Information-Theoretic Integration of Sensing and Communication for Active Robot Networks

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***Abstract*—This paper presents an information-theoretic approach to sensor placement that incorporates communication capacity into an optimal formulation. A new formulation is presented that maximizes the information rate achievable by a set of sensors communicating wirelessly to a single collection node. Shannon capacity and the standard radio propagation model are used to model the throughput achievable by a sensor configuration. Likewise, the d-optimality criterion from the active sensing literature is used to model information gain provided by range and bearing sensors. The combiniation of information-theoretic measures leads to a metric equivalent to the expected information rate achievable by the system. Sensor positions are selected that optimize this measure.**

I. Introduction

A robot sensor network (RSN) is a member of a broad class of multi-agent robotic systems that consist of individual nodes capable of some combination of sensing, communication, and actuation. Sensing provides data and information about the environment in order to enable perception by the RSN. Communication facilitates the collection and transmission of the sensed data and information between individual nodes, enabling coordination over large areas. Actuation allows nodes to change their state in order to improve the performance of individual tasks or mission-level objectives. Future robotic sensor networks are expected to provide ubiquitous perception for a wide variety of applications. These applications include in-situ volumetric sensing [1], multi-target surveillance [2], chemical plume detection and tracking [3], autonomous underwater networks [4], [5], and networked communication relaying [6].

Controlled mobility in robot sensor networks, often referred to as *active sensing*, takes advantage of robot mobility to optimize or improve information gathering activities. For some applications, such as exploration, persistent surveillance, and area coverage, proactive motion and cooperation between multiple vehicles can improve performance in terms of execution time, search efficiency, and system flexibility or robustness. For other applications, such as bearing-only (or range-only) localization and target tracking or chemical plume tracking, controlled mobility by a single robot or multi-robot collaboration is necessary to provide basic sensing observability. Autonomous sensing tasks often impose conflicting constraints on node mobility which make the planning and control of robot sensor networks challenging. For instance a searching task may be evaluated according to the expected time to find the target. This leads to an inherent tension. Communications requirements favor moving nodes closer together while task requirements (such as searching) favor moving nodes further apart in order to better meet the task goals.

The exploitation of mobility for maintaining networked communication in the context of multi-objective distributed control has important ramifications for the deployment of robot sensor networks and has only recently begun to receive attention in the literature. In order to develop planning methods for robot sensor networks, communication objectives have been explicitly incorporated into some larger multi-objective frameworks [7]–[10]. Most often these systems assume a predefined minimum communication radius and add a relative position or line-of-sight constraint between cooperating vehicles [7], [8], [10], [11]. This approach has been applied to the deployment of mobile ground vehicles in complex buildings [8], [11]; between unmanned aerial vehicles performing coordinated search missions [10]; and for distributed coverage control of sensor networks [7], [12].

A handful of works address communication-reactive control strategies for ad-hoc networks. Li and Rus [13] are one of the first to address deliberate trajectory changes to make message sending between two communicating nodes possible. Movement control for fault tolerance was investigated in [10] by moving a subset of nodes to new locations in order to achieve biconnectivity in the network graph. A self-adaptive distributed feedback control scheme has been designed to obtain desirable network properties such as connectivity and power efficiency [9]. Although the goal is to optimize these network parameters, the performance metrics are transformed into position constraints based on a basic energy cost function.

A key component of active sensing systems is the dependence of information-gathering on robot motion and the ability to predict in advance the effect of robot motion on the quality of information that is collected [14]–[16]. For broad area sensing tasks such as exploration and surveillance, information is often described in terms of coverage area. In contrast, for model-based sensing tasks (e.g. feature localization, target tracking, and diffuse target tracking) information-theoretic concepts such as mutual information, Fisher information, and entropy can be used to quantify the sensitivity of the sensing task to robot motion.

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The development of planning and control algorithms for active sensing typically falls into one of three broad categories. Algorithms in the first category focus on geometric representations of the sensing tasks and solve coverage or tracking problems. Coverage control consists of placing or distributing a set of sensors to optimize the percentage of a given environment that can be seen by at least one sensor [7], [12]. The main drawback of this category of algorithms is failure to consider uncertainty in sensor information. The second category of algorithms, often referred to as adaptive sampling, focus on the directed sampling of a scalar field in order to obtain certain statistical properties or to locate sources, contours, and boundaries [5], [16]–[18]. The main drawback of these adaptive sampling approaches is their reliance on local information only and failure to develop and consider models of the underlying phenomena being measured.

