

Speed Adaptive Probabilistic Flooding in Cooperative Emergency Warning

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ABSTRACT

Vehicular Ad Hoc Networks (VANETs) are emerging as the preferred network design for intelligent transportation systems and are envisioned to be useful in road safety and commercial applications. A significant issue in VANETs is the design of an effective broadcast scheme which can facilitate the fast and reliable dissemination of critical safety messages to neighbouring vehicles in case of an unexpected event, such as a traffic accident. Towards this goal, we propose a novel Speed Adaptive Probabilistic Flooding algorithm (SAPF)¹. Its decision to rebroadcast a message is based on a probability, evaluated using the speed of the vehicle. The algorithm enjoys a number of benefits relative to other approaches: it is simple to implement and does not introduce additional communication burden as it requires local information only, it does not rely on the existence of a positioning system which may not always be available and above all, mitigates the effect of the broadcast storm problem, typical when utilizing blind flooding. Our results indicate that the proposed algorithm outperforms blind flooding, especially in cases of heavy congestion. The SAPF algorithm achieves high reachability and unlike blind flooding, it also maintains low latency as the density of vehicles in the road network increases.

Keywords: VANETs, DSRC, Flooding, SAPF

1. INTRODUCTION

Vehicular Ad Hoc Networks are emerging as the preferred network design for ITS technologies. Vehicles employ wireless communication to form Ad Hoc networks which are envisioned to accommodate the new generation of cooperative road safety applications. A major challenge in such an application is the

design of an efficient broadcast scheme which will facilitate the fast and reliable dissemination of the early warning message to the approaching vehicles.

A straightforward solution is flooding [1], an approach which involves each vehicle rebroadcasting the message whenever it receives it for the first time. However, blind flooding is known to generate a large number of redundant messages leading to contention and unnecessary collisions. This is known as the Broadcast Storm Problem [1]. Several techniques have been proposed in literature to alleviate this problem [2-11]. The main idea has been to reduce the number of nodes rebroadcasting the message without affecting the total number of nodes receiving the message. The various proposed solutions then differ in the method with which this restricted set of nodes is chosen.

Specifically for VANETs, the most popular approach has been to choose vehicles which lie on the boundary of the transmission range of the vehicle transmitting each message [12-14]. However, this method assumes the availability of a positioning system, such as GPS, which may not always be available. Our main contribution in this work is to develop a new broadcast scheme which does not rely on the existence of a positioning system, and only relies on local information. In order to reduce the number of vehicles rebroadcasting the message we employ probabilistic flooding and in addition, we adaptively regulate the rebroadcast probabilities, based on the vehicle speeds, to optimally reduce message delivery delays caused by increased contention, in areas with high vehicle densities. We refer to the proposed scheme as Speed Adaptive Probabilistic Flooding (SAPF). The main idea is that each vehicle upon receiving an emergency message for the first time, decides to rebroadcast the message with probability p whose value is calculated as a function of the speed of the vehicle. This function aims at mapping the speed of the vehicle to the optimal rebroadcast probability value which coincides with what is known as the 'critical' probability. As we show in this paper probabilistic flooding in VANETs is characterized by phase transition phenomena which dictate optimal values for the rebroadcast probability at the onset of the transitions. At these optimal values high reachability is achieved with the minimum possible latency. These optimal values, however, depend on the vehicle densities which in turn are known to depend on the vehicle speeds [21]. In this paper, we investigate these phase transition phenomena and the forementioned relationships, and derive a piecewise linear function relating the rebroadcast probability with the vehicle speed. This function constitutes the basis of the proposed SAPF scheme.

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The algorithm enjoys a number of benefits relative to other approaches: it is simple to implement and does not introduce additional communication burden as it requires local information only, it does not rely on the existence of a positioning system which may not always be available and above all, mitigates the effect of the broadcast storm problem, typical when utilizing blind flooding. Our results indicate that the proposed algorithm outperforms blind flooding, especially in cases of heavy congestion. The SAPF algorithm achieves high reachability and unlike blind flooding, maintains low latency as the density of vehicles in the road network increases.

