriction evolved from transmitter power limits and demands the application of advanced technology and flexible management, since a simple transmitter power cap does not easily support the current high demand for spectrum and its dynamic usage [3]. It also pays more attention on receiver, since it does not make sense to slow down the spectrum utilization for the sake of ancient receivers. The idea of "tolerable interference level" shares some similarities with the ideas we discuss in this paper. [4]

No matter how we define spectrum rights, the value that a primary user obtains from their license depends on (1) their ability to earn revenues from that investment and (2) capital gains associated with its increasing value. The former depends on the ability of the license holder to deliver a suite of services to end users at a contracted quality level. The latter depends on the license holder's ability to defend the parameters of the license. Interference Rights are defined by explicit management of interference. In essence, they authorize primary users to manage both aspects of their license. They can earn revenues from bandwidth that has not been fully utilized (due to old receivers, perhaps) while maintaining the license parameters granted under the license.

In this paper, we are explicitly considering cooperative secondary sharing according to the typology proposed by Weiss and Lehr [5]. In this approach, sharing a single spectrum hole among multiple secondary users is part of the outcome to be negotiated. One likely outcome of this negotiation, and the one that is the focus of this paper, is that the interference right will be granted to a single secondary user, so that sharing among secondary users will not be considered here.

The paper is organized as follows. Section II introduces the concept of Interference Rights and how to formulate Interference Rights. Section III illustrates spectrum trading in an Interference Rights regime, including trading procedures, participants, operation and enforcement. Section IV provides a case study to illustrate this approach. Section V concludes the paper and outlines future research.

II. Interference Rights

A. The Concept of Interference Rights

Under the Interference Rights regime, "license holders can write interference rights on their licenses, which can be traded, combined, or exchanged with other users." [6] Interference Rights explicitly allow secondary users to interfere with the existing services offered by a primary user to a certain level. It degrades the resource (spectrum), but does not necessarily decrease the Quality of Services (QoS), since it is possible, within limits, to design receivers (albeit at higher cost) that are less sensitive to prevailing interference and noise. Further, Interference Rights allow different secondary users to overlay with each other. In contrast, the traditional exclusive usage rights regime requires an "all or nothing" kind of sharing, so primary users can only share spectrum when they do not operate on it.

We will show that there are several advantages of Interference Rights. First, license holders have the authority to control the spectrum on their own instead of having to follow

This work was supported in part by the U.S. National Science Foundation under Grant 1149422

CROWNCOM 2012, June 18-20, Stockholm, Sweden
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DOI 10.4108/icst.crowncom.2012.248690

regulators' instructions on how to share the spectrum with other users. In other words, they will trade the spectrum if the gain is higher than not trading. Therefore, licensees have more motivations to trade spectrum. Second, buyers of Interference Rights have flexibility in deploying services. There is no limitation on technology, service types, transmitter location, and so on, as long as they remain within the parameters of the right. Third, Interference Rights has less regulatory involvement, and then reduces the transaction cost for trading parties. Fourth, Interference Rights may also lead to infrastructure development. If primary users are satisfied with the revenue collected from selling spectrum, they tends to upgrade their receivers and other equipments in order to free up more spectrum. Last but not least, it may also lead to a heterogeneous network architecture that combines microcell with picocell and femtocell, in order to maximum the spectrum utilization and social welfare. The spectrum efficiency is not the throughput alone, since there may be retransmission. The spectrum efficiency should be the ability that receivers could successfully interpret desired information.

B. The Formulation of Interference Rights

We propose that Interference Rights have five basic dimensions: Power density for both primary and secondary users, frequency band, area, and duration[1]. It can be written as

$$C_n \triangleq \{S_n,\ P_n,\ F_n, A_n, T_n\}$$

1) Secondary Users' Power Density (S_n): defines the maximum interference (and noise) power that can be tolerated by the primary user in electrospace. Clearly, this value changes with primary receivers' capabilities. In other words, when the primary users' receivers are less sensitive to interference and noise, this value would be higher (i.e., a primary user could sell more interference rights). The electrospace that can be used by secondary user also varies with the surrounding propagation and electromagnetic environment, both natural and manmade.