The final category of active sensing approaches explicitly formulates sensing objectives in terms of information-theoretic criteria and then optimizes accordingly. This approach is generally referred to as *optimal input design* and exploits the influence of input parameters such as sensor mobility on observability or information measures of nonlinear and distributed (model-based) estimation algorithms [2], [14], [19]. The concept of using system inputs to improve the quality of information sensed by the system has been applied to a variety of problems. Example applications include: target tracking [2], [20], [21]; collaborative perception by unmanned ground and aerial vehicle teams [15]; vision-based navigation and control [22]; hierarchical reconnaissance by teams of unmanned aircraft [23] ; plume tracking by multiple autonomous underwater vehicles [5]; and cooperative radio source localization [24].

Finally, while information-theoretic concepts have been applied to the planning and control of robot sensor networks in some capacity, few efforts have been made to integrate the effects of both communication and sensing into a single framework. One approach was developed for adaptive sampling by multiple autonomous underwater vehicles [5]. Algorithms developed for this application combine one-step look-ahead optimization of the Kullblack-Liebler divergence of estimates of scalar fields with network flow control algorithms that weight link utility based on the Shannon-Hartley Capacity [25]. In this work mobility control is limited to sensing while the network routing schemes respond to the communication dependence on position. A second approach considers cross-layer design in mobile sensor networks [26]. The Shannon-Hartley law is restated to give received data uncertainty as a function of node separation. This formulation combines measurement uncertainty with communication uncertainty in order to minimize the overall uncertainty in an estimation process. The main limitation of this approach is failure to consider error correction coding on the data transmission. With error correction, data certainty is exchanged with communication throughput.

This paper describes an information-theoretic approach to integrating communication and sensing into a single robot sensor network planning framework.

II. System Models

A robot sensor network is defined as a set R of autonomous robots further divided into sets $M \subseteq R, S \subseteq R$, and $C \subseteq R$. The first set consists of *mobile* robots, the second consists of *sensing* robots capable of measuring data from the environment or targets, and the third contains robots capable of wireless multi-hop *communication* with other robots in the network. It is obvious that if a robot $r_i \notin C$ then it is not useful to the network, so we will assume $C = R$ (since we will not be considering robot failures). However, not every robot must move or sense. Finally, define the set $B \subseteq C$ of *base* nodes to whom communication is directed, i.e. which serve as data sinks in the robot sensor network. At this point no assumption is made on $|B|$ so there can be more than one base node in a robot sensor network. Specification of data flows and network paths from particular sources to destination (base) nodes will be application-specific.

Let $r_i \in R$ denote the i^{th} robot with state (position and velocity) $\mathbf{x}_i = [x_i, y_i, \dot{x}_i, \dot{y}_i]^T$ whose dynamics are governed by $\dot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i, \mathbf{u}_i, \mathbf{w}_i)$ where $\mathbf{u}_i$ is the control input and $\mathbf{w}_i$ is a disturbance or process noise. This work considers two types of robot motion, either general first order motion, i.e. the input is the velocity command

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} u_{x,i} \\ u_{y,i} \end{bmatrix} \qquad \sqrt{u_{x,i}^2 + u_{y,i}^2} \leq v_{max} \tag{1}$$

or the nonholonomic kinematic model

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\psi}_i \end{bmatrix} = \begin{bmatrix} u_{1,i} cos(\psi_i) \\ u_{1,i} sin(\psi_i) \\ u_{2,i} \end{bmatrix} \qquad \begin{matrix} v_{min} \leq u_{1,i} \leq v_{max} \\ \\ |u_{2,i}| \leq \omega_{max} \end{matrix} \tag{2}$$

The target set T is defined to contain all targets t_l in the environment that could be sensed by the network R. Targets are described by the state vector $\mathbf{x}_l = [x_l, y_l, \dot{x}_l, \dot{y}_l]^T$ with dynamics $\dot{\mathbf{x}}_l = \mathbf{f}_l(\mathbf{x}_l, \mathbf{u}_l)$. Target state estimates $\hat{t}_l$ with covariance matrices P_l are maintained by some sensor fusion system.

Typical sensors on mobile robots measure the range and bearing to target objects. These measurements come from a variety of sensors including monocular computer vision, sonar, radar, and time-of-arrival systems. Given a target located at position $\mathbf{p}_l$, the measurement obtained by a robot at position $\mathbf{p}_i$ is:

$$\mathbf{z}_{il} = h(\mathbf{p}_i, \mathbf{p}_l, \mathbf{v}) = \begin{bmatrix} \sqrt{(x_l - x_i)^2 + (y_l + y_i)^2} \\ \arctan\left(\frac{y_l - y_i}{x_l - x_i}\right) \end{bmatrix} + \mathbf{v} \tag{3}$$

where $\mathbf{v}$ is zero-mean Gaussian noise with covariance

$$E[\mathbf{v}\mathbf{v}^T] = \mathbf{R} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}. \tag{4}$$

Different sensors can be modelled by taking different values of the variance terms. For instance, bearings-only sensing occurs when $\sigma_r^2 \to \infty$. Additionals sensor can be modelled by making σ_r^2 and σ_θ^2 dependent on the range and bearing to the target.