The paper is organized as follows. In section 2, we present the rationale behind our design choices and the adopted design methodology and in section 3 we present the system architecture and implementation details of the proposed SAPF algorithm. In section 4, we evaluate the performance of the proposed scheme using simulations and finally in section 5 offer our conclusions and future research directions.

2. Design Rationale

In this section we present the rationale behind our design choices and the adopted design methodology. Our objective has been to design an effective broadcast scheme which facilitates reliable and fast delivery of emergency messages to neighboring vehicles in case of an unexpected event such as a traffic accident. Probabilistic flooding was chosen as the candidate solution in order to mitigate the effects of the broadcast storm problem encountered when using blind flooding. In order, however, to achieve optimal performance, there is a need to set the rebroadcast probability at each vehicle adaptively, based on the vehicle speed. Our reasoning behind this is the following: low vehicle speeds in a freeway setting imply high vehicle density [21], which then implies that high reachability can be achieved by choosing relatively low rebroadcast probability values. Probabilistic flooding in Mobile Ad Hoc Networks is known to yield phase transition phenomena. As the rebroadcast probability increases, there exists a threshold value beyond which reachability suddenly approaches 100 percent with high probability [18-20].

This critical value is the rebroadcast probability of choice as it guarantees high reachability and low delay. If higher rebroadcast probabilities are chosen high reachability is still achieved, however, delays may increase due to more vehicles rebroadcasting the emergency messages. However, the critical probability depends on the node density. In a vehicular setting, the vehicle density is also known to relate to the vehicle speed. So, by suitable choice of the rebroadcast probability as a function of the vehicle speed optimal performance can be achieved at all traffic densities. In the remainder of this section, we show the method adopted to derive the desired relationship between the rebroadcast probability and the vehicle speed. We first use simulations to find the critical probability at various vehicle densities and then use known relationships between the vehicle density and the speed to derive the desired relationship between the rebroadcast probability and the vehicle speed.

All the simulation experiments were conducted using an integrated platform combining two simulators, VISSIM, a traffic simulator, and OPNET Modeler, a network simulator. The assumed setting is the following: A vehicle that detects an

unexpected road hazard becomes an abnormal vehicle (AV) and transmits instantly an early warning message (EWM) to warn approaching vehicles of the imminent danger. Following the specifications of the 802.11b standard, the transmission range of all vehicles is set to 300 meters. Upon reception of the EWM for the first time a vehicles decides to rebroadcast the message with probability p , or decides not rebroadcast the message with probability $1-p$.

The performance metrics that we consider in our study are the number of nodes which receive the transmitted emergency message, protocol overhead, and the latency. The latency is defined as the time interval between the instant the message is transmitted by the vehicle detecting the road hazard and the instant that the last vehicle in the network receives the message. We use two measures for the protocol overhead, the number of backoffs and the number of rebroadcasts. A node enters a backoff state before each transmission and after the medium is sensed to be non-idle and is an indicative measure of the protocol contention.

The chosen test site is a two lane highway which spans a distance of 6Km. We conducted a number of experiments to reflect different scenarios. Our objective has been to examine the behavior of the above performance metrics as we change the rebroadcast probability and the vehicle density. We have considered rebroadcast probability values in the range 0 to 1 in steps of 0.1. To generate scenarios with different vehicle densities we considered different rate of vehicles entering the chosen test site. We considered the following penetration rates: 5 vehicles per kilometer per lane, 10 per kilometer per lane, 20 per kilometer per lane, 30 per kilometer per lane, 40 per kilometer per lane and 50 per kilometer per lane. All values obtained are averages over 10 simulation experiments.

Fig. 1 shows the percentage of vehicles receiving the emergency warning message as we change the rebroadcast probability and the vehicle penetration rate. For a particular rate it is evident that there exists a critical threshold probability beyond which the number of vehicles receiving the message suddenly increases and stays almost constant. This sudden change is compatible with phase transition phenomena observed in the literature of random graphs and percolation theory. We also observe that as the penetration rate and thus the vehicle density increases, the critical threshold decreases.

