Universal Linear Pricing for Multiple Access and Broadcast Channels under QoS Requirements

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ABSTRACT

In this paper, we address the problem of power allocation in multiple access (MAC) and broadcast channels (BC) with linear pricing framework to ensure that each user in the interference network can achieve its utility requirement. In the framework the system optimizer maximizes the system utility with the pricing function. The existence of the universal linear pricing mechanism is characterized. Algorithms for solving the linear pricing problems in MAC and BC are proposed. The sufficient condition for linear pricing in MAC with successive interference cancelation (SIC) and its best decoding order are analyzed. The relation between the order of channel states, the optimal weights and prices, and the resulting cost is characterized. Moreover, we show that it is possible for users to cheat in their channel states in order to achieve better rates. Finally, we argue that there cannot be an incentive compatible mechanism design for this universal linear pricing scheme.

Keywords

Linear pricing, utility requirement, power allocation, cheating

1. INTRODUCTION

In wireless systems, game theoretic analysis has often been used to investigate the problems of system utility optimization and efficient resource allocation. We study a user centric wireless communication system. A framework equipped with a regulator providing the individual pricing parameters for each user could be applied in order to fulfill the utility requirement of each user. It is recognized in [6] that pricing on service and application layer needs to be considered with physical layer techniques. Pricing includes certain cost function of transmit power which improves the energy efficiency and proposes a possible scheme to map the result of utility maximization to any desired point in the region.

There have been some previous work concerning universal pricing mechanism for interference coupled systems [4]. The impact of interference coupling on the convexity of certain utility functions and problems is characterized in [2]. The Pareto efficiency of a pricing policy in terms of the transmit power and the Nash equilibria achieved under pricing are characterized by using supermodularity in [15]. In order to obtain Pareto improvement of the non-cooperative power control game where users maximize their utility of quality of service (QoS), pricing of transmit powers is introduced in [16]. They show that pricing is especially helpful in a heavily loaded system. In [8], the network uses two pricing strategies: global pricing that maximizes the revenue, and minimax pricing that trades off the revenue for a more even resource allocation. [3] described the properties and implementations of non-manipulable and efficient social choice functions in interference wireless systems. The mechanism based on d'Aspremont and Gerard-Varet (AGV) mechanism for an incentive compatible spectrum sharing game is proposed in [18]. In chapter [14], the authors focus on the role of prices on decentralized resource allocation among competing users in communication networks. The impact on network performance played by prices among users and between users and service providers is explored. A utility-based power control framework is presented in [19] by using a softened SIR requirement and adding a penalty on power consumption. This framework shows some degree of cooperation and improves system convergence. In [13], the authors summarize and explain the main results on signal-to-interference based power control algorithms in wireless communications. A distributed power control scheme for wireless ad hoc networks is considered in [10] where the users voluntarily cooperate with each other by announcing a price in order to compensate the other users' interference instead of playing a noncooperative game. In the current paper, the general system utility is proposed as a social welfare function as the difference between a general utility and the linear pricing term. When the utility function is jointly concave in the transmit power, universal linear pricing mechanism is possible to achieve all the given desired point in the utility region for each user.

The main contribution and organization of the paper is as follows. In Section 2, a general utility function which is a weighted SINR-based utility function is proposed for gen-

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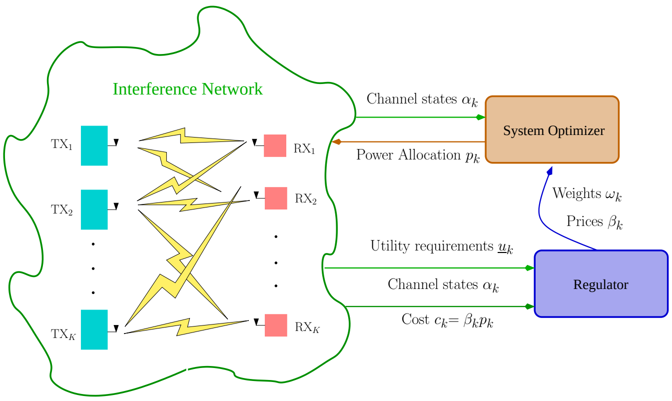

Figure 1: System Model

eral interference networks. A linear pricing model is presented in order to achieve all the desired utility requirement of each user in the wireless system. In Section 3, results in Lemma 1 show the necessary and sufficient condition of universal linear pricing. In Section 4, the algorithm of solving the universal linear pricing problem for the uplink MAC with SIC and the downlink BC with Dirty-paper precoding (DPC) is presented. Lemma 2 shows the condition of the utility for MAC with SIC to be jointly concave with regard to the weights. The best SIC decoding order concerning to minimize the sum transmit power is investigated in Lemma 3. According to these two lemmas, the mechanism to reorder the users is proposed. The cost terms are analyzed for different SIC decoding orders. In Section 5 a cheating problem is discussed. The users may have the willing to cheat for their channel states since the power allocated to the cheated channel is changed. The rates achieved by real channels after cheating are investigated. Finally, we argue that there exists no incentive compatible universal linear pricing scheme for MAC/BC with individual utility requirements.

2. PRELIMINARIES

Denote matrices and vectors by bold capital letters $\boldsymbol{A}$ and bold lowercase letters $\boldsymbol{x}$, respectively. x_k is the k^{th} component. $\boldsymbol{x} \geq 0$ implies that $x_k \geq 0$ for all the components k. The matrix inverse and transpose are denoted by $[\cdot]^{-1}$ and $[\cdot]^T$, respectively. $\text{diag}(A)$ is the vector with diagonal entries of $\boldsymbol{A}$. The set of non-negative reals is denoted as $\mathbb{R}_+$. Denote $\mathcal{F}$ as a set of functions.

In a wireless system, consider K transmitters with source messages are transmitting with power[1] $\boldsymbol{p} = [p_1, \cdots, p_K]^T$, and at least K sinks are interested in their messages.. We consider a general utility function

$$u(\boldsymbol{p}, \boldsymbol{\omega}) = \sum_{k=1}^{K} \omega_k g_k \left(\frac{p_k}{I_k(\boldsymbol{p})} \right), \tag{1}$$

where ω_k is the weight for user k, $\boldsymbol{\omega} = [\omega_1, \cdots, \omega_K]$ and ω_k is usually between zero and one and $\sum \omega_k = 1$. $I_k(\boldsymbol{p})$ is from the set of simple linear interference (plus noise) functions $\mathcal{I}(\boldsymbol{p})$, i.e.,

$$I_k(\boldsymbol{p}) = \boldsymbol{a}^T \cdot \boldsymbol{p} + \sigma_n^2, \tag{2}$$

where the vector $\boldsymbol{a}$ depends on the concrete system scenario and contains the effective channel coeffients, and σ_n^2 is additive noise power.

For example, in the MAC with K users, at the base station (BS), SIC is used. Define Π as a set of SIC decoding orders $\Pi := \{\pi^1, \cdots, \pi^N\}$, where in each decoding order $\boldsymbol{\pi}^i := [\pi_1^i, \cdots, \pi_K^i]$ and $N = K!$. For a certain decoding/precoding order $\boldsymbol{\pi}^i$, the interference function is $\boldsymbol{I}(\boldsymbol{p}) = [I_{\pi_1^i}(\boldsymbol{p}), \cdots, I_{\pi_K^i}(\boldsymbol{p})]$ with

$$I_{\pi_k^i}(\boldsymbol{p}) = \sum_{l=k+1}^{K} \alpha_{\pi_l^i} p_{\pi_l^i} + \sigma_n^2, \tag{3}$$

where α_k is the complex channel state of user k, $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_K]$.

The general interference function possesses the properties of conditional positivity, scale invariance and monotonicity with respect to the power allocation and strict monotonicity with respect to the noise component [5]. g_k is some SINR-based utility function. And we assume $g_k \in Conc$.

Definition 1. [4] *Conc* is the family of all strictly monotonic increasing, continuous functions g, such that $g(x)$ is concave.

In an interference wireless system, each user k is mainly interested in maximizing its own utility, but not the entire system utility. As shown in Fig. 1, the regulator chooses linear prices $\boldsymbol{\beta} = [\beta_1, \cdots, \beta_K]$ with the knowledge of channel states $\boldsymbol{\alpha}$ in order to achieve all the desired points of QoS requirement for each user k. The prices $\boldsymbol{\beta}$ serve for both the system revenue and the cost paid by the links. And the system maximizes the system utility $\tilde{u}(\boldsymbol{p}, \boldsymbol{\beta}, \boldsymbol{\omega})$ given below with the linear pricing mechanism.

$$\tilde{u}(\boldsymbol{p}, \boldsymbol{\beta}, \boldsymbol{\omega}) = u(\boldsymbol{p}, \boldsymbol{\omega}) - \sum_{k=1}^{K} \beta_k p_k. \tag{4}$$

We denote the solution to this system optimization problem as the optimal power allocation

$$p^*(\boldsymbol{\beta}, \boldsymbol{\omega}) = \arg\max_{\boldsymbol{p} \geq 0} \tilde{u}(\boldsymbol{p}, \boldsymbol{\beta}, \boldsymbol{\omega}). \tag{5}$$

The fee $c_k = \beta_k p_k$ of link k is paid by the links to the regulator either directly or via the system optimizer, see Figure 1.