For robot sensor networks, some form of ad-hoc wireless networking provides intra-robot communication. The graph-theoretic concept of network links is overly simplistic since radio frequency (RF) propagation depends on many unknown parameters beyond relative position such as multi-path, environmental fading, noise, and interference. In particular, an RF communication link can be characterized by the signal-to-noise-and-interference ratio (SNIR) defined as

$$S_{ij}(\mathbf{p}_i, \mathbf{p}_j) = \frac{P_{ij}(\mathbf{p}_i, \mathbf{p}_j)}{N(\mathbf{p}_i)} \tag{5}$$

where $P_{ij}(\mathbf{p}_i, \mathbf{p}_j)$ is the power received by robot r_i at position $\mathbf{p}_i \in \Re^2$ from the transmission of robot r_j located at $\mathbf{p}_j \in \Re^2$. $N(\mathbf{p}_i)$ is the environmental noise seen by robot i at location $\mathbf{p}_i$. For simplification of notation, let $S_{ij} = S_{ij}(\mathbf{p}_i, \mathbf{p}_j)$. It should be pointed out that in this paper $P_{ij} \neq P_{ji}$ and that $N(\mathbf{p}_i) \neq N(\mathbf{p}_j)$, giving $S_{ij} \neq S_{ji}$.

Radio propagation is oftened specified using the standard empirical radio propagation model

$$P_{ij}(\mathbf{p}_i, \mathbf{p}_j) = \frac{P_0 d_0}{|\mathbf{p}_i - \mathbf{p}_j|^\alpha} + v \tag{6}$$

where P_0 is the reference power, d_0 is the reference distance, $2 \leq \alpha \leq 6$ is the propagation decay exponent which is assumed constant for a given environment, and v is a randomly-distributed noise factor representing radio variability which is ignored here. The noise $N(\mathbf{p}_i)$ that a receiver has on its input is dependent upon the quality of the receiver electronics, the noise temperature of the system, the local RF environment of the node, and adversarial or inadvertent jamming. When the noise source is another radio node (e.g due to jamming or a malfunctioning robot) at position $\mathbf{p}_n$ the SNIR becomes

$$S_{ij}(\mathbf{p}_i, \mathbf{p}_j) = \frac{P_0 d_0}{P_n d_n}\Big(\frac{|\mathbf{p}_i - \mathbf{p}_n|}{|\mathbf{p}_i - \mathbf{p}_j|}\Big)^\alpha. \tag{7}$$

The traditional graph-theoretic model used to describe network communication in robot networks assumes some fixed range within which communication is possible at some rate and beyond which no communication is possible. The model in reality is richer than this. In wireless communication broadcast energy is received by all nodes within some environment and the notion of a link is not well defined. Furthermore, many digital wireless interfaces such as IEEE 802.11 have multiple data rates [27]. When communicators are close the communication rate is high (e.g. 54 Mbps in 802.11g). When communicators are far the communication rate is lower (802.11g has 14 different rates between 54 and 1 Mbps). The rate decision is based on the signal power to noise power ratio and packet success measurements made over the channel. This phenomenon is an example of a general principle - the well known Shannon-Hartley Law [25] which provides the theoretical maximum rate at which information passes error free over a channel.

III. Information-Theoretic Metrics

This section presents information-theoretic metrics for modelling sensing and communication. In particular, the concepts of Fisher Information [28] and Shannon Capacity [25] are introduced to describe position-dependent bounds on expected sensing and communication capabilities respectively. These two concepts are then combined into a single metric of expected information rate (EIR).

A. Fisher Information

Consider a discrete time nonlinear estimation problem. Let the state vector $\mathbf{x}_k$ evolve according to the deterministic state equation

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) \tag{8}$$

where k is the discrete sample time and $\mathbf{u}_k$ is the vector of deterministic inputs. Furthermore, let $\mathbf{y}_k^i$ denote the i^{th} measurement of the system with