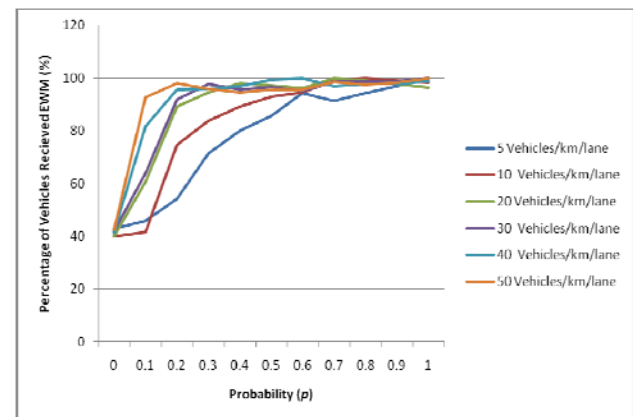


Figure 1: Percentage of Nodes Received EWM vs. Probability

In Fig. 2, we have plotted the critical probability vs. the corresponding penetration rates. We observe an almost linear decrease of the critical probability as the penetration rate increases up to a saturation point (50 per kilometer per lane) after which the critical probability stays constant. This saturation is due to the fact that after some value of the penetration rate the vehicle density remains almost constant since the capacity of the particular road has been reached. For a particular section of the road there is a maximum number of vehicles which can be present simultaneously in that section.

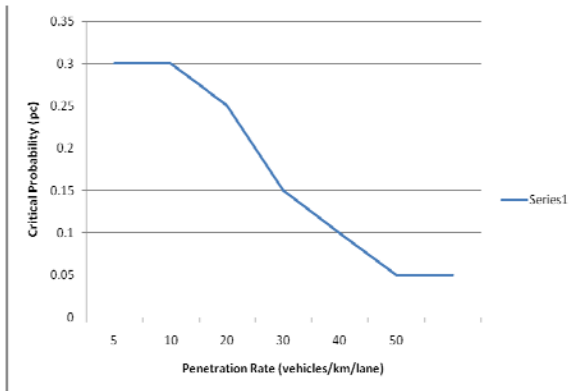


Figure 2: Critical Probability vs. Vehicle Densities

Our next objective is to find the desired rebroadcast probability value at each vehicle density. The desired value is the one which achieves high reachability but minimizes the overhead and thus the delays. In Fig. 3, we present the number of backoffs reported as we vary the rebroadcast probability and the penetration rate of the vehicles. We observe similar behavior at all rates with the number of backoffs increasing as we increase the rebroadcast probability. This is expected as higher rebroadcast probabilities imply a larger number of vehicles contending for the common medium.

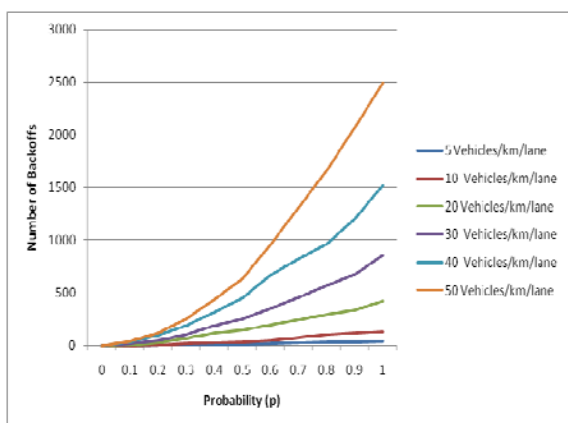


Figure 3: Number of Nodes Backoffs vs. Probability

The greater contention observed as we increase the penetration rate is caused primarily due to the larger number of vehicles

trying to rebroadcast the message. This is verified in Fig. 4 where we plot the number of rebroadcasts versus the penetration rate. We observe that at all penetration rates as we increase the rebroadcast probability the number of vehicles rebroadcasting the message increases as well.