2.1 Problem Statement

Assume each user k has a utility requirement $\underline{u}_k$ which needs to be fulfilled, $\underline{\boldsymbol{u}} = [\underline{u}_1, \cdots, \underline{u}_K]$. The pricing problem is to find the universal linear pricing parameter $\boldsymbol{\beta^*}$ for given $\boldsymbol{\omega}$ and $\underline{\boldsymbol{u}} \in \mathcal{F}(\boldsymbol{\alpha})$, s.t.,

$$g_k \left(\frac{p_k^*(\boldsymbol{\beta}^*, \boldsymbol{\omega})}{I_k(\boldsymbol{p}^*(\boldsymbol{\beta}^*, \boldsymbol{\omega}))} \right) = \underline{u}_k \tag{6}$$

for all $k \in K$.

[1] The sources as well as sinks could be colocated resulting in MAC or BC.

$\mathcal{F}(\boldsymbol{\alpha})$ is the feasibility region for channel states $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_K]$:

$$\mathcal{F}(\boldsymbol{\alpha}) = \bigcup_{\boldsymbol{p} \geq 0} \left(g_1(p_1/I_1(\boldsymbol{p})), \cdots, g_K(p_K/I_K(\boldsymbol{p}))\right). \quad (7)$$

Definition 2. A *universal pricing scheme* finds a pricing vector $\boldsymbol{\beta}$ for all channels $\boldsymbol{\alpha}$ and all weights $\boldsymbol{\omega}$ and their feasible utility requirements $\underline{\boldsymbol{u}} \in \mathcal{F}(\boldsymbol{\alpha})$.

3. UNIVERSAL PRICING

LEMMA 1. *Let $g_1, \cdots, g_K \in Conc$. And assume $I_1(\boldsymbol{p}), \cdots, I_K(\boldsymbol{p})$ are linear interference functions. If and only if $u(\boldsymbol{p}, \boldsymbol{\omega})$ is jointly concave in $\boldsymbol{p}$ for all $\boldsymbol{\alpha} \in \mathbb{R}_+^K$, and $\boldsymbol{\omega} > 0$, then there exists a universal linear pricing mechanism.*

PROOF. "$\Rightarrow$": Assume $u(\boldsymbol{p}, \boldsymbol{\omega})$ is jointly concave in $\boldsymbol{p}$, then the optimization problem

$$\max_{\boldsymbol{p} \geq 0} \tilde{u}(\boldsymbol{p}, \boldsymbol{\beta}, \boldsymbol{\omega}) = \max_{\boldsymbol{p} \geq 0} \left(u(\boldsymbol{p}, \boldsymbol{\omega}) - \sum_{k=1}^{K} \beta_k p_k \right) \quad (8)$$

has a unique solution characterized by the first order optimality condition:

$$\frac{\partial}{\partial p_l} u(\boldsymbol{p}^*, \boldsymbol{\omega}) - \beta_l = 0 \quad (9)$$

if $p_l^* > 0$.

Let us assume that $\underline{u}_k \in \mathcal{F}(\boldsymbol{\alpha})$ is achieved by a certain power allocation $\underline{\boldsymbol{p}}$, i.e.,

$$g_k(\underline{p}_k/I_k((\underline{\boldsymbol{p}}))) = \underline{u}_k \quad (10)$$

for all $k \in K$. For positive utility requirements, the required power p_k^* is always positive and thereby justifying (9).

Then choose a pricing parameter

$$\beta_l^*(\underline{\boldsymbol{p}}) = \frac{\partial}{\partial p_l} u(\boldsymbol{p}, \boldsymbol{\omega}) \Big|_{\boldsymbol{p} = \underline{\boldsymbol{p}}}$$

in order to achieve the necessary power allocation.

"$\Leftarrow$": It is proved in Theorem 1 in [4]. □

Remark 1. The result of Lemma 1 is related to Theorem 1 in [1]. The difference is that here the optimal power allocation is determined by a system utility (1) but not the (possibly unique) outcome of a noncooperative game.

Remark 2. Note that there might occur cases in which the individual utility function does belong to a natural competitive user utility (NCUU) function as defined in [2]. However, if the channel realizations or the interference is by chance chosen to provide a jointly concave system utility function, then all rates can be achieved by linear pricing. This behavior could change if another channel realization leads to a non-concave system utility function.

4. APPLICATIONS

Consider a number of K users wanting to transmit/receive information to/from a BS in the uplink MAC and downlink BC, respectively. All users as well as the BS are equipped with a single transmit or receive antenna. The received signal at the BS for MAC is

$$y = \sum_{k=1}^{K} h_k x_k + n. \quad (11)$$

And the received signal at each user in BC is

$$y_k = h_k \sum_{l=1}^{K} x_l + n. \quad (12)$$

where x_k is the transmit signal of user k, n is the additive white Gaussian noise with zero-mean and variance of σ_n^2. The channel gain of user k is denoted by $\alpha_k = \mid h_k \mid^2$.

For the remainder of this paper, we specialize the utility requirement $\underline{u}_k$ as the rate for each user k, i.e., $g_k\left(\frac{p_k^*(\boldsymbol{\beta}^*, \boldsymbol{\omega})}{I_k(\boldsymbol{p}^*(\boldsymbol{\beta}^*, \boldsymbol{\omega}))}\right)$ $= \log(1 + \text{SINR})$. It is possible to extend the utility requirement to other performance measures.

4.1 MAC with Successive Interference Cancelation

It is well known that with SIC at the BS, the capacity region of the single antenna Gaussian MAC can be achieved [7]. Assume that (1) The BS decides the best decoding order $\boldsymbol{\pi}^i = \{\pi_1^i, \cdots, \pi_K^i\}$ with perfect knowledge of the channel states $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_K]$. The best decoding order will be determined later; (2) The regulator gives the weight ω_k according to the best SIC decoding order and chooses the optimal universal linear pricing parameter β_k^* in order to achieve the utility requirement $\underline{u}_k$ for each user k; (3) The system optimizer maximizes the system utility $\tilde{u}(\boldsymbol{p}, \boldsymbol{\beta}, \boldsymbol{\omega})$ with the given pricing parameter β_k and weight ω_k; (4) Each user k pays the cost $\beta_k p_k$ to the regulator.

Let us assume a SIC decoding order as $\pi^1 = [K \to K-1 \to \cdots \to 1]$, then the rate function without pricing for each user k is

$$r_k(\boldsymbol{p}) = \log\left(1 + \frac{\alpha_k p_k}{1 + \sum_{l=1}^{k-1} \alpha_l p_l}\right) \geq \underline{u}_k. \quad (13)$$

Obviously the individual user rate depends on the SIC decoding order. From (8), the system maximizes the system utility for the MAC with SIC

$$\begin{aligned} \max_{\boldsymbol{p} \geq 0} \tilde{u}(\boldsymbol{p}, \boldsymbol{\beta}, \boldsymbol{\omega}) &= \max_{\boldsymbol{p}} \sum_{k=1}^{K} \omega_k \Bigg(\log\left(1 + \sum_{m=1}^{k} \alpha_m p_m\right) \\ &- \log\left(1 + \sum_{m=1}^{k-1} \alpha_m p_m\right) \Bigg) - \beta_k p_k. \quad (14) \end{aligned}$$

In general, the optimal power allocation is characterized by the first order optimality conditions

$$\begin{aligned}\frac{\partial}{\partial p_l} &= \omega_l \frac{\alpha_l}{1+\sum_{m=1}^{l} p_m \alpha_m} - \beta_l \\ &+ \sum_{k=l+1}^{K} \omega_k \left(\frac{\alpha_l}{1+\sum_{m=1}^{k} \alpha_m p_m} - \frac{\alpha_l}{1+\sum_{m=1}^{k-1} \alpha_m p_m} \right) \\ &= 0. \end{aligned} \tag{15}$$

Calculate the power allocation and substitute it into (13), then the linear pricing parameter β_k can be derived. For illustration, we now perform a case study.

4.1.1 Two-User Case

For simplicity and illustration, we investigate the special case with two users first and assume $\omega_1 \neq \omega_2$.