$$\mathbf{z}_k^i = h(\mathbf{x}_k, \mathbf{u}_k^i, \mathbf{v}_k) \tag{9}$$

where $h(\mathbf{x}_k, \mathbf{u}_k^i, \mathbf{v}_k)$ is the measurement function and $\mathbf{v}_k$ is zero-mean, spatially uncorrelated, white Gaussian measurement noise with covariance $E[\mathbf{v}_k \mathbf{v}_k^T] = \mathbf{R_k}$. Such a model could represent a target tracking application where the state vector is the target state and the output is the bearing angle and rate or the model could represent a distributed parameter system where the unknown states are diffusion coefficients and the outputs are concentration levels of some field. Finally, assume some unbiased a priori statistical information about $\mathbf{x}_0$ is given with estimate error covariance $E[(\overline{\mathbf{x}}_0 - \mathbf{x}_0)(\overline{\mathbf{x}}_0 - \mathbf{x}_0)^T] = \mathbf{P}_0$ where $\overline{\mathbf{x}}_k$ is the state estimate at sample time k. Given the initial estimate $\overline{\mathbf{x}}_0$ and measurement history $\mathbf{Z}_k = [\mathbf{z}_0, \cdots, \mathbf{z}_k]$ where $\mathbf{z}_k = [\mathbf{z}_k^1, \cdots, \mathbf{z}_k^n]$, the estimate $\overline{\mathbf{x}}_k$ and its covariance matrix $\mathbf{P}_k$ can be determined.

Estimation of the state vector $\overline{\mathbf{x}}_k$ from the measurement history can be achieved many different ways and is beyond the discussion of this paper. However, when the estimate output, and more importantly the posterior probability distribution, is a function of the state history, careful selection of the system input can improve the quality of the estimator performance. Since the estimated state vector $\overline{\mathbf{x}}_k$ is a random variable, the estimation process returns a probability distribution function (pdf) describing $\overline{\mathbf{x}}_k$ and the quality of the estimation is often expressed as the second moment (the covariance matrix) or the Shannon entropy of the pdf. Ideally, optimal sensing is achieved by minimizing the uncertainty (or maximizing the quality) of $\overline{\mathbf{x}}_{k+T}$ at some later time $k+T$. However, it is impossible to calculate the estimated state vector in advance since it is a random variable and it is impractical to simulate the estimation process for many nonlinear problems. Instead, the Cramer-Rao Lower Bound (CRLB) gives the sensitivity of the estimation process at a given state and provides a limit on the best possible result that can be obtained.

The Cramer Rao Lower Bound gives the following limit on the performance of any nonlinear estimator

$$\mathbf{P}_k \geq \mathbf{J}_k^{-1} \tag{10}$$

where $\mathbf{J}_k$ is the Fisher Information Matrix (FIM) defined as

$$\mathbf{J}_{k,ij} = E\Big[-\frac{\delta^2 \ln p(z,x)}{\delta x^i \delta x^j}\Big] \tag{11}$$

where $\mathbf{J}_{k,ij}$ is the entry in the i^{th} row and j^{th} column of $\mathbf{J}_k$, $p(z,x)$ is the joint probability density function of (z,x), and the expectation E is with respect to both $z$and x. For an unknown, deterministic variable the FIM at time $k+1$ can be found by the recursive relationship [14]

$$\mathbf{J}_{k+1} = \mathbf{\Phi}_k^{-1} J_k \mathbf{\Phi}_k + \mathbf{J_z} \tag{12}$$

where

$$\mathbf{J}_z = \sum_{i=1}^{n} \mathbf{H}_{k,i}^T \mathbf{R}_{k,i}^{-1} \mathbf{H}_{k,i} \tag{13}$$

$$\mathbf{\Phi}_k = \frac{\delta \mathbf{f}}{\delta \mathbf{x}}(\mathbf{x}_k, \mathbf{u}_k) \tag{14}$$

$$\mathbf{H}_{k,i} = \frac{\delta \mathbf{h}}{\delta \mathbf{x}}(\mathbf{x}_k, \mathbf{u}_{k,i}) \tag{15}$$

and n refers to the number of independent sensors measuring the system.

For the case of no prior information ($\mathbf{J}_0 = \mathbf{0}$), a static target ($\mathbf{\Phi}_k = \mathbf{I}$), and the measurement equation given by Eq. 9, the FIM becomes

$$\mathbf{J}_k = \sum_{j=1}^{k} \sum_{i=1}^{n} \begin{bmatrix} \frac{c^2\theta_i}{\sigma_r^2} + \frac{s^2\theta_i}{r_i^2\sigma_\theta^2} & \frac{c\theta_i s\theta_i}{\sigma_r^2} - \frac{c\theta_i s\theta_i}{r_i^2\sigma_\theta^2} \\ \frac{c\theta_i s\theta_i}{\sigma_r^2} - \frac{c\theta_i s\theta_i}{r_i^2\sigma_\theta^2} & \frac{s^2\theta_i}{\sigma_r^2} + \frac{c^2\theta_i}{r_i^2\sigma_\theta^2} \end{bmatrix} \tag{16}$$

where $c\theta_i$ and $s\theta_i$ represent $\cos\theta_i$ and $\sin\theta_i$, respectively.