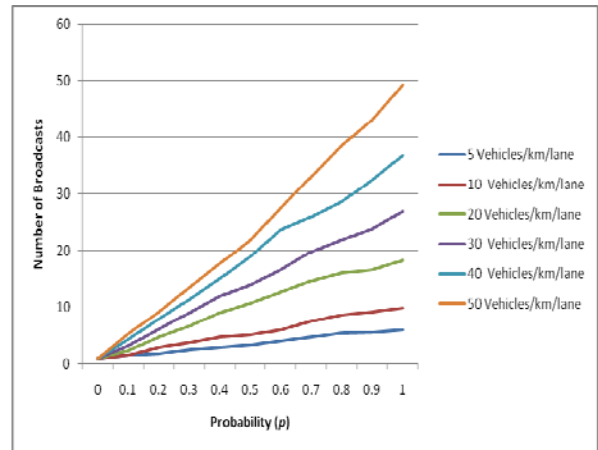


Figure 4: Number of Nodes Broadcasts vs. Probability

Finally in Fig. 5, we show plots of the message propagation delay. The graphs indicate that at all penetration rates the delay exhibits a monotonically increasing pattern. This is also expected as higher contention causes collisions which delay message delivery.

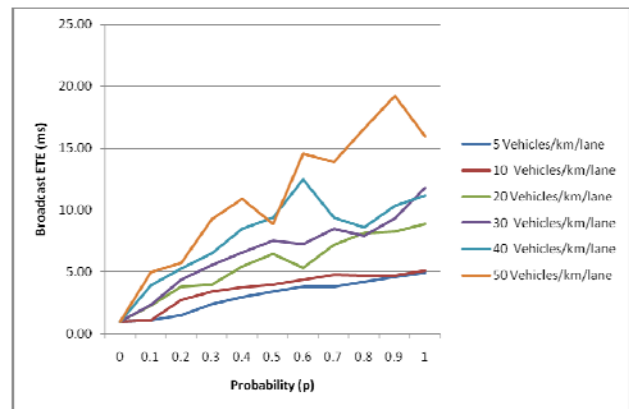


Figure 5: Broadcast ETE vs. Probability

What the above graphs indicate is that as we increase the rebroadcast probability beyond the critical probability, the reachability remains high while the latency increases. So, for a particular node density the critical probability is the optimal choice for the rebroadcast probability which ensures the fastest message delivery to almost all vehicles. The next step is to find the relationship which allows each vehicle to calculate this optimal value based on its own speed. We derive the relationship by composition of the mapping shown in Fig. 2 with the vehicle speed vs. density relationship shown in Fig. 6 [21]. The latter was

obtained from field data. The resulting function, mapping the speed to the rebroadcast probability is shown in Fig. 7. We consider a linear approximation of the graph also shown in Fig. 7 whose equation is given by:

$$p = 0.0557v - 0.003 \quad (1)$$

where p denotes the rebroadcast probability and v the velocity of the vehicles. In addition, for speeds above 100km/h (speed limit) we set the rebroadcast probability to 1. The reason is that beyond such speeds it is impossible to estimate the density based on speed information only and so we adopt a rather aggressive rebroadcast policy in order to ensure message delivery to all vehicles. On the other hand for speed values below 15km/h, we assume that the network has almost reached its capacity and so we consider a constant rebroadcast probability of 0.05. The resulting rebroadcast probability vs. speed function used in the proposed SAFPF scheme is shown in Fig. 8.