RESULT 1. *For decoding order* $[1 \to 2]$*, the optimal power allocation with respect to* $\boldsymbol{\omega}$ *and* $\boldsymbol{\beta}$ *is* $p_1^{*1\to2}(\beta_1^*) = \frac{\omega_1}{\beta_1} - \frac{\alpha_2(\omega_1-\omega_2)}{\alpha_2\beta_1-\alpha_1\beta_2}$ *for user 1 and* $p_2^{*1\to2}(\beta_2^*) = \frac{\alpha_1(\omega_1-\omega_2)}{\alpha_2\beta_1-\alpha_1\beta_2} - \frac{1}{\alpha_2}$ *for user 2. The pricing parameters are* $\beta_1^{*1\to2} = \frac{\alpha_1\omega_1}{2^{\underline{u}_1+\underline{u}_2}}$ *for user 1 and* $\beta_2^{*1\to2} = \frac{\alpha_2}{2^{\underline{u}_2}}\left(\frac{\omega_1}{2^{\underline{u}_1}} + \omega_2 - \omega_1\right)$ *for user 2.*

DERIVATION 1. *For user 1, the rate requirement for SIC decoding order of* $[1 \to 2]$ *is fulfilled by* $\underline{u}_1 = r_1(p_1^*(\beta_1)) = \log\left(1 + \frac{\alpha_1 p_1^*(\beta_1)}{1+\alpha_2 p_2^*(\beta_2)}\right)$*. For user 2, the rate requirement is fulfilled by* $\underline{u}_2 = r_2(p_2^*(\beta_2)) = \log(1+\alpha_2 p_2^*(\beta_2))$*.*

Therefore, the power needed to achieve the rate requirement is

$$p_1^*(\beta_1) = \frac{2^{\underline{u}_1}-1}{\alpha_1}(1+\alpha_2 p_2^*(\beta_2)) = \frac{2^{\underline{u}_2}(2^{\underline{u}_1}-1)}{\alpha_1}, \tag{16}$$

$$p_2^*(\beta_2) = \frac{2^{\underline{u}_2}-1}{\alpha_2}. \tag{17}$$

The optimal power allocation solves this maximization problem by $\frac{\partial}{\partial p_1} = \frac{\omega_1\alpha_1}{1+\alpha_1 p_1+\alpha_2 p_2} - \beta_1 = 0$ *and* $\frac{\partial}{\partial p_2} = \frac{\omega_1\alpha_2}{1+\alpha_1 p_1+\alpha_2 p_2} - \frac{(\omega_1-\omega_2)\alpha_2}{1+\alpha_2 p_2} - \beta_2 = 0$*. Now we obtain the power allocation* p_k^* *with respect to the weight* ω_k *as well as the pricing parameter* β_k*. Substitute* p_k^* *into (16) and (17), the pricing parameter* β_k^* *is observed. The case for the decoding order* $[2 \to 1]$ *is analogue.*

Fig. 2 shows the sum of the cost terms $c^{\pi^i} = \sum_l \beta_l p_l$ for both decoding orders for the 2-user MAC. It can be seen that for the weights $\omega_1 = \frac{1}{3}$, $\omega_2 = \frac{2}{3}$ and equal channels $\alpha_1 = \alpha_2$, the sum cost term for decoding order $[1 \to 2]$ is higher than $[2 \to 1]$. This will be analyzed later in Subsection 4.1.5.

4.1.2 K-User Case

Now we investigate the scenario where K users are transmitting signal to the BS. First, assume all weights ω_i are pairwise disjunct $\omega_1 \neq \cdots \neq \omega_K$.

THEOREM 1. *The universal linear pricing parameter* $\boldsymbol{\beta}$ *for* K *user MAC and the SIC decoding order* $\boldsymbol{\pi}^i = [\pi_i^i \to \pi_2^i \to \cdots \to \pi_K^i]$ *is given by*

$$\boldsymbol{\beta} = \boldsymbol{A}^{-1} \cdot 2^{\boldsymbol{s}}, \tag{18}$$

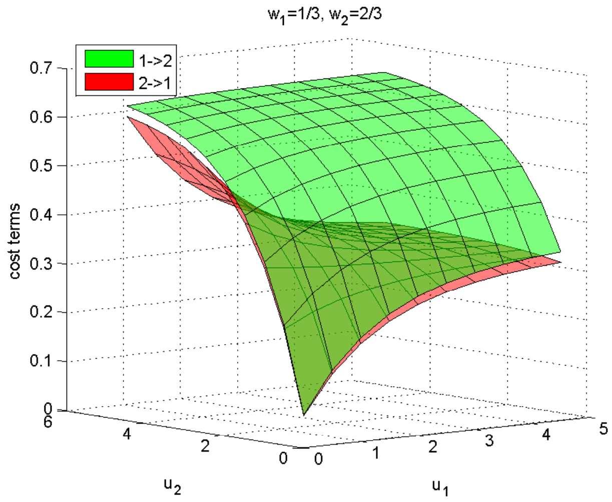

Figure 2: Cost terms for 2-user MAC with different SIC decoding order

where the matrix of different channels is denoted by $\boldsymbol{A}$

$$\boldsymbol{A} = \begin{bmatrix} \alpha_{\pi_{K-1}^i} & -\alpha_{\pi_K^i} & 0 & \cdots & 0 \\ 0 & \alpha_{\pi_{K-2}^i} & -\alpha_{\pi_{K-1}^i} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \alpha_{\pi_0^i} \end{bmatrix}.$$

And the vector $\boldsymbol{s}$ *is given by*

$$\boldsymbol{s} = \begin{bmatrix} \log\left((\omega_{\pi_K^i} - \omega_{\pi_{K-1}^i})\alpha_{\pi_K^i}\alpha_{\pi_{K-1}^i}\right) - \underline{u}_{\pi_K^i} \\ \log\left((\omega_{\pi_{K-1}^i} - \omega_{\pi_{K-2}^i})\alpha_{\pi_{K-1}^i}\alpha_{\pi_{K-2}^i}\right) - \underline{u}_{\pi_K^i} - \underline{u}_{\pi_{K-1}^i} \\ \vdots \\ \log(\omega_{\pi_1^i}\alpha_{\pi_1^i}) - \underline{u}_{\pi_K^i} - \cdots - \underline{u}_{\pi_1^i} \end{bmatrix}.$$

The power allocation for the K*-user MAC with the SIC [9] decoding order* $\boldsymbol{\pi}^i = [\pi_1^i \to \pi_2^i \to \cdots \to \pi_K^i]$ *is*

$$p_{\pi_k^i} = \frac{\tilde{\underline{u}}_{\pi_k^i} - 1}{\alpha_{\pi_k^i}} \cdot \prod_{j=k+1}^{K} \tilde{\underline{u}}_{\pi_j^i}, \tag{19}$$

where $\tilde{\underline{u}}_i = 2^{\underline{u}_i}$*,* $\alpha_{\pi_0^i} = 1$ *and* $\omega_{\pi_0^i} = 0$*.*

PROOF. It is sufficient to consider the case with SIC decoding order $[K \to \cdots \to 1]$. In order to obtain the universal pricing of $\boldsymbol{\beta}$, set $X_l = 1 + \sum_{k=1}^{l} \alpha_k p_k$. Note that $\omega_{K+1} = 0$ and $\alpha_{K+1} = 1$, (15) can be written as

$$\begin{aligned}\frac{\partial}{\partial p_l} &= \alpha_l \cdot \Big(\frac{\omega_l - \omega_{l+1}}{X_l} + \frac{\omega_{l+1} - \omega_{l+2}}{X_{l+1}} + \cdots \\ &+ \frac{\omega_{K-1} - \omega_K}{X_{K-1}} + \frac{\omega_K}{X_K}\Big) - \beta_l = 0. \end{aligned} \tag{20}$$

Since $\frac{\partial}{\partial p_K} = \frac{\omega_K \alpha_K}{X_K} - \beta_K = 0$, $X_K = \frac{\alpha_K \omega_K}{\beta_K}$. Insert X_K into

(20), we get $X_l = \frac{(\omega_l-\omega_{l+1})\alpha_l\alpha_{l+1}}{\alpha_{l+1}\beta_l-\alpha_l\beta_{l+1}}$. Therefore,

$$\log(X_l) = \overbrace{\underbrace{\log((\omega_l-\omega_{l+1})\alpha_l\alpha_{l+1})}_{\tilde{c_l}}}^{c_l} - \log\underbrace{(\alpha_{l+1}\beta_l-\alpha_l\beta_{l+1})}_{D_l}. \quad (21)$$

Since ω_l and α_l are given numbers, the first item on the right handside in (21) is constant number c_l. Denote $\boldsymbol{D} = [D_1, \cdots, D_K]$, $\boldsymbol{D} = \boldsymbol{A}\cdot\boldsymbol{\beta}$.

From (13), the rate of each user l is

$$r_l = \begin{cases} \log(\frac{X_l}{X_{l-1}}) & : \text{otherwise} \\ \log(X_l) & : l = 1 \end{cases} \quad (22)$$

Then we obtain $2^{r_1} = X_1$ and $2^{r_l} = \frac{X_l}{X_{l-1}}$ for $1 < l \leq K$, thus $X_l = \prod_{k=1}^{l} 2^{r_k} = 2^{\sum_{k=1}^{l} r_k}$. Therefore, $\log(X_l) = \sum_{k=1}^{l} r_k = \tilde{X}_l$. From (21), $\tilde{\boldsymbol{X}} = \boldsymbol{c} - \log(\boldsymbol{A}\cdot\boldsymbol{\beta})$, i.e. $2^{\boldsymbol{s}} = \boldsymbol{A}\cdot\boldsymbol{\beta}$, where $\boldsymbol{s} = \boldsymbol{c} - \tilde{\boldsymbol{X}}$. $\boldsymbol{A}^{-1}$ always exists because $\omega_k > 0, \alpha_k > 0$ for all $k, 1 \leq k \leq K$.