2) Primary User's Power Density (P_n) : It is also important for primary users to state the power density level that they will impose on the same electrospace as defined for secondary users' power density. This is required so that the secondary users know what they can expect for their communication system. Traditionally, regulators only demand that secondary users not create "harmful interference" for primary users. In fact, secondary users also have a Signal to Interference and Noise Ratio ($SINR$) requirement and it is a key point for them to decide whether to buy the interference right or not. This value considers the secondary users' needs and reduces the transaction cost for a trade.

3) Frequency Band (F_n): defines the frequency range on which the spectrum users can operate. Spectrum providers could free up some frequency bands for trading or allow spectrum users to transmit in the same band. Transmission in the same band may lead to more interference, but spectrum users could control the interference level by lowering their transmission power. Transmitting in different frequency band avoids the interference to some extent, however the actual performance level depends on both the transmitter filter and receivers' sensitivity.

4) Test area (A_n): determines the geographical area that secondary users could operate in. This could be regions in which primary users have few receivers.

5) Duration (T_n) : is the period of time in which the secondary users may operate. The duration for Interference Rights can ranges from seconds to hours. Therefore, we may need a fluid method for trading.

Primary users may also set other requirements in the contract to control their resource and QoS more tightly. For example, three possible extensions that could help primary users to define Interference Rights are Probability, Technology, and Penalties.

6) Probability: Primary users can set prices based on the ex ante estimated probability that a secondary user affects their existing services, since probabilistic rights parameters reflect the changing radio propagation environment [7]. For example, a secondary user that is estimated to affect 10% of existing transmission would pay less than users that would impact 20%. This probability could be determined by geographic boundary, period of time, and power density. Setting probability-based prices would be economically efficient but it relies on the correct estimate of ex ante interference

7) Technology: is another aspect that impacts the interference. Spectrum users with higher QoS requirements may consider specifying the type of technology as well as the channel type used by the secondary user. For example, it is known that upload channel has different characteristics than download channel; likewise, CDMA has different spectrum usage than OFDM.

8) Penalties: for violating the terms of Interference Rights may be determined in the negotiated contract. This is like an insurance policy on the investment that both primary and secondary users make and helps reduce transaction costs. As in financial markets, trustworthy spectrum users should benefit from better terms and lower prices in future trading. Other penalties may include "black-listing" certain primary or secondary users [6].

III. SPECTRUM TRADING

Summarizing Section II, Interference Rights allow a secondary user to emit a maximum amount of RF energy into the primary user's electrospace. The terms and parameters of this are set explicitly and are accepted by both the primary and secondary users. In this section, we outline how Interference Rights could work in practice.

A. Trading Procedures

1) Define Interference Rights: The first step is that the primary user needs to set the parameters for Interference Rights. They do so with full knowledge of their transmission parameters, receivers' sensitivity and distribution, QoS

[1] Primary users could add additional dimensions or attributes.

requirements, and with an awareness of how this affects their future service levels. If primary users do not have a high QoS expectation or the service level does not largely impact their revenue, they may choose to set a higher interference threshold, which may attract more buyers and result in a higher price.

2) Place Interference Rights into the Market: After the primary users define the parameters forInterference Rights, they may advertise and/or offer it in a spectrum market. Thus, Interference Rights share similar features with options in financial markets (see Table I). In financial options, asset owners can create (or "write") options and sell them in an options exchange. In this paper, we pursue the analogy of Interference Rights with financial options and assume a market that treats them in a similar manner. That is, we assume the existence of a liquid market.Spectrum providers who have unused spectrum could write Interference Rights that work more or less like a "covered call option[2]", and (secondary) spectrum users who are ready to operate buy it. In purchasing Interference Rights, secondary users have the right (but not the obligation) to put a maximum amount of RF power into a defined electrospace over a certain interval. Because it is a call option, the buyer of the option has the right to purchase the underlying asset at a set price (the strike price). The strike price is thus analogous to the RF power in the predefined electrospace in the Interference Rights.