B. Shannon Capacity

The well known Shannon-Hartley Law [25] states that the Shannon channel capacity $C_{ij}(\mathbf{p}_i, \mathbf{p}_j)$, which is the theoretical maximum rate at which information passes error free over a channel from node j at position $\mathbf{p}_j$ to node i at position $\mathbf{p}_i$, is equal to

$$C_{ij}(\mathbf{p}_i, \mathbf{p}_j) = B \log_2 \left(1 + S_{ij}(\mathbf{p}_i, \mathbf{p}_j)\right), \tag{17}$$

where B is the bandwidth of the channel and $S_{ij}(\mathbf{p}_i, \mathbf{p}_j))$ is the signal-to-noise-and-interference ratio of the channel. Like the FIM, the Shannon capacity represents a theoretical upper bound on the capacity of a communication channel. However, modern coding techniques can come to within fractions of a dB of achieving this rate. The Shannon-Hartley Law Eq. (17) is a continuous abstraction to the real channel which uses discrete power levels and can be used to predict (and therefore plan) the performance of a wireless communication link.

Signal power generally decreases with increasing distance, so the Shannon-Hartley law captures the relationship between distance and communication rate when combined with the empirical radio propagation model. Combining the empirical radio model with the Shannon-Hartley Law yields a function for channel capacity that is now dependent on node positions

$$C_{ij}(\mathbf{p}_i, \mathbf{p}_j) = B \log_2 \Big(1 + \frac{P_0 d_0}{P_n d_n} \Big(\frac{|\mathbf{p}_i - \mathbf{p}_n|}{|\mathbf{p}_i - \mathbf{p}_j|}\Big)^{\alpha}\Big). \tag{18}$$

This relationship provides a provides an information-theoretic approach to integrating communication objectives into a robot sensor network control framework that moves beyond the traditional graph representation. Using the Shannon-Hartley Law, communication performance, e.g. link throughput, can be included as an objective function in motion planning algorithms or as a constraint due to information processing demands. Shannon capacity captures the dependence of the channel throughput on node separation and environmental effects such as noise. Planning algorithms can be developed that not only consider connectivity but also data rates and delay. Finally, use of the Shannon Capacity to model communication also enables integration of data ferrying or data mule concepts into a single framework [29].

C. Expected Information Rate

The expected information rate (EIR) metric combines Fisher Information and Shannon Capacity to model the total information gained by a distributed robot sensor network. Wireless networked communication is modeled using the Shannon Capacity to estimate the throughput of a link between two robot nodes. Specifically, given two robot nodes, the throughput of the link from node j to node i is given by

$$T_{ij}(\mathbf{p}_i, \mathbf{p}_j) = k_0 \cdot k_n(N_{ij}) \cdot C_{ij}(\mathbf{p}_i, \mathbf{p}_j) \tag{19}$$

where C_{ij} is the position-dependent Shannon capacity, k_0 is a scaling factor that accounts for the error correction coding and is typically taken to be $k_0 = 1$, and $k_n(N_{ij})$ is a scaling factor that depends on the number of other nodes in the neighborhood N_{ij} of node j and node i. This second scaling factor accounts for the collision avoidance procedures used at the media access control layer of the typical network hierarchy. Standard network protocols like IEEE 802.11b (WiFi) use media access control to ensure that surrounding nodes do not interfere with the communication. These protocols reduce the throughput of a multi-hop chain relative to the single link capacity. These collision avoidance protocols also make wireless communication a fundamentally asynchronous process as only one node in the neighborhood can transmit at a time. The scale factor k_n can be as high as 1 when only two nodes communicate directly or as low as 1/6 for dense multi-hop networks [30].

For a base robot b_i located at position $\mathbf{p}_i$ the expected information rate from data produced by robot r_j at position $\mathbf{p}_j$ of a target at position $\mathbf{p}_t$ is determined by combining throughput (Eq. 19) with the measurement component of the FIM (Eq. 13). In particular, if b_{data} bytes of data are transmitted, then the expected information rate is

$$\dot{\mathbf{I}}_i = \frac{T_{ij}(\mathbf{p}_i, \mathbf{p}_j)}{b_{data}} \cdot \mathbf{J}_z(\mathbf{p}_j, \mathbf{p}_t). \tag{20}$$

The first term on the right hand side gives the expected rate at which messages can be received while the second gives the amount of information about the estimation process received per message. When measurements are taken and transmitted by n robots the EIR becomes

$$\dot{\mathbf{I}}_i = \sum_{j=1}^{n} \frac{T_{ij}(\mathbf{p}_i, \mathbf{p}_j)}{b_{data}} \cdot \mathbf{J}_z(\mathbf{p}_j, \mathbf{p}_t). \tag{21}$$