This proves the universal linear pricing parameter $\boldsymbol{\beta}$ in (18).

From (10) and (13), the rate requirement $\underline{u}_k$ for each user with SIC decoding order $[\pi_1^i \to \pi_2^i \to \cdots \to \pi_K^i]$ is achieved by certain power allocation $\boldsymbol{p}$ where

$$\underline{u}_k = \log\left(1 + \frac{\alpha_{\pi_k^i} p_{\pi_k^i}}{1 + \sum_{l=k+1}^{K} \alpha_{\pi_l^i} p_{\pi_l^i}}\right). \quad (23)$$

Compute the power allocation $\boldsymbol{p}$ in the SIC decoding order $\boldsymbol{\pi}^i$ as a function of utility requirement $\underline{\boldsymbol{u}}$ and the channel states $\boldsymbol{\alpha}$,

$$\begin{aligned} p_{\pi_k^i} &= \frac{2^{\underline{u}_{\pi_k^i}} - 1}{\alpha_{\pi_k^i}} \cdot (1 + \sum_{l=k+1}^{K} \alpha_{\pi_l^i} p_{\pi_l^i}) \\ &= \frac{2^{\underline{u}_{\pi_k^i}} - 1}{\alpha_{\pi_k^i}} \cdot 2^{\sum_{j=k+1}^{K} \underline{u}_{\pi_j^i}}. \end{aligned} \quad (24)$$

This proves the second statement in Theorem 1. ☐

Second, consider the case where all weights ω_i are equal.

RESULT 2. *The optimal prices $\boldsymbol{\beta}$ and power allocation $\boldsymbol{p}$ for the K user case MAC with equal weights and the SIC decoding order $[K \to \cdots, \to 1]$ are*

$$\begin{aligned} \beta_l &= \frac{\alpha_l}{2^{\sum_{k=1}^{K} \underline{u}_k}}, \quad (25) \\ p_l &= \frac{2^{\sum_{k=1}^{l} \underline{u}_k} - 2^{\sum_{k=1}^{l-1} \underline{u}_k}}{\alpha_l}. \end{aligned}$$

DERIVATION 2. *When considering the equal weight, the pricing problem is easier to characterize, since the optimization problem of (14) becomes*

$$\begin{aligned} \max_{\boldsymbol{p}\geq 0, \boldsymbol{\pi}} \tilde{u}(\boldsymbol{p}, \boldsymbol{\beta}, \boldsymbol{\omega}) &= \max_{\boldsymbol{p}} \sum_{k=1}^{K} \log(1 + \sum_{m=1}^{k} \alpha_m p_m) \\ &- \log(1 + \sum_{m=1}^{k-1} \alpha_m p_m) - \beta_k p_k \\ &= \max_{\boldsymbol{p}\geq 0, \boldsymbol{\pi}} \log X_K - \sum_{k=1}^{K} \beta_k p_k. \end{aligned} \quad (26)$$

The solution of the optimization problem is

$$\begin{aligned} \frac{\partial}{\partial p_l} &= \frac{\alpha_l}{1 + \sum_{k=1}^{K} \alpha_k p_k} - \beta_l \\ &= \frac{\alpha_l}{X_K} - \beta_l = 0 \end{aligned} \quad (27)$$

if $p_l > 0$.

The optimization holds for $p_l = \frac{2^{\sum_{k=1}^{l} \underline{u}_k} - 2^{\sum_{k=1}^{l-1} \underline{u}_k}}{\alpha_l}$ and $\beta_l = \frac{\alpha_l}{X_k}$. This result is similar to equation (6) in [9]. In a similar way, the cases in which some weights are equal can be solved.

4.1.3 Condition for Jointly Concave Utility for MAC with SIC

LEMMA 2. *For certain decoding order $\boldsymbol{\pi}^i$, if $\boldsymbol{\pi}^i = [\pi_1^i \to \pi_2^i \to \cdots \to \pi_{K-1}^i \to \pi_K^i]$, then a sufficient condition for a jointly concave utility function $u(\boldsymbol{p}, \boldsymbol{\omega})$ irrespective of the channel realizations $\boldsymbol{\alpha}$ is*

$$\omega_{\boldsymbol{\pi}_K^i} \geq \omega_{\boldsymbol{\pi}_{K-1}^i} \geq \cdots \geq \omega_{\boldsymbol{\pi}_2^i} \geq \omega_{\boldsymbol{\pi}_1^i}. \quad (28)$$

PROOF. Recall the utility function $u(\boldsymbol{p}, \boldsymbol{\omega})$ for the K users in MAC with SIC. First consider the SIC decoding order $\boldsymbol{\pi}^1 = [K \to K-1 \to \cdots \to 2 \to 1]$,

$$\begin{aligned} u(\boldsymbol{p}, \boldsymbol{\omega}) &= \sum_{k=1}^{K} \omega_k \log(1 + \frac{\alpha_k p_k}{1 + \sum_{m=1}^{k-1} \alpha_m p_m}) \\ &= \omega_K \log(1 + \sum_{m=1}^{K} \alpha_m p_m) + \cdots \\ &+ (\omega_{k-1} - \omega_k) \log(1 + \sum_{m=1}^{k-1} \alpha_m p_m) + \cdots \\ &+ (\omega_1 - \omega_2) \log(1 + \alpha_1 p_1). \end{aligned} \quad (29)$$

(29) is the sum of weighted concave functions. Since all weights are non-negative the overall function is concave, too. For the SIC decoding order $\boldsymbol{\pi}^1$, if the weights are ordered as $\omega_1 \geq \omega_2 \geq \cdots \geq \omega_{K-1} \geq \omega_K$, is the utility $u(\boldsymbol{p}, \boldsymbol{\omega})$ jointly concave.

It is analogue for any given decoding order $\boldsymbol{\pi}^i = [\boldsymbol{\pi}_1^i \to \boldsymbol{\pi}_2^i \to \cdots \to \boldsymbol{\pi}_{K-1}^i \to \boldsymbol{\pi}_{\boldsymbol{K}}^i]$. ☐

4.1.4 Choosing Best Decoding Order

The idea for the best SIC decoding order is not to compare the system utilities for different decoding orders but to minimize the sum transmit power with different decoding orders.

LEMMA 3. *The best SIC decoding order depends on the channel state $\boldsymbol{\alpha}$. In order to maximize the system utility function $\tilde{u}(\boldsymbol{p},\boldsymbol{\beta},\boldsymbol{\omega})$ fulfilling the rate requirement $\underline{\boldsymbol{u}}_{\boldsymbol{k}}$ with minimum sum power, the decoding order $\boldsymbol{\pi}^i = [\pi_1^i \to \pi_2^i \to \cdots \to \pi_{K-1}^i \to \pi_K^i]$ is induced by*

$$\alpha_{\pi_1^i} \geq \alpha_{\pi_2^i} \geq \cdots \geq \alpha_{\pi_{K-1}^i} \geq \alpha_{\pi_K^i}. \tag{30}$$

If the order of weights for some users does not fit the order of channels, e.g., if the order of channel states is $\alpha_K \geq \cdots \geq \alpha_{k+1} \geq \alpha_k \geq \cdots \geq \alpha_1$, but the weight $\omega_k < \omega_{k+1}$ for user k and $k+1$, then it is sufficient to use the unweighted sum utility maximization as (26) at the system maximizer.

PROOF. Since the rate requirements $\underline{\boldsymbol{u}}$ are fixed for different decoding orders, the basic idea to prove the first statement in Lemma 3 is to find the best decoding order which consumes the lowest sum transmit power. It is sufficient to consider the power allocation $\boldsymbol{p}^{k+1\to k}$ and $\boldsymbol{p}^{k\to k+1}$ for two users $k+1$ and k with the decoding order $k+1 \to k$ and $k \to k+1$, respectively [12]. Assume a decoding order $\pi^1 = [K \to \cdots \to 1]$ and $\alpha_K \geq \cdots \geq \alpha_{k+1} \geq \alpha_k \geq \cdots \geq \alpha_1$. From the power allocation of (19),

$$\begin{aligned} p_{k+1}^{k+1\to k} &= \frac{2^{\underline{u}_{k+1}}-1}{\alpha_{k+1}} \cdot 2^{\sum_{i=1}^{k} \underline{u}_i}, \\ p_k^{k+1\to k} &= \frac{2^{\underline{u}_k}-1}{\alpha_k} \cdot 2^{\sum_{i=1}^{k-1} \underline{u}_i}. \end{aligned} \tag{31}$$

$$\begin{aligned} p_{k+1}^{k\to k+1} &= \frac{2^{\underline{u}_{k+1}}-1}{\alpha_{k+1}} \cdot 2^{\sum_{i=1}^{k-1} \underline{u}_i}, \\ p_k^{k\to k+1} &= \frac{2^{\underline{u}_k}-1}{\alpha_k} \cdot 2^{\sum_{i=1}^{k-1} \underline{u}_i} \cdot 2^{\underline{u}_{k+1}}. \end{aligned} \tag{32}$$