TABLE I. ANALOGY OF FIANANCIAL OPTION AND INTERFERENCE RIGHT

Financial Option	Trading in Interference Right
Strike Price	Power density permitted in time/space/frequency
Expiration	Expiration
Premium	Price

3) Buying Interference Rights: The spectrum license that issued by FCC does not transfer or change when Interference Rights are initiated and bought. The buyer of Interference Rights only has the opportunity to operate in the electrospace as specified by the option (contract). Ideally, spectrum users should not need the regulators' permission before trading Interference Rights. In reality, they may need to register with regulator before entering the spectrum market. Therefore, the regulatory involvement is minimal in trading with Interference Right so the transaction cost is reduced.

B. Participants in the Trading and their behaviors

1) Selling party: The dominant spectrum selling party consists of license holders who underutilize the spectrum whose licenses they hold. However, the selling party is not restricted to license holders. Sellers could also be secondary users who cannot use entire Interference Rights that they purchased from primary users. Under this circumstance, secondary users may subdivide their own transmission from the original Interference Rights, and sell the remainders to others. This division could be made in terms of time, power density, and coverage. It futher maximizes the spectrum access opportunities and liquid the market, since every user who has spare spectrum can sell it for monetary conpensation. Note that this adds complexity to enforcement in the event of breach of contract or illicit activity. For that reason, it is reasonable to expect that regulators would have an interest in monitoring these markets.

2) Buying party: Users that want to operate on certain electrospace to provide all kinds of services but do not have spectrum are the dominant buying party for spectrum trading. Note that license holders could also buy Interference Righs to increase their service levels, experiment with new applications, and expand coverage.

C. Operations

After a spectrum trade takes place, both parties operate on the spectrum according to the contract. The contract only determines the affected electrospace and the duration as well as the price. In the most general case, the contract does not include transmission parameters such as the air interface, transmitter density, location, and power caps. Thus, secondary users are free to choose parameters on their own. They could transmit using a single station with higher power, or set a number of transmitters with lower power to get a larger geographic coverage. It is also possible for them to buy Interference Rights from different license holders and aggregate them to create a unique service area. Sensing is not required for operation, although it may be helpful to optimize spectrum utilization under the contract.

D. Enforcement

Enforcement remains a challenge for Dynamic Spectrum Access (DSA) systems of all kinds. Without enforcement, spectrum users cannot guarantee the value of the resource. Lack of effective enforcement means that users have limited recourse in the event of contract breaches or illicit activity, effectively raising the transaction costs. It is quite likely that this is one of the key reasons why DSA and spectrum trading are not commonplace today. Enforcement benefits every party in the wireless communication industry. Traders have more guarantees on contracts when the enforcement is implemented. Regulators can ensure no one violates the policy by monitor the spectrum usage. Licensees could also assure their license terms with enforcement system. It is an important issue for spectrum trading, one that justifies a paper on its own, so we will not address it further here.

IV. CASE STUDY

The fundamental difference between exclusive usage rights and Interference Rights is that the former are more deterministic while the latter are more stochastic. Primary users do not surrender the entire electrospace to secondary users under Interference Rights. Instead, they retain a portion of the electrospace for their own purposes, albeit with a reduced QoS.

[2] In a "call option", the buyer of the call has the right, but not the obligation to buy the underlying asset at the strike price at or before a certain time. The seller has the obligation to sell the underlying asset if the buyer exercies the option. When the options are "covered", the seller owns the underlying asset

In this way, customers for primary users still get some (if not all) of the services even though primary users released electrospace to others. We use TV white space as an example to illustrate our spectrum trading model.