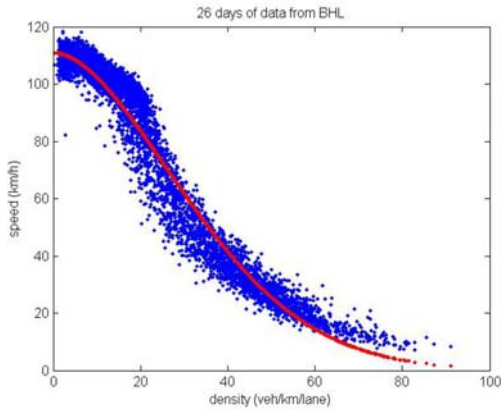


Figure 6: Vehicle speed vs. Density [21]

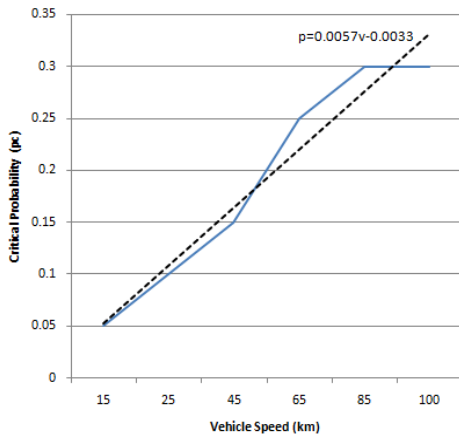


Figure 7: Critical Probability vs. Vehicle speed

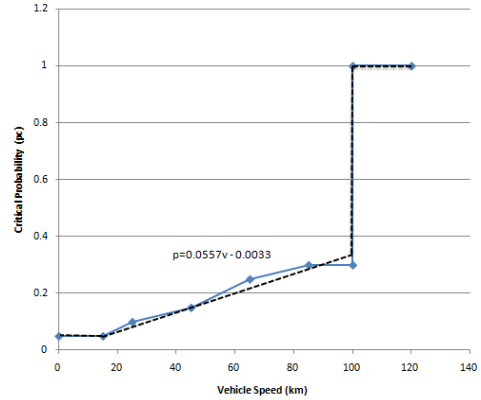


Figure 8: Critical Probability vs. Vehicle speed

3. System Overview and Implementation

In this section, we present implementation details of the SAFPF algorithm. The setting that we consider is similar to the one presented in section II. The AV (vehicle that detects an unexpected road hazard) is the originator of the EWM message and remains the originator of the message until the message expires or is destroyed. Every message has the following fields: IP address of the originator (AV), destination address (set to broadcast), a time to live field (TTL), and sequence number field. All these fields are set by the AV at the time of the creation of the EWM message. The only field that can be modified by other vehicles is the time to live field, TTL. The TTL field is decreased by one every time the EWM is rebroadcasted. As soon the TTL reaches a zero value the message is destroyed. The TTL depicts the number of hops the EWM can be transmitted in the network. Each vehicle maintains a table of recently received messages. Each message is identified by a combination of the source IP address and the sequence number. Whenever a vehicle receives a new message, it checks whether this message has been recently received through matching of the identifiers in the relevant table. If no matching is found, the message is classified as being received for the first time and is made available for rebroadcast upon decision of the probabilistic algorithm. The identifier of the message is also added to the table of recently received messages. Based on the function of Fig. 8 derived in the previous section, the speed adaptive probabilistic flooding algorithm is as follows:

Speed Adaptive Probabilistic Flooding Algorithm

1. upon reception of EWM s at node n :
2. if EWM received first time and it is not the originator of message s and $TTL > 0$ then
 - a. if (veh. speed ≥ 15 km && veh. speed ≤ 100 km) then broadcast EWM s with probability $p = 0.0557x - 0.0033$
 - b. else if (veh. speed < 15 km) then broadcast EWM s with probability $p = 0.05$
 - c. else if (veh. speed > 100 km) then broadcast EWM s with probability $p = 1$
3. end if

For the implementation of the SAFPF algorithm we used OPNET Modeler simulation engine by making appropriate modifications to an 802.11 WLAN station model which excludes implementations of layers above the MAC layers such as TCP/IP. Table 1 demonstrates the WLAN parameters chosen for all nodes.