Now compare the sum power $\sum_i p_i^{k+1\to k}$ and $\sum_i p_i^{k\to k+1}$. Define $N = \sum_i p_i^{k+1\to k} - \sum_i p_i^{k\to k+1}$.

$$N = 2^{\sum_{i=1}^{k-1} \underline{u}_i}(2^{\underline{u}_{k+1}}-1)(2^{\underline{u}_k}-1)\left(\frac{1}{\alpha_{k+1}} - \frac{1}{\alpha_k}\right). \tag{33}$$

Since the rate requirement $\underline{\boldsymbol{u}} > 0$, $2^{\underline{u}_k} > 1$, $2^{\underline{u}_{k+1}} > 1$ and $2^{\sum_{i=1}^{k-1} \underline{u}_i} > 0$. With the assumption $\alpha_{k+1} > \alpha_k$, $N < 0$. Therefore, decoding order $k+1 \to k$ consumes lower transmit power than decoding order $k \to k+1$. For any arbitrary decoding orders $\boldsymbol{\pi} \neq [K \to \cdots \to 1]$ with the channel states $\alpha_K \geq \cdots \geq \alpha_{k+1} \geq \alpha_k \geq \cdots \geq \alpha_1$, reordering the successive two neighbor indices lowers the sum transmit power. It is analogue for any other orders of channel states $\boldsymbol{\alpha}$ and π^i.

This proves the first statement in Lemma 3.

Deduced by (30), the SIC decoding order for the order of channel states $\alpha_K \geq \cdots \geq \alpha_{k+1} \geq \alpha_k \geq \cdots \geq \alpha_1$ is $[K \to \cdots \to 1]$. If $\omega_k < \omega_{k+1}$, then $\omega_k - \omega_{k+1} \leq 0$, using $\omega_k = \omega_{k+1}$ maximizes the utility function $\boldsymbol{u}(\boldsymbol{p},\boldsymbol{\omega})$. This proves the second statement in Lemma 3. ☐

4.1.5 Cost Analysis

Figure 2 illustrates the cost terms $c^{\pi^i} = \sum_l \beta_l p_l$ for different SIC decoding orders. Now we will analyze the relationship between cost terms and the SIC decoding order. From (18), for a certain decoding order $\pi^i = [\pi_1^i \to \pi_2^i \to \cdots \to \pi_K^i]$, the pricing parameters for the K-user MAC can be written as

$$\boldsymbol{\beta} = \begin{bmatrix} \alpha_{\pi_1^i}\left(\frac{\omega_{\pi_1^i}}{\prod_{j=1}^K \underline{\tilde{u}}_{\pi_j^i}}\right) \\ \alpha_{\pi_2^i}\left(\frac{\omega_{\pi_2^i}-\omega_{\pi_1^i}}{\prod_{j=2}^K \underline{\tilde{u}}_{\pi_j^i}} + \frac{\omega_{\pi_1^i}}{\prod_{j=1}^K \underline{\tilde{u}}_{\pi_j^i}}\right) \\ \vdots \\ \alpha_{\pi_K^i}\left(\frac{\omega_{\pi_K^i}-\omega_{\pi_{K-1}^i}}{\underline{\tilde{u}}_{\pi_K^i}} + \frac{\omega_{\pi_{K-1}^i}-\omega_{\pi_{K-2}^i}}{\prod_{j=K-1}^K \underline{\tilde{u}}_{\pi_j^i}} + \cdots + \frac{\omega_{\pi_1^i}}{\prod_{j=1}^K \underline{\tilde{u}}_{\pi_j^i}}\right) \end{bmatrix}.$$

It can be seen that by multiplying the power allocation in (19), the cost terms c^{π^i} are independent of the channel states $\boldsymbol{\alpha}$.

The cost term c^{π^i} is

$$\begin{aligned} c^{\pi^i} &= (\omega_{\pi_K^i} - \omega_{\pi_{K-1}^i})\frac{\underline{\tilde{u}}_{\pi_K^i}-1}{\underline{\tilde{u}}_{\pi_K^i}} + \\ &+ (\omega_{\pi_{K-1}^i} - \omega_{\pi_{K-2}^i})\left(\frac{\underline{\tilde{u}}_{\pi_{K-1}^i}-1}{\underline{\tilde{u}}_{\pi_{K-1}^i}} + \frac{\underline{\tilde{u}}_{\pi_K^i}-1}{\prod_{K-1}^K \underline{\tilde{u}}_{\pi_j^i}}\right) + \cdots \\ &+ \omega_{\pi_1^i}\left(\frac{\underline{\tilde{u}}_{\pi_1^i}-1}{\underline{\tilde{u}}_{\pi_1^i}} + \frac{\underline{\tilde{u}}_{\pi_2^i}-1}{\prod_{j=1}^2 \underline{\tilde{u}}_{\pi_j^i}} + \cdots + \frac{\underline{\tilde{u}}_{\pi_K^i}-1}{\prod_{j=1}^K \underline{\tilde{u}}_{\pi_j^i}}\right). \end{aligned} \tag{34}$$

LEMMA 4. *The cost terms c^{π^i} are only dependent on the weights $\boldsymbol{\omega}$ and the utility requirements $\underline{\boldsymbol{u}}$ of each user for different decoding orders. If the order of weights is $\omega_1 \geq \cdots \geq \omega_K$, then the regulator can charge highest from the SIC decoding order $[K \to \cdots \to 1]$.*

PROOF. It is sufficient to compare the cost terms $c^{k+1\to k}$ and $c^{k\to k+1}$ of two successive users k and $k+1$ with the decoding order $k+1 \to k$ and $k \to k+1$, respectively. Assume the weights for each user are ordered by $\omega_1 \geq \cdots \geq \omega_K$, which induce the SIC decoding order as $[K \to \cdots \to 1]$. By changing the decoding order of two successive users $k+1$ and k, the corresponding pricing parameters are

$$\begin{aligned} \beta_{k+1}^{k+1\to k} &= \alpha_{k+1} \cdot \left(\frac{\omega_{k+1}-\omega_{k+2}}{\prod_{i=i}^{k+1} \underline{\tilde{u}}_i} + \cdots + \frac{\omega_K}{\prod_{i=1}^K \underline{\tilde{u}}_i}\right), \\ \beta_k^{k+1\to k} &= \alpha_k \cdot \left(\frac{\omega_k-\omega_{k+1}}{\prod_{i=i}^{k} \underline{\tilde{u}}_i} + \cdots + \frac{\omega_K}{\prod_{i=1}^K \underline{\tilde{u}}_i}\right). \end{aligned} \tag{35}$$

$$\begin{aligned} \beta_{k+1}^{k\to k+1} &= \alpha_{k+1} \cdot \left(\frac{\omega_{k+1}-\omega_k}{\prod_{i=i}^{k-1} \underline{\tilde{u}}_i \cdot \underline{\tilde{u}}_{k+1}} + \frac{\omega_k-\omega_{k+2}}{\prod_{i=i}^{k+1} \underline{\tilde{u}}_i} \cdots \right. \\ &\left. + \frac{\omega_K}{\prod_{i=1}^K \underline{\tilde{u}}_i}\right), \\ \beta_k^{k\to k+1} &= \alpha_k \cdot \left(\frac{\omega_k-\omega_{k+2}}{\prod_{i=i}^{k+1} \underline{\tilde{u}}_i} + \cdots + \frac{\omega_K}{\prod_{i=1}^K \underline{\tilde{u}}_i}\right). \end{aligned} \tag{36}$$

Note that $\omega_{K+1} = 0$, $\underline{\tilde{u}}_0 = 1$, and $\underline{\tilde{u}}_i = 2^{\underline{u}_i}$. Now we compare the cost terms $c^{k+1\to k}$ and $c^{k\to k+1}$. Define $M = c^{k+1\to k} - c^{k\to k+1}$, where $c^{k+1\to k} = \beta_{k+1}^{k+1\to k} p_{k+1}^{k+1\to k} + \beta_k^{k+1\to k} p_k^{k+1\to k}$ and $c^{k\to k+1} = \beta_{k+1}^{k\to k+1} p_{k+1}^{k\to k+1} + \beta_k^{k\to k+1} p_k^{k\to k+1}$. From (31) and (32), the difference between the cost terms of the two decoding orders for user $k+1$ and k is

$$M = \frac{(\underline{\tilde{u}}_k - 1)(\underline{\tilde{u}}_{k+1}-1)}{\underline{\tilde{u}}_k \cdot \underline{\tilde{u}}_{k+1}}(\omega_k - \omega_{k+1}). \tag{37}$$

Since $\underline{\boldsymbol{u}} \geq 0$, $\tilde{\underline{\boldsymbol{u}}} \geq 1$. With $\omega_k \geq \omega_{k+1}$, $M \geq 0$. Therefore. the cost term for decoding order $k+1 \rightarrow k$ is higher than decoding order $k \rightarrow k+1$. For any arbitrary decoding order $\boldsymbol{\pi} \neq [K \rightarrow \cdots \rightarrow 1]$ with the weights $\omega_1 \geq \cdots \geq \omega_K$, reordering the successive two neighbor indices increases the cost term. It is analogue for any other orders of weights $\boldsymbol{\alpha}$ and decoding orders π^i. □

It is of interest for the regulator to design the individual weights $\boldsymbol{\omega}$ in order to achieve the unique power allocation with concave utility function, which indeed coincides with the highest charge from the users. Fig. 2 illustrates this for the 2-user MAC.