Finally, to convert the EIR, which is a matrix, into a scalar metric for optimization, we take the D-optimality (determinant) criterion [28]

$$J_{eir,i} = det(\dot{\mathbf{I}}_i). \quad (22)$$

The EIR as defined by Eq. 21 requires centralized computation since the total information rate is based on the sum of individual measurement components. Although contribution of each individual robot can be computated locally, the net value can only be assessed based on the final summation. In order to generate a decentralized solution, a sub-optimal approach is needed. In this case, knowing the location of the base node, individual robots can calculate the D-optimality criterion of the EIR assuming they were the only nodes. In other words, the local criterion for the j^{th} robot is

$$J_{eir,ij} = \frac{T_{ij}(\mathbf{p}_i, \mathbf{p}_j)}{b_{data}} \cdot det(\mathbf{J}_z(\mathbf{p}_j, \mathbf{p}_t)). \quad (23)$$

and the total objective function is

$$J_{d-eir,i} = \sum_{j=1}^{n} \mathbf{J}_{eir,ij}. \quad (24)$$

IV. Application of the EIR to Control of Robot Sensor Networks

Several applications of robot sensor network control using the expected information rate are given in this section. For each example, the EIR is calculated based on true state information for robot and target positions. In practice, these variables are all estimated from sensor data. However, application of the EIR with true state values provides insight into the algorithms.

A. Bearings-only Localization

Consider optimal sensor placement for planar stand-off bearings-only localization of a stationary target by two cooperating robots capable of motion, sensing, and communication ($S = M = C = R$ with $|C| = 2$). It is well known that the optimal configuration for this problem is to position the robots 90 degrees apart relative to the target on a circle with radius equal to the minimum standoff distance [26]. This result only considers the information gained at a single sample time and does not consider the update rate achievable by the communication system.

To calculate the EIR for this scenario, it is assumed that each vehicle transmits data to the other and fusion only occurs when data is received from the other robot. In this case, either robot can be considered the base robot and the EIR becomes

$$\dot{\mathbf{I}} = \frac{B}{b_{data}} \cdot \log_2\left(1 + \frac{P_0 \cdot d_0}{|\mathbf{p}_1 - \mathbf{p}_2|^{\alpha}}\right) \cdot \sum_{i=1}^{2} \begin{bmatrix} \frac{c^2\theta_i}{\sigma_r^2} + \frac{s^2\theta_i}{r_i^2\sigma_\theta^2} & \frac{c\theta_i s\theta_i}{\sigma_r^2} - \frac{c\theta_i s\theta_i}{r_i^2\sigma_\theta^2} \\ \frac{c\theta_i s\theta_i}{\sigma_r^2} - \frac{c\theta_i s\theta_i}{r_i^2\sigma_\theta^2} & \frac{s^2\theta_i}{\sigma_r^2} + \frac{c^2\theta_i}{r_i^2\sigma_\theta^2} \end{bmatrix} \quad (25)$$

Since the range from each robot to the target is specified, the D-optimality criterion of Eq. 25 is a function of the angular separation of the two robots relative to the target only (not shown).

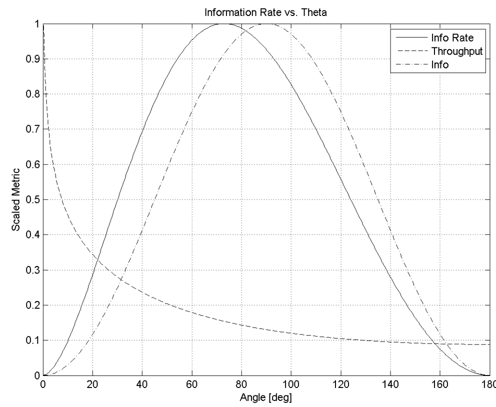

Fig. 1. Throughput, det(FIM), and EIR for $\alpha = 2.0$

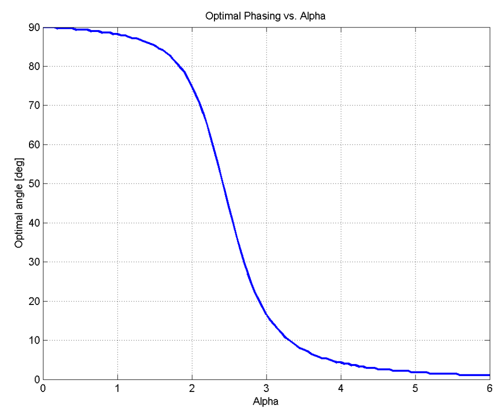

Fig. 2. Simulation Results

Figure 1 shows plots of the EIR metric (Eq. 25) for the stand-off bearings-only geolocalization problem, the D-optimality criterion of the FIM, and the throughput of the link between the two vehicles as a function of their relative angular separation. The dashed lines show the inherent tradeoff between sensor information, which improves as separation increases to an optimal at 90 degrees, and communication, which monotonically falls off as the separation angle (and distance) increases. The combined effect (solid line) is to reduce the optimal separation relative to the case when communication is ignored. Figure 2 shows how the optimal separation varies as a function of the radio decay exponential α. As α is decreased, radio propagation and hence throughput decreases more as a function of distance. Since the information gained by the sensing is unchanged, the robots must stay closer together in order to optimize the overall epected information rate.