4. Performance Evaluation

In this section, we evaluate the performance of the SAPF algorithm using an integrated platform combining two simulators, the traffic simulator VISSIM and OPNET Modeler. We have used VISSIM to generate traces of the vehicles involved in our scenarios and furnished into the OPNET Modeler simulation model. The topologies of the simulation model used in our study are based on a section of the Nicosia-Limassol highway in Cyprus. The first chosen test site (Scenario 1) is a two lane straight line highway which spans a distance of 6Km without any exits or entries in the highway, Fig. 9 which is the same topology that we have used in section II. The second chosen site (Scenario 2) has been taken from a particular section of the highway where high congestion and traffic accidents have been observed frequently even with low penetration rates, Fig. 10. Fig. 10 shows the entry and exit lanes in the highway that has been chosen for our study. It is apparent from the Fig. 10 that the 3rd vehicle has collided with the 4th vehicle between the exit lane and the main lane of the highway. Our objective has been to evaluate the performance of the SAPF algorithm with respect to chosen performance metrics and compare it with the Blind Flooding (BF) algorithm under different scenarios. We conducted a number of experiments, in order to evaluate the performance of the SAPF scheme in scenarios reflecting three types of congestion: light congestion, medium congestion and heavy congestion. We emulate the different congestion levels in the chosen topology as follows: For the first scenario, the vehicles in the considered two lane freeway propagate abiding to an original speed limit in the range 90-120km/h. In order to emulate an unexpected event which generates congestion, at the 3rd kilometer of the freeway we suddenly change the speed limit to lie in the range 30-60km/h. We then create different levels of congestion by varying the penetration rate. We considered three penetration rate values, 2000veh/hour, 5000veh/hour and 50000veh/hour reflecting light, medium and heavy congestion conditions respectively.

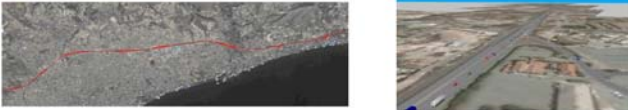


Figure 9: Snapshot of the reference model generated on the VISSIM Simulator

For the second scenario the three different congestion levels are based on real measurements. The levels of congestion are categorized in same order as in the first scenario light, medium and heavy congestion.

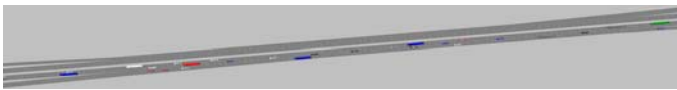


Figure 10: Snapshot of the section of Nicosia-Limasol

4.1 Results

In this section we present the results obtained and we extract useful conclusions regarding the performance of the SAPF algorithm comparing it with the BF algorithm under light, medium and heavy congested traffic. Our results indicate that the SAPF algorithm outperforms the BF algorithms as it maintains high reachability and low delays at all congestion levels in both scenarios.

4.1.1 Scenario 1

Fig. 11 shows the percentage of vehicles receiving the emergency warning message for the three penetration rates under consideration. It is evident, that both algorithms in all cases perform equally well achieving reachability values higher than 92%. The SAPF algorithm in fact achieves reachability values greater than 97% which is more than satisfactory. In Fig. 12 we present the number of backoffs generated by the two algorithms under the three aforementioned scenarios. We observe that at light congestion conditions the two algorithms report similar values. At more severe congestion conditions, however, the SAPF algorithm significantly outperforms the BF algorithm as it consistently reports significantly smaller number of backoffs. As congestion becomes more severe, blind flooding generates increasing number of backoffs while the SAPF scheme maintains approximately the same numbers.