4.1.6 Reordering Mechanism

Lemma 3 shows that the best SIC decoding order π^i is determined by the order of channel states $\boldsymbol{\alpha}$. Lemma 2 provides the order of individual wights $\boldsymbol{\omega}$ induced by a given SIC decoding order as a sufficient condition for the utility function $u(\boldsymbol{p}, \boldsymbol{\omega})$ to be jointly concave. Therefore, in order to ensure that the system works with a unique solution and minimum sum power, the regulator could set the individual weight ω_k according to the order of channel states $\boldsymbol{\alpha}$ and reorder the k^{th} user accordingly, i.e., the SIC decoding order and weights are alternated by the change of channel states. The resulting order also provides highest charge to the regulator.

Assume that the channel states are ordered as $\alpha_{\pi_1^i} \geq \alpha_{\pi_2^i} \geq \cdots > \alpha_{\pi_{K-1}^i} \geq \alpha_{\pi_K^i}$ which induce the SIC decoding order as $\boldsymbol{\pi}^i = [\pi_1^i \rightarrow \pi_2^i \rightarrow \cdots \rightarrow \pi_{K-1}^i \rightarrow \pi_K^i]$. Set the weights in order $\omega_{\pi_K^i} \geq \omega_{\pi_{K-1}^i} \geq \cdots > \omega_{\pi_2^i} \geq \omega_{\pi_1^i}$ to ensure a jointly concave utility function $u(\boldsymbol{p}, \boldsymbol{\omega})$. Reorder the user with channel state $\alpha_{\pi_1^i}$ as the K^{th} user, the user with channel state $\alpha_{\pi_2^i}$ as the $K-1^{th}$ user, and so on. Then the SIC decoding order is shifted to $\boldsymbol{\pi}^1 = [K \rightarrow \cdots \rightarrow 1]$. It is analogue for different decoding orders. Therefore, any fixed SIC decoding order could be obtained by simply reordering the users with the order of their channel states.

4.2 Broadcast Channel with DPC

Known as the duality between MAC and BC, with the same total transmit power, MAC and BC can achieve the same rate [11]. This duality holds provided that the decoding order of SIC in the uplink MAC is the reverse of the DPC order in the downlink BC [17]. Using this interesting duality, we analyze the universal linear pricing problem in BC.

The general utility function for BC is

$$u(\boldsymbol{q}, \boldsymbol{\omega}) = \sum_{k \in K} \omega_k g_k \left(\frac{q_k}{I_k(\boldsymbol{q})} \right), \tag{38}$$

where $\boldsymbol{q}$ is the transmit power allocation in BC. Note that the interference function $\boldsymbol{I}(\boldsymbol{q})$ here for BC is different from in MAC. For a certain DPC precoding order $\tilde{\boldsymbol{\pi}}^i = [\tilde{\pi}_1^i \rightarrow \cdots \rightarrow \tilde{\pi}_K^i]$, the interference function for BC is

$$I_{\tilde{\pi}_k^i} = \alpha_{\tilde{\pi}_k^i} \sum_{j=k+1}^{K} q_{\tilde{\pi}_j^i} + \sigma_n^2. \tag{39}$$

The regulator chooses linear pricing parameters $\boldsymbol{\beta'} = [\beta_1', \cdots, \beta_K']$ and the system utility is

$$\tilde{u}(\boldsymbol{q}, \boldsymbol{\beta'}, \boldsymbol{\omega}) = u(\boldsymbol{q}, \boldsymbol{\omega}) - \sum_{k=1}^{K} \beta_k' q_k. \tag{40}$$

4.2.1 Two-User Case in BC

Similar to the analysis in MAC, we consider the special case of two users in the BC first.

RESULT 3. *For BC with DPC precoding order as* $[1 \rightarrow 2]$ *according to the SIC decoding order in MAC as* $\pi^1 = [2 \rightarrow 1]$, *the optimized power allocation with respect to the utility requirement* $\underline{u}_k$ *are* $q_1^* = (2^{\underline{u}_1} - 1)(\frac{1}{\alpha_1} + \frac{2^{\underline{u}_2}-1}{\alpha_2})$ *for user 1 and* $q_2^* = \frac{2^{\underline{u}_2}-1}{\alpha_2}$ *for user 2. And the pricing parameters are* $\beta_1^{*'} = \frac{\omega_1\alpha_1\alpha_2}{2^{\underline{u}_1}(\alpha_2-\alpha_1+\alpha_1 2^{\underline{u}_2})}$ *for user 1 and* $\beta_2^{*'} = \frac{\omega_1\alpha_1\alpha_2(1+2^{\underline{u}_1})}{2^{\underline{u}_1}(\alpha_2-\alpha_1)+\alpha_1 2^{\underline{u}_1+\underline{u}_2}} - \frac{\omega_2\alpha_2}{2^{\underline{u}_2}}$ *for user 2. For the DPC precoding order as* $[2 \rightarrow 1]$, *the calculation of optimized power allocation and pricing parameters are similar.*

DERIVATION 3. *According to the MAC and BC duality, the sum transmit power in BC is*

$$\begin{aligned} \sum_i q_i &= \sum_i p_i, \\ q_1 + q_2 &= \frac{2^{\underline{u}_1} - 1}{\alpha_1} + \frac{2^{\underline{u}_1}(2^{\underline{u}_2} - 1)}{\alpha_2} \\ &= \frac{2^{\underline{u}_1}(\alpha_2 - \alpha_1) + \alpha_1 2^{\underline{u}_1 + \underline{u}_2} - \alpha_2}{\alpha_1\alpha_2}. \end{aligned} \tag{41}$$

Solve the optimization problem in (40), $\frac{\partial}{\partial q_1} = \frac{\omega_1\alpha_1}{1+\alpha_1 q_1+\alpha_1 q_2} - \beta_1 = 0$ *and* $\frac{\partial}{\partial q_2} = \frac{\omega_1\alpha_1}{1+\alpha_1 q_1+\alpha_1 q_2} - \frac{\omega_1\alpha_1}{1+\alpha_1 q_2} + \frac{\omega_2\alpha_2}{1+\alpha_2 q_2} - \beta_2 = 0$. *Hence, knowing the sum power in (41), the optimal pricing parameters* $\beta_1^{*'}$ *and* $\beta_2^{*'}$ *are solved.*

4.2.2 K-User Case

Now, we investigate the universal linear pricing problem in BC for general cases. Due to the duality between MAC and BC, the rate requirement for each user k in BC is the same as in MAC as $\underline{\boldsymbol{u}}_k$.

LEMMA 5. *Assume the DPC precoding order* $[K \rightarrow K-1 \rightarrow \cdots \rightarrow 2 \rightarrow 1]$, *the pricing parameters* $\boldsymbol{\beta^{*'}}$ *given by regulator for BC are*

$$\beta_l^{*'} = \frac{\omega_l \alpha_l}{Z_l} + \sum_{m=l+1}^{K} \alpha_m \omega_m \left(\frac{1}{Z_m} - \frac{1}{Y_m} \right), \tag{42}$$

where

$$Y_l = 1 + \alpha_l \sum_{1}^{l-1} q_i,$$

$$Z_l = 1 + \alpha_l \sum_{1}^{l} q_i = Y_l + \alpha_l q_l,$$

here $\omega_{K+1} = 0$.

PROOF. From (40), the system utility for BC is

$$\max_{\boldsymbol{q}\geq 0} \tilde{u}(\boldsymbol{q},\boldsymbol{\beta}',\boldsymbol{\omega}) = \max_{\boldsymbol{q}\geq 0} \sum_{k=1}^{K} \omega_k \log\left(1+\frac{\alpha_k q_k}{1+\alpha_k \sum_{j=1}^{k-1} q_j}\right) - \sum_{k=1}^{K} \beta'_k q_k. \quad (43)$$

$$\frac{\partial}{\partial q_l} = \sum_{m=l+1}^{K} \left(\frac{\omega_m \alpha_m}{Z_m} - \frac{\omega_m \alpha_m}{Y_m}\right) + \frac{\omega_l \alpha_l}{Z_l} - \beta_l = 0. \quad (44)$$

Then the rate requirement $\underline{\boldsymbol{u}}$ is

$$\underline{u}_l = \log\left(1+\frac{\alpha_l q_l}{1+\alpha_l \sum_{m=1}^{l-1} q_m}\right) = \log\left(\frac{Z_l}{Y_l}\right) \quad (45)$$

To solve Y_l and Z_l, the power given in Chapter 10.3.2 in [17] could be used.

$$\boldsymbol{q} = (\boldsymbol{D}_a - \boldsymbol{B})^{-1}\mathbf{1},$$

where $\boldsymbol{D}_a := \text{diag}\left(\frac{1}{a_1},\cdots,\frac{1}{a_K}\right)$, and $\mathbf{1}$ is the vector of all 1's. And $\boldsymbol{B}$ have components of α_k,

$$a_k := \frac{2^{\underline{u}_k}-1}{2^{\underline{u}_k}\alpha_k}. \quad (46)$$

By inserting $\boldsymbol{D}_a$ into Y_l and Z_l, the optimal pricing parameters $\boldsymbol{\beta'}$ are solved. □

4.3 Contrary Example

MAC without SIC is one of the contrary example for our pricing mechanism. Because the interference function for MAC without SIC is $I_k(\boldsymbol{p}) = \sum_{l\neq k} \alpha_l p_l + \sigma^2$, where σ^2 is the noise power.