TV white spaces occur because of the spatial and frequency separation between stations. This separation was used so that users at the fringes of each station's service area can get unambiguous signal reception; that is, it is possible for those users to receive the signal from exactly one station in any particular channel. The push to utilize this "fallow" spectrum has coincided with the transition from analog to digital TV and with increasing substitutes for video consumption, such as satellite, cable and Internet TV. Therefore, the profit model for TV broadcaster has changed, relying less on over-the-air transmission than in earlier eras. However, TV broadcasters still have a precious resource, spectrum, from which they could gain economic profits.

In an attempt to strike a balance between efficient spectrum use and consideration of broadcasters' business models, the FCC has created some rules for unlicensed operations in TV white spaces[3]. At a high level, these rules allow transmissions in these white spaces on a non-interfering basis. In this case study, we consider an expanded scenario, where broadcasters could choose to accept *some* interference in exchange for payment; i.e., they could write Interference Rights. Specifically, we will examine the revenues and costs for both primary and secondary users. Since residential customers of TV broadcasters are grouped into clusters, we assume that there are areas that do not consume over-the-air TV, instead consuming programming over one of the alternative media. For the purposes of this paper, we imagine Interference Rights being sold for a sector of the entire coverage area, or a ring on the edge of that area; other spatial models may exist as well.

A. Operation Flexibility

We begin by analyzing $SINR$, since it determines transmission parameters and is closely related to cost and revenue. The $SINR$ depends on three factors. The first one is the distance (distance between primary users' transmitter (Tx) and receiver (R) and distance between primary users' receiver and secondary users' transmitter (Ts)). The second one is primary and secondary users' transmission power. The third one is transmitter density. We simplify the example by only using only one secondary transmitter. Figures 1-3 illustrate this idea using a free space propagation model. We use three levels of transmission power for both users, and plot the SINR as a function of the distance between Ts and Tp.

In the scenario depicted in Figure 1, the primary user does not have many customers in certain section of their serrvice area, but they have significant user populations at the border of their service area. The scenario for Figure 3 is that the primary users' service population decreases as a function of distance. Figure 2 is an intermediate example.

[3] Federal Communications Commission. (2008). *In the Matter of Unlicensed Operation in TV Broadcast Bands.* 2nd Report and Order and Memorandum Opinion and Order, Federal Communications Commission, Washington DC. Retrieved from http://hraunfoss.fcc.gov/edocs_public/attachmatch/FCC-08-260A1.pdf.

From these three examples, we will show that Interference Rights are more flexible in spectrum sharing than exclusive rights from the following perspectives.

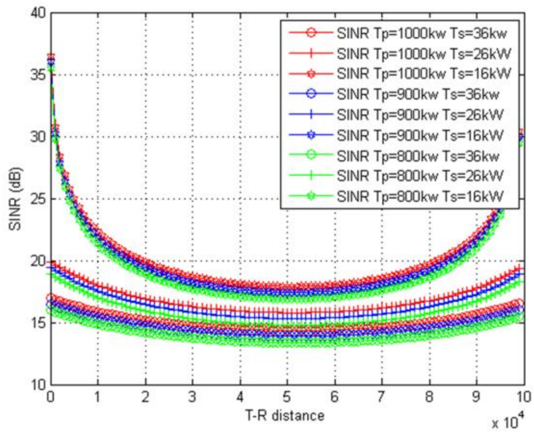

Figure 1. Ts at center (50km away from Tp)