B. Optimal Sensor Placement for Target Tracking with Single Base Station

The second application of the EIR metric considers target tracking by a group of robots with a single base station. A multicast communication topology is assumed such that every node communicates directly with the base node r_1. We assume

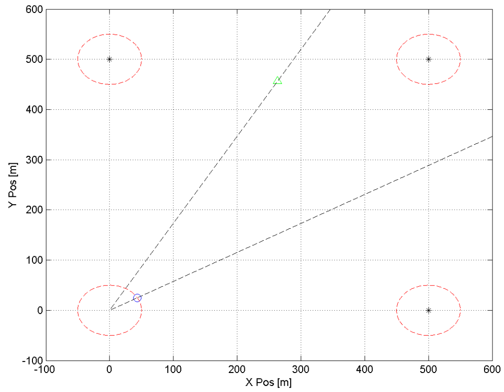

Fig. 3. Optimal positions for α = 2.0

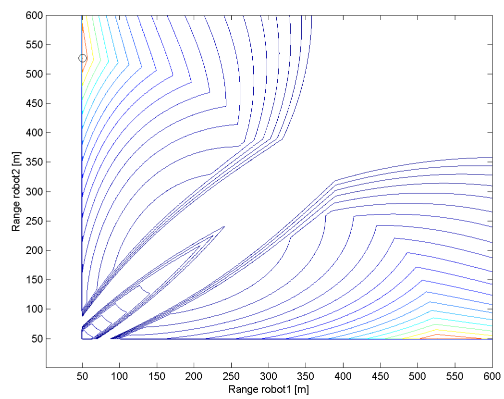

Fig. 4. Contour plot of EIR

a total of n robots using range and bearing sensors to track m targets. A standoff distance of $r_{off} = 50$ meters is imposed between the robots and all targets and base robots.

For the case of multiple robots, the EIR for a single target becomes

$$\dot{\mathbf{I}}_t = \sum_{i=2}^{n} \frac{B}{b_{data}} \cdot \log_2 \left(1 + \frac{P_0 \cdot d_0}{|\mathbf{p}_1 - \mathbf{p}_i|^{\alpha}}\right) \cdot \begin{bmatrix} \frac{c^2\theta_i}{\sigma_r^2} + \frac{s^2\theta_i}{r_i^2\sigma_\theta^2} & \frac{c\theta_i s\theta_i}{\sigma_r^2} - \frac{c\theta_i s\theta_i}{r_i^2\sigma_\theta^2} \\ \frac{c\theta_i s\theta_i}{\sigma_r^2} - \frac{c\theta_i s\theta_i}{r_i^2\sigma_\theta^2} & \frac{s^2\theta_i}{\sigma_r^2} + \frac{c^2\theta_i}{r_i^2\sigma_\theta^2} \end{bmatrix}. \quad (26)$$

For the case of multiple targets the optimization objective is

$$J_{m-eir} = \min[\det \dot{\mathbf{I}}_1, \cdots, \det \dot{\mathbf{I}}_t, \cdots, \det \dot{\mathbf{I}}_m] \quad (27)$$

and we wish to find

$$[\mathbf{p}_1, \cdots, \mathbf{p}_i, \cdots, \mathbf{p}_n] = \arg\max J_{m-eir}. \quad (28)$$

Note, the optimization problem defined in Eq. 28 includes the position of the base node. This position could just as easily be fixed.

The first example considers a scenario with a fixed base at the origin, three targets at positions [500, 0], [0, 500], and

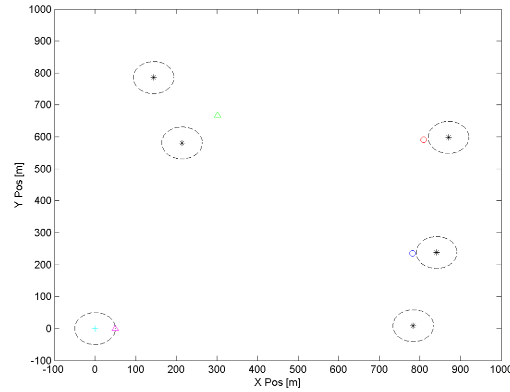

Fig. 5. Optimal position of 4 robots tracking 5 targets

[500,500] meters, and two robots. In order to simplify the problem the positions of the robots are restricted to lie along the lines passing through the origin with slopes $\tan(\pi/6)$ and $\tan(\pi/3)$. The optimization problem is solved using the fmincon function in Matlab. Figure 3 shows the optimal position for this scenario. One robot is close to the base station in order to minimize communication while the other moves close to two of the targets in order to improve sensing. Figure 4 shows a contour plot of the EIR for this case.