Then the utility function $u(\boldsymbol{p},\boldsymbol{\omega}) = \sum_{k\in K} \omega_k g_k(\frac{p_k}{I_k(\boldsymbol{p})})$ is no longer jointly concave in general, it becomes the $\mathcal{NU}$ function in [4]. There is no universal linear pricing holding for these functions in general. For example, if $u_1 = \log\left(1+\frac{\alpha_1 p_1}{1+\alpha_2 p_2}\right)$ and $u_2 = \log\left(1+\frac{\alpha_2 p_2}{1+\alpha_1 p_1}\right)$, the eigenvalues for the Hessian matrix with $\alpha_1 = \alpha_2 = 1$ and $p_1 = p_2 = 1$ are 0.25 and -0.194444. Therefore, $\tilde{u} = u_1 + u_2$ is not jointly concave in p_1 and p_2. It is jointly concave if and only if the Hessian matrix of the system utility $\tilde{u}(\boldsymbol{q},\boldsymbol{\beta},\boldsymbol{\omega})$ is negative definite.

5. INCENTIVE COMPATIBLE MECHANISM DESIGN

In the game theoretic sense, announcing one's true preference for its own payoff measured on a certain *utility* might not be in the best interest of rational users. This is where the system designer, in our case the regulator, has to intelligently conjure a mechanism to design a game such that the individual user's objectives do not contradict with the system objectives. Here the theory of *mechanism design* comes into play.

5.1 Cheating Problem

It is possible for user k to manipulate the universal linear pricing scheme by reporting a channel $\hat{\alpha}_k$ instead of the true α_k. If user k reports $\hat{\alpha}_k < \alpha_k$ a higher rate is achieved. We perform a case study as well.

- Case 1: Assume a decoding order $[K \to K-1 \to \cdots \to 2 \to 1]$ for both channel vectors $\boldsymbol{\alpha} = [\alpha_1,\cdots,\alpha_K]$ and $\hat{\boldsymbol{\alpha}} = [\alpha_1,\cdots,\alpha_{k-1},\hat{\alpha}_k,\alpha_{k+1},\cdots,\alpha_K]$, respectively.

The power allocation $p_k(\hat{\boldsymbol{\alpha}})$ which is chosen by the linear pricing scheme satisfies the rate requirements $\underline{\boldsymbol{u}}$ for the reported channels $\hat{\boldsymbol{\alpha}}$, i.e.,

$$\begin{aligned}
\underline{u}_1 &= \log(1+\alpha_1 p_1(\hat{\boldsymbol{\alpha}})), \\
&\vdots \\
\underline{u}_k &= \log\left(1+\frac{\hat{\alpha}_k p_k(\hat{\boldsymbol{\alpha}})}{1+\sum_{l<k}\alpha_l p_l(\hat{\boldsymbol{\alpha}})}\right), \\
&\vdots \\
\underline{u}_K &= \log\left(1+\frac{\alpha_K p_K(\hat{\boldsymbol{\alpha}})}{1+\sum_{l<K,l\neq k}\alpha_l p_l(\hat{\boldsymbol{\alpha}})+\hat{\alpha}_k p_k(\hat{\boldsymbol{\alpha}})}\right)
\end{aligned} \quad (47)$$

We interpret the power allocation as a function of $\hat{\boldsymbol{\alpha}}$, i.e., $\boldsymbol{p}(\hat{\boldsymbol{\alpha}})$ solves (47). The actual rate achieved after cheating for each user k is $r_k(\hat{\boldsymbol{\alpha}})$.

LEMMA 6. *By cheating only the own power allocation does change. e.g., If $\hat{\alpha}_k > \alpha_k$ ($\hat{\alpha}_k < \alpha_k$), then $p_k(\hat{\boldsymbol{\alpha}}) < p_k(\boldsymbol{\alpha})$ ($p_k(\hat{\boldsymbol{\alpha}}) > p_k(\boldsymbol{\alpha})$), and $p_l(\hat{\boldsymbol{\alpha}}) = p_l(\boldsymbol{\alpha})$ for all $l \neq k$.*

PROOF. The second statement $p_l(\hat{\boldsymbol{\alpha}}) = p_l(\boldsymbol{\alpha})$ for all $l < k$ follows directly from (47) because $\underline{u}_1,\cdots,\underline{u}_{k-1}$ do not depend on the channel of user k.

Consider the rate requirement $\underline{u}_k$ for user k and (24) and solve for $p_k(\hat{\boldsymbol{\alpha}})$,

$$p_k(\hat{\boldsymbol{\alpha}}) = \frac{2^{\underline{u}_k}-1}{\hat{\alpha}_k}(1+\sum_{l=1}^{k-1}\alpha_l p_l) = C\cdot\hat{\alpha}_k^{-1}, \quad (48)$$

where C is a constant because it is also independent on the channel of user k.

Since (48) is monotonic decreasing in $\hat{\alpha}_k$, if $\hat{\alpha}_k > \alpha_k$, then $p_k(\hat{\boldsymbol{\alpha}}) < p_k(\boldsymbol{\alpha})$. If $\hat{\alpha}_k < \alpha_k$, then $p_k(\hat{\boldsymbol{\alpha}}) > p_k(\boldsymbol{\alpha})$.

This proves the first statement in Lemma 6.

It is important to note that $p_k(\hat{\boldsymbol{\alpha}})\cdot\hat{\alpha}_k = C$ is constant and independent of $\hat{\alpha}_k$. Therefore, the interference terms in $\underline{u}_l$ with $l > k$ are independent of $\hat{\alpha}_k$, i.e., for all $l > k$,

$$I_l(\hat{\boldsymbol{\alpha}}) = \sum_{m<l,m\neq k}\alpha_m p_m + \sigma_n^2 + \hat{\alpha}_k p_k(\hat{\boldsymbol{\alpha}}) = I_l(\boldsymbol{\alpha}). \quad (49)$$

This solves the second statement in Lemma 6. □

COROLLARY 1. *The actual rate $r_l(\hat{\boldsymbol{\alpha}})$ achieved after cheating deviates from the rate requirement $\underline{u}_l$ for user $l \geq k$, while the rate $r_l(\hat{\boldsymbol{\alpha}})$ remains the same as $\underline{u}_l$ for $l < k$. If $\hat{\alpha}_k < \alpha_k$, then the actual rate*

1. $r_k(\hat{\boldsymbol{\alpha}}) > \underline{u}_k$,
2. $r_l(\hat{\boldsymbol{\alpha}}) < \underline{u}_l$ *for all $l > k$ and*

3. $r_l(\hat{\boldsymbol{\alpha}}) = \underline{u}_l$ *for all* $l < k$.

And vice versa.

PROOF. The actual rate $r_k(\hat{\boldsymbol{\alpha}})$ achieved by the power allocation $p_k(\hat{\boldsymbol{\alpha}})$ for user k with the real channel α_k is

$$r_k(\hat{\boldsymbol{\alpha}}) = \log\left(1 + \frac{\alpha_k p_k(\hat{\boldsymbol{\alpha}})}{1 + \sum_{l=1}^{k-1} \alpha_l p_l}\right). \quad (50)$$

And compare with the rate requirement $\underline{u}_k$ achieved by user k calculated in (47),

$$r_k(\hat{\boldsymbol{\alpha}}) = \log\left(1 + \frac{\alpha_k}{\hat{\alpha}_k}(2^{\underline{u}_k} - 1)\right). \quad (51)$$

If $\hat{\alpha}_k > \alpha_k$, then $r_k(\boldsymbol{\alpha}) < \log(1 + 2^{\underline{u}_k} - 1) = \underline{u}_k$. If $\hat{\alpha}_k < \alpha_k$ then $r_k(\boldsymbol{\alpha}) > \log(1 + 2^{\underline{u}_k} - 1) = \underline{u}_k$.

This proves the first statement.