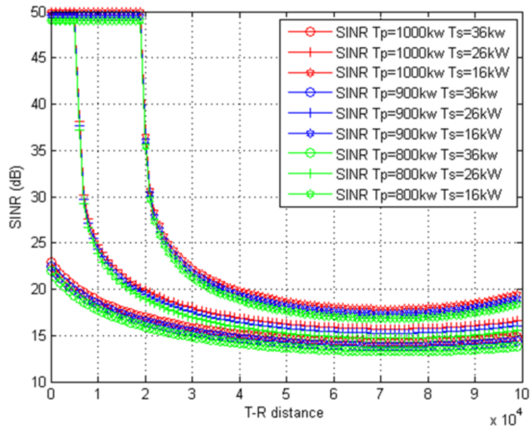

Figure 2. Ts is 79km away from Tp

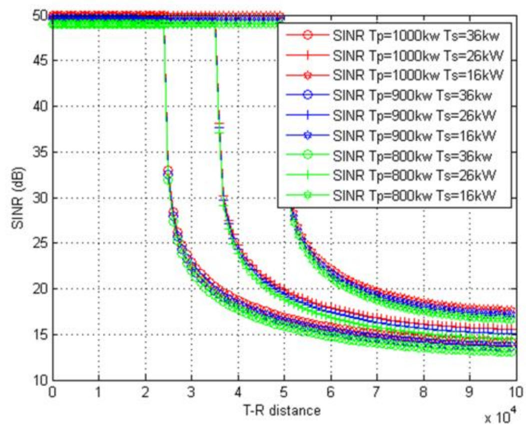

Figure 3. Ts at edge (100km away from Tp)

1) Primary users can adjust the spectrum sharing pattern with their customer distribution characteristics. For example, primary users who do not have many users in the area between 40-60km from the transmitter are modelled by the first case; and those whodo not have many users in area between 60-

80km from the transmitter fit the second case; finally, primary users with few users in a ring around the edge of their service area are modelled by the third case.

2) Using power density allows secondary users to set up their transmission parameters to comply with primary users' requirements. It is clear from figures that same $SINR$ can be achieved by different transmission power and locations. This flexibility will be increased when the transmitter density is higher.

B. Reservation Price

For broadcasters in the US (the primary users), revenues come from advertisements sold during programming. The magnitude of these revenues is determined by the number of viewers and their demographics. Direct costs, for broadcasters, are from the development and production of programming as well as for broadcasting operations. For the sector model (model (a) above), selling Interference Rights to a sector where few over-the-air customers exist could allow the broadcaster to earn a new revenue stream and reduce costs if it is feasible to cease broadcasting into that region. Broadcasters would have to weigh this revenue streams with lost revenues due to customers whose service quality diminishes. Therefore, the revenue and cost for DTV broadcasters (primary users) when the secondary user is not present can be expressed as

$$\mathrm{R_P} = \mathrm{N_v} \times \mathrm{a} \tag{1}$$

$$C_T = C_p + C_b + C_F \tag{2}$$

where, R_P is the total revenue for primary user, N_v is the number of viewers, a indicates the income per viewer. C_T is the total cost for primary user, C_p is the cost of programming which is a sunk cost, C_b is the cost of broadcasting and C_F are fixed costs, such as overhead. The revenue and Cost for primary users when secondary users are absent are the same for exclusive usage right and interference right.

When secondary users come into play, the revenues and costs for primary users under Interference Rights change as follows:

$$R_P^S = N_v' \times a + I_s \tag{3}$$

$$C_P^S = C_P + C_b' + C_F \tag{4}$$

where, R_P^S is primary user's total revenue when secondary users present, N_v' is the new number of viewers ($N_v \geq N_v'$ due to intereference from secondary use), I_s is the income for spectrum trading. C_P^S is primary users' total cost when share spectrum with secondary users, C_b' is the cost of broadcasting ($C_b \geq C_b'$).