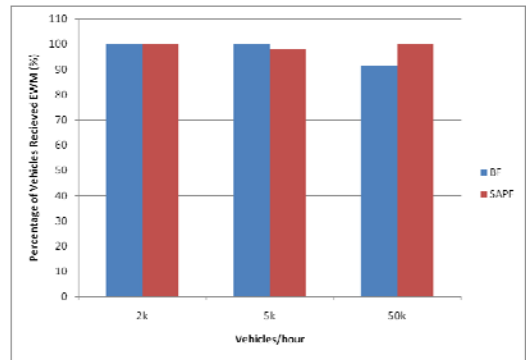


Figure 11: Percentage of Nodes Received EWM vs. Probability

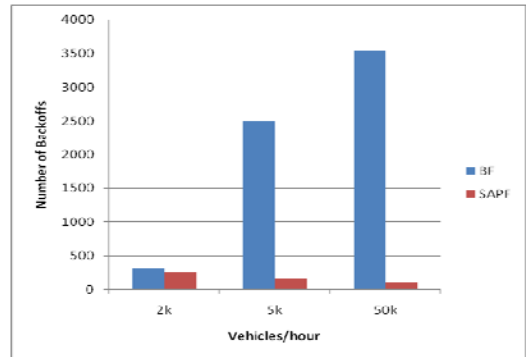


Figure 12: Number of Nodes Backoffs vs. Probability

The greater contention observed, as we increase the traffic density, is caused primarily due to the larger number of vehicles trying to rebroadcast the message. This is demonstrated in Fig. 13 where the number of rebroadcasts reported is shown for the three considered scenarios. The picture is almost identical to the one in Fig. 14. In Fig. 14 we show plots of the message propagation delay. This graph highlights the true superiority of the SAPF scheme compared to the BF algorithm. As the congestion becomes heavier, the BF algorithm takes more time to deliver the messages to all vehicles. The SAPF algorithm, on the other hand, manages to maintain almost constant latency values at all congestion conditions.

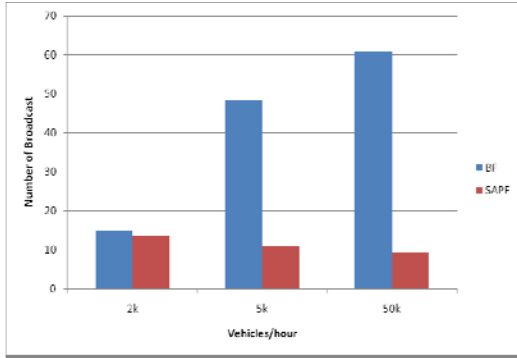


Figure 13: Number of Nodes Broadcasts vs. Probability

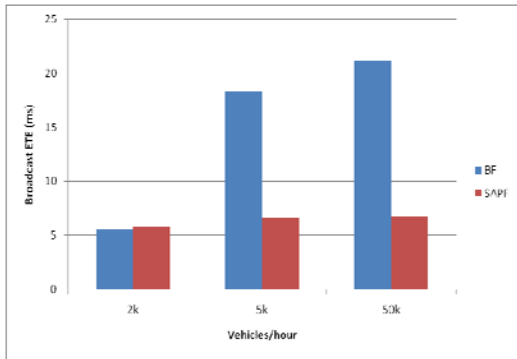


Figure 14: Broadcast ETE vs. Probability

In the heavy congestion case for example we observe that blind flooding reports a delay of approximately 22ms while the SAPF reports a delay of approximately 7ms. This significant reduction in the delay is of great importance in the considered applications as it gives the drivers or the automatic controller additional time to respond effectively.

4.1.2 Scenario 2

Fig. 15 depicts the percentage of vehicles receiving the emergency warning message for the three penetration rates under consideration. Both algorithms, SAPF and BF exhibit high reachability in all penetration values. SAPF algorithm outperforms the BF algorithm in almost all penetration rates, except at the medium congestion where the BF algorithm performs slightly better, however, in figures 16, 17 and 18 SAPF algorithm outperforms BF algorithm significantly. Based on Fig. 16 and Fig. 17 which they represent the protocol overhead, it is obvious that SAPF algorithm generates much less contention in the network in all penetration rates.