The actual rate achieved by each user $l, l \neq k$ is

$$r_l(\hat{\boldsymbol{\alpha}}) = \log\left(1 + \frac{\alpha_l p_l}{1 + \sum_{m=1}^{k-1} \alpha_m p_m + \sum_{k+1}^{i-1} \alpha_m p_m + \alpha_k \hat{p}_k}\right), \quad (52)$$

for all $l > k$. And

$$r_l(\hat{\boldsymbol{\alpha}}) = \log\left(1 + \frac{\alpha_l p_l}{1 + \sum_{m=1}^{l-1} \alpha_m p_m}\right), \quad (53)$$

for all $l < k$.

If $\hat{\alpha}_k > \alpha_k$, then $p_k(\hat{\boldsymbol{\alpha}}) < p_k(\boldsymbol{\alpha})$ and $\alpha_k p_k(\hat{\boldsymbol{\alpha}}) < \hat{\alpha}_k p_k(\hat{\boldsymbol{\alpha}})$. Comparing with (47), $r_l(\hat{\boldsymbol{\alpha}}) > \underline{u}_l$ for all $l > k$, and vice versa.

This proves the second statement.

Because (53) is independent of α_k and $p_k(\hat{\boldsymbol{\alpha}})$, the actual rate $r_l(\hat{\boldsymbol{\alpha}})$ achieved by each user $l < k$ is the same as the rate requirement $\underline{u}_l$. This proves the third statement. □

- Case 2: If $\hat{\alpha}_k < \alpha_k$ then from $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_{k-1} \leq \alpha_k \leq \alpha_{k+1} \leq \cdots \leq \alpha_K$, we arrive at some different order in which $\hat{\alpha}_k$ is moved l places to the left, i.e., $\alpha_1 \leq \cdots \leq \alpha_{k-l-1} \leq \hat{\alpha}_k \leq \alpha_{k-l} \cdots \leq \alpha_{k-1} \leq \alpha_{k+1} \leq \cdots \leq \alpha_K$.

The decoding order is changed accordingly to

$$\pi = [K \to K-1 \to \cdots \to k+1 \to k-1 \to \cdots \to \\ \to k-l \to k \to k-l-1 \to \cdots \to 2 \to 1].$$

The rate requirement is achieved by allocating the power $p_k(\hat{\boldsymbol{\alpha}})$ with the reported channels $\hat{\boldsymbol{\alpha}}$,

$$\underline{u}_k(\hat{\boldsymbol{\alpha}}) = \log\left(1 + \frac{\hat{\alpha}_k p_k(\hat{\boldsymbol{\alpha}})}{1 + \sum_{m=1}^{k-l-1} \alpha_m p_m}\right). \quad (54)$$

Comparing the power allocation $p_k(\hat{\boldsymbol{\alpha}})$ and $p_k(\boldsymbol{\alpha})$ yields

$$\begin{aligned} p_k(\boldsymbol{\alpha}) &= \frac{2^{\underline{u}_k} - 1}{\alpha_k}\left(1 + \sum_{l<k} \alpha_l p_l\right) \\ p_k(\hat{\boldsymbol{\alpha}}) &= \frac{2^{\underline{u}_k} - 1}{\hat{\alpha}_k}\left(1 + \sum_{m<k-l} \alpha_m p_m\right). \end{aligned} \quad (55)$$

LEMMA 7. *If user k reports a lower channel $\hat{\alpha}_k < \alpha_k$ and makes the SIC decoding order at the BS change as in case 2, then the actual rate $r_k(\hat{\boldsymbol{\alpha}})$ achieved by user k is larger than the rate requirement $\underline{u}_k$.*

PROOF. The actual rate $r_k(\hat{\boldsymbol{\alpha}})$ achieved by the power allocation $p_k(\hat{\boldsymbol{\alpha}})$ for user k with the real channel α_k after changing the SIC decoding order is

$$r_k(\hat{\boldsymbol{\alpha}}) = \log\left(1 + \frac{\alpha_k p_k(\hat{\boldsymbol{\alpha}})}{1 + \sum_{m=1}^{k-l-1} \alpha_m p_m}\right). \quad (56)$$

Comparing with the rate requirement in (54), when $\hat{\alpha}_k < \alpha_k$,

$$\begin{aligned} r_k(\hat{\boldsymbol{\alpha}}) &= \log\left(1 + \frac{\alpha_k}{\hat{\alpha}_k}(2^{\underline{u}_k} - 1)\right) \\ &> \log(1 + 2^{\underline{u}_k} - 1) = \underline{u}_k(\hat{\boldsymbol{\alpha}}). \end{aligned} \quad (57)$$

Therefore, it is advantageous for user k to report a smaller channel $\hat{\alpha}_k$ than its true channel α_k. □

Remark 3. The result is intuitive because if the BS receives a smaller channel gain from user k, it needs to compensate this by either increasing the power p_k for user k (see in Case 1), or helping by changing the SIC decoding order π, s.t., user k receives less interference.

5.2 Incentive Compatible Mechanism

Assume user k cheats for a lower channel state $\hat{\alpha}_k$. According to Corollary 1 and Lemma 7, the rate $r_k(\hat{\boldsymbol{\alpha}})$ achieved by user k on the real channel α_k after cheating is higher than its utility requirement $\underline{u}_k$. Thus an incentive compatible mechanism in order to prevent users from cheating their channels should be designed. The achievable rate $r_j(\hat{\boldsymbol{\alpha}})$ after cheating a lower channel $\hat{\alpha_k}$ is derived by

$$\begin{cases} \underline{u}_j & \text{if } j < k \\ \log\left(1 + \frac{\alpha_j}{\hat{\alpha}_j}(2^{\underline{u}_j} - 1)\right) & \text{if } j = k \\ \log\left(1 + (2^{\underline{u}_j} - 1)\frac{1 + \sum_{l<j, l\neq k} p_l \alpha_l + p_k \hat{\alpha}_k}{1 + \sum_{l<j, l\neq k} p_l \alpha_l + p_k \alpha_k}\right) & \text{if } j > k \end{cases}. \quad (58)$$

In order to prevent cheating, we have to assume that mobiles have transferable utilities by some side payments. Define the transfer for user k as a general function of his reported type $\hat{\alpha}_k$ as $\phi(\hat{\boldsymbol{\alpha}})$.Then the expected utility $\bar{u}_k(\hat{\boldsymbol{\alpha}})$ is given by the sum of the achievable rate of user k and the transfer $\phi(\hat{\boldsymbol{\alpha}})$

$$\bar{u}_k(\hat{\boldsymbol{\alpha}}) = \log\left(1 + \frac{\alpha_k}{\hat{\alpha}_k}(2^{\underline{u}_k} - 1)\right) - \phi(\hat{\boldsymbol{\alpha}}). \quad (59)$$

For $\bar{u}_k(\hat{\boldsymbol{\alpha}})$ to achieve maximum at the true channel state α_k, it should fulfill the following conditions

$$\lim_{\hat{\alpha}_k \to 0} \bar{u}_k(\hat{\boldsymbol{\alpha}}) = 0 = \lim_{\hat{\alpha}_k \to \infty} \bar{u}_k(\hat{\boldsymbol{\alpha}}), \tag{60}$$

$$\frac{\partial \bar{u}_k(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_k} \begin{cases} < 0 & \text{if } \hat{\alpha}_k > \alpha_k \\ = 0 & \text{if } \hat{\alpha}_k = \alpha_k \\ > 0 & \text{if } \hat{\alpha}_k < \alpha_k, \end{cases} \tag{61}$$

$$\frac{\partial^2 \bar{u}_k(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_k^2} < 0. \tag{62}$$

However, we argue that there does not exist such an incentive compatible universal linear pricing scheme for MAC/BC with individual utility requirements. Consider if the first decoded user cheats for a smaller channel $\hat{\alpha}_{\pi_1^i}$, the system optimizer will allocate more power to him/her. From Lemma 6, all the power allocated to other users remains the same. Since there is no sum power or individual power constraint in the system model in order to satisfy the utility requirements of each user and Lemma 4 shows that the cost terms c^{π^i} are independent of the channel states, cheating of the first decoded user does not produce any influence on other users, but only increase his/her own achievable rate. There is no incentive compatible mechanism to prevent cheating at least for the first decoded user.

6. CONCLUSION AND OUTLOOK

We propose a linear pricing framework in which a general system utility function is optimized under the utility requirements for each user in the uplink MAC as well as in the downlink BC. For the MAC with SIC, we characterize the condition for the system utility to be jointly concave which supports the universal linear pricing. Furthermore, we propose the algorithm of the pricing parameters to achieve the utility requirement for each user. The best decoding order for SIC in MAC which minimizes the sum transmit power is completed. A reordering mechanism for the K users with regard to the order of $\boldsymbol{\alpha}$ is proposed so that the SIC decoding order can be fixed. In the downlink BC, due to the duality to MAC, the universal linear pricing algorithm is also proposed. It is possible that the users have the willing to cheat for a smaller channel state instead of the real one in order to achieve a higher rate. The cheating cases are thoroughly discussed as well as the argument of the nonexistence of the incentive compatible mechanism that prevents users from cheating.

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