In the case of exclusive usage rights, primary users must vacate the channel to enable secondary users to operate. Therefore, cost and revenue for primary users under exclusive rights are:

$$R_P^E = I_S^E \tag{5}$$

$$C_P^E = C_P + C_b' + C_F \tag{6}$$

As we stated before, the incentive for primary user to trade spectrum with secondary users is that they are better off from doing so, i.e., Whether this outcome obtains depends on $\mathrm{N_v}$ and $\mathrm{N_v'}$, which, in turn, depends on the signal strength and SINR at the receiver. For the purposes of this paper, we use simple, high level calculations to arrive at a first order estimate of the subscriber loss.

$$R_P^S - C_P^S \geq R_P - C_P \tag{7}$$

$$R_P^E - C_P^E \geq R_P - C_P \tag{8}$$

We calculate the channel capacity using the Shannon Hartley theorem $\mathrm{C} = \mathrm{B} \times \log_2(1 + \mathrm{SINR})$, where, C is channel capacity, B is the channel bandwidth (which is 6 MHz in this case). $\mathrm{C_b}$ (and also $\mathrm{C_b'}$) is determined by transmission power, duration and electricity cost.

$$\mathrm{C_b} = \mathrm{b} \times \mathrm{T_P} \times \mathrm{T_n} \tag{10}$$

where, b indicates the cost of transmission in kWh, T_P is the transmission power in kW, T_n is the contract during in h.

For digital television, the threshold of visibility occurs at $SNR = 15\ dB$ [8]. Below 15 dB, the picture is frozen. The picture is satisfactory but does not reach the entire coverage area when $15\ dB < SNR \leq 27\ dB$. The ATSC suggests $SNR = 27\ dB$ as the minimum to provide adequate viewing over a broadcaster's coverage area. For the purpose of our calculation, we assume the number of viewers N_v (and also N_v') distribute as

$$N_v = \begin{cases} 0, & SINR \leq 15\ dB \\ 0.3C, & 15\ dB < SINR \leq 27\ dB \\ 0.5C, & SINR \geq 27\ dB \end{cases} \tag{11}$$

From this, we can determine the minimum price a primary user would accept (i.e., their reservation price) from a secondary user. A primary user would have to earn at least as much from the transaction with the secondary user as they would have earned without secondary use, or:

$$I_s \geq (N_v - N_v') \times a - b \times T_n \times (T_P - T_P') \tag{12}$$

$$I_S^E \geq N_v \times a - b \times T_n \times (T_P - T_P'') \tag{13}$$

I_s is the reservation price for a primary user under Interference Rights regime, and I_S^E is the reservation price for a secondary user under exclusive rights regime. If we assume the reservation price is equal to the cost in each case, $I_s \leq I_S^E$, since $N_v' \geq 0$ && $T_n \geq 0$ && $(T_P - T_P') \geq 0$ && $(T_P - T_P'') \geq 0$. Therefore, the reservation price for interference right is always less or equal to that for exclusive usage right. In essence, under exclusive usage rights, if the primary users' original ≤ 15 , $R_S^E = 0$ since the primary user does not earn advertising revenue anyway. However, as long as the original $SINR > 15$, the reservation price must cover all potential advertisement cost less the savings from not broadcasting, since the primary user will clear up the entire channel for secondary users.

This has some practical implications in general. If the price paid by a secondary user were equal in the interference right and exclusive right regime, then the primary user would profit more from interference rights. Alternately, the lower reservation price means that the primary user would be potentially willing to take a lower price from a secondary user, which could serve to stimulate demand for secondary use.

Looking a bit deeper, we can consider three main cases depending on the primary users' original $SINR$ as described in Table 2. If $SINR \leq 15$, there is no advertising income for the primary user, therefore the reservation price is zero. When the primary users' original $SINR$ is in the range $(15,27]$, the reservation price for spectrum trading must the lost advertising (partial or total) revenue. Finally, when the original $SINR > 27$, the reservation price is equal to the partial advertising loss when the new $SINR \leq 27$. However, if the new $SINR \geq 27$ is, the reservation price is zero, since the primary users do not lose any customers.