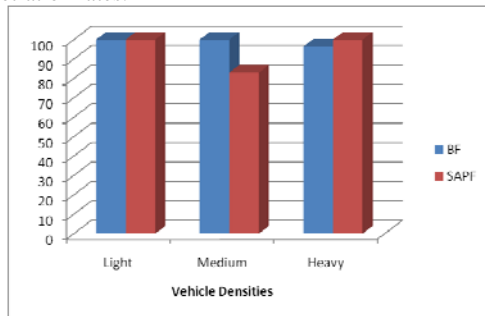


Figure 15: Percentage of Nodes Received EWM vs. Probability

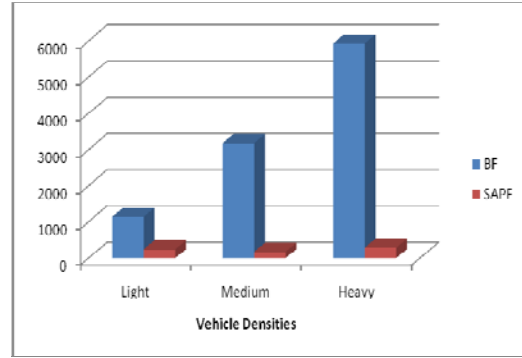


Figure 16: Number of Nodes Backoffs vs. Probability

Finally Fig. 18, represents the propagation delay. It is obvious from both of the scenarios that SAPF algorithm maintains a rather constant delay among different vehicle penetration rates and scenarios. This can be observed in Fig. 18 and Fig. 14 where the SAPF algorithm maintains its end-to-end delay around 7ms in comparison with the BF algorithm where its end-to-end delay varies from 11ms to 30ms. This stress the advantage of SAPF algorithm compared to the BF algorithm in VANETs.

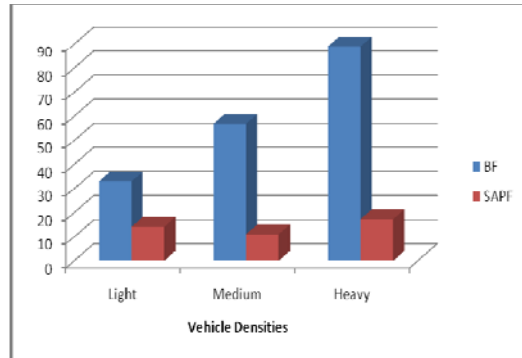


Figure 17: Number of Nodes Broadcasts vs. Probability

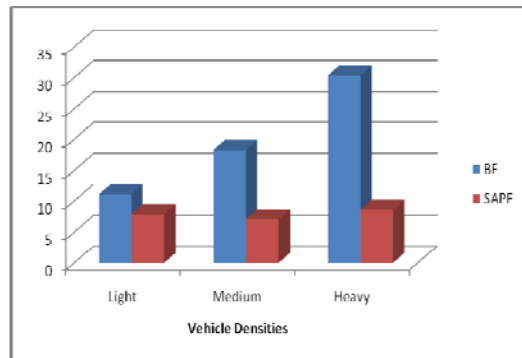


Figure 18: Broadcast ETE vs. Probability

5. Conclusions

In this paper we present a novel broadcast scheme for VANETs which does not rely on the existence of a positioning system and relies only on local information. It is shown through simulations to work effectively in a number of scenarios outperforming blind flooding. The scheme employs probabilistic flooding to reduce the number of vehicles rebroadcasting emergency messages and in addition it adaptively regulates the rebroadcast probabilities, based on the vehicle speeds, to

optimally reduce message delivery delays caused by increased contention, in areas with high vehicle densities. Our initial results indicate that the proposed algorithm outperforms blind flooding, especially in cases of heavy congestion. The SAPF algorithm achieves high reachability and unlike blind flooding, maintains low latency as the density of vehicles in the road network increases. We aim at further evaluating the performance of the proposed scheme, also comparing with other proposed algorithms, also considering more complex scenarios, including city and other road environments, and if required modify the adaptive probability component of the algorithm accordingly. In addition, we are working on verifying our simulation results with analytical results, using tools from random graph theory.

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