TABLE II. PRIMARY USERS' RESERVATION PRICE FOR DIFFERENT SINR REGION (INTERFERENCE RIGHTS)

	$SINR_O \le 15$	$15 < SINR_O \le 27$	$SINR_O > 27$
$SINR_N \le 15$	None	Advertising Loss	Advertising Loss
$15 < SINR_N \le 27$	N/A	Advertising Loss	Advertising Loss
$SINR_N > 27$	N/A	N/A	None

Figure 4 shows the broadcaster's (primary user's) reservation price by SINR using the above parameters. In producing these figures, we assume that the viewers are uniformly distributed throughout the service area and secondary user's transmitter is located 70km away from the primary user's transmitter. The reservation price requirements vary with the SINR "region". Red lines represent a primary user's transmission of 1000kw and it does not change when secondary user start transmission; the secondary user's transmission power is 36kw. The original SINR is different for each red line. It is clear that reservation price is higher when the original SINR is large. The blue lines illustrate the case when the primary user's transmission power is reduced by 100kw, so that the primary user saves some money on broadcasting (largely depends on electricity consumption), which further decreases the reservation price. Finally, the green lines, illustrate the situation when the primary users' transmission power is 1000kw and the secondary user's transmission power is reduced to 16kw. It largely increases primary user's SINR. The reservation price for SINR > 27 is 0.

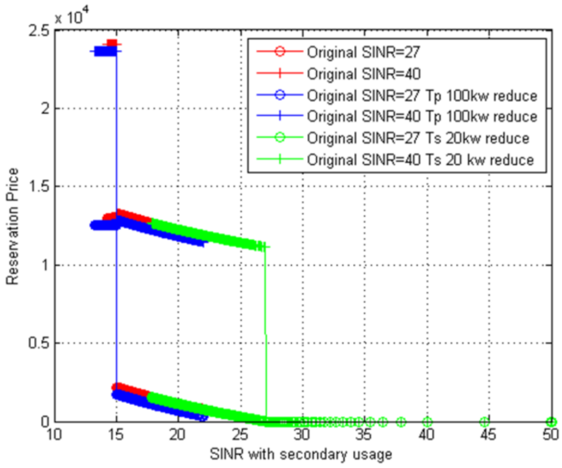

Figure 4. Reservation Price

The secondary users' profit comes from providing wireless services to its customers, which, we assume is also a function of channel capacity. Their cost includes the cost for spectrum and their operations cost. Therefore, the spectrum value (i.e., their maximum willingness-to-pay) for secondary user can be estimated by

$$V_S = e \times C \times r - C_b \tag{14}$$

where, V_S is the spectrum value for secondary user, r is the revenue that secondary users can get per customers, $e \in (0, 1)$ indicates the penetration of secondary users' services. e is positively correlated with secondary users' $SINR$.

When the secondary users' spectrum value is larger than primary users' reservation price (i.e., $V_S \ge I_S$), spectrum trades could occur. When this inequality does not hold, voluntary spectrum trades are unlikely. The difference between these two metrics reflects the bargaining dynamics for the resource and is outside the scope of this paper.

V. CONCLUSION

In this paper, we have presented the concept of Interference Right as instruments analogous to covered financial options that may (but need not) be written by license holders. This is different from the notion of Spectrum Usage Rights (SURs) [9] in that these are completely determined privately, without the intervention of the regulator. We illustrate this idea by two case studies. The first one focuses on spectrum trading procedures and features and the second one shows the economic efficiency that can be gained by this method. As seen from our first-order analysis, the trading in Interference Right soffers some advantages to exclusive usage rights from three perspectives. First, it allows users to overlay with each other that increase the spectrum access opportunities. Second, traders negotiate the Interference Right on their own, which reduces the transaction costs and gives more incentives for both parties. Third, the financial option-like trading method reflects the resource value more realistically. This proof-of-concept evaluation indicates that additional research is warranted to further develop the concept and analyze its implications, especially to quantify context acquisition costs [10]. More sophisticated modeling, such as that found in [11], should be performed to understand this kind of bargaining more fully.

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