A second scenario considers 5 randomly distributed targets, 4 communication robots, and 1 base robot positioned at the origin. The same standoff constraints as the previous example are still applied. Figure 5 shows the resulting optimal positions. In this case 1 robot stays near the base (lower left), two robots are placed at the standoff distance away from two targets (right side) and the last robot is placed between two targets (top left).

C. Optimal Communication Chain or Relay

A third application of the information-theoretic approach described here is optimal chaining or relaying using controlled mobility of the robot sensor network. In this scenario, the robot sensor network consists of one base node and one sensing node whose motion control strategies are not known or influenced by the rest of the network. The remaining robots are communication nodes whose goal is to self-organize to form a chain that provides an optimal multi-hop link between the base and sensing nodes.

The end-to-end throughput from source node r_1 to base node r_n is limited by the lowest throughput of any node, hence $T_{1,n} = \min[T_{1,2}, T_{2,3}, \cdots, T_{n-1,n}]$. When transmission power of all nodes is equal $T_{i,j} = T_{j,i}$ and round trip throughput is equivalent to $\frac{T_{1,n}}{2}$. With uneven power transmission, throughput between two links is asymmetric, i.e. $T_{i,j} \neq T_{j,i}$. Thus, in order to accomplish optimal chaining in this scenario, a new variable $R_{i,j}$ called the node throughput (as opposed to link throughput) is defined $R_{i,j} = \min[T_{i,j}, T_{j,i}]$ and the round trip throughput $R_{1,n} = \min[R_{1,2}, R_{2,3}, \cdots, R_{n-1,n}]$.

The mobility of the intermediate relay nodes can now be

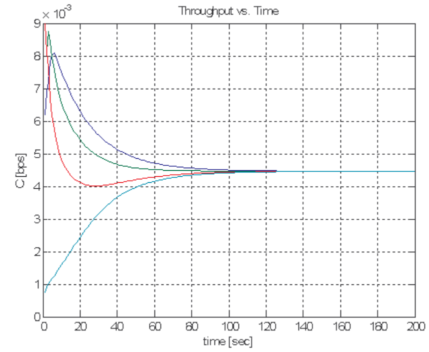

Fig. 6. Link throughput versus time

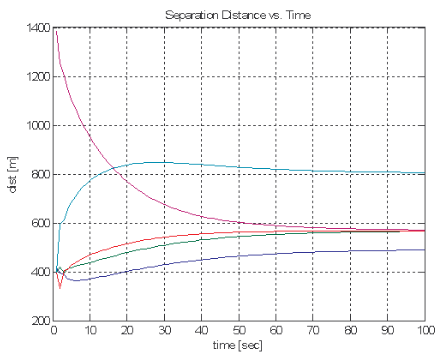

Fig. 7. Separation distance versus time

determined to optimize the round trip performance of this chain using the EIR. In this case, an ordering of the robots is assumed and a decentralized scheme results in which each successive robot r_i treats the previous robot r_{i-1} and the next robot r_{i+1} as base nodes. Since the information content is independent of robot motion, the EIR reduces to a function of the local node throughputs $R_{i,i\pm1}$.

Consider a one-dimensional example in which four communication robots provide a link between two stationary robots where robots have relative transmission powers $P_{0,i} = (3/4, 1, 1, 4, 2, 1)$. Assuming the end robots are stationary, a linear chain that provides the optimal round-trip throughput can be achieved by the decentralized velocity control law

$$u_i = \dot{x}_i = -\lambda(R_{i,i-1} - R_{i,i+1}) \quad \forall i = [2, \cdots, n-1]. \quad (29)$$

Figure 6 shows that this decentralized control scheme (Eq. 29) balances throughput across the robot team, maximizing the minimum value over all vehicles, and hence maximizing the total throughput. Since the transmission powers of each robot is different, the final sepration distances are not equal (Fig. 7) Thus, a position-based solution that simply distributes the robots evenly between source and destination would be sub-optimal. The decentralized control scheme presented above can be extended to more realistic scenarios involving planar or three-dimensional motion, complex vehicle models, and environmental effects such as noise. If the structure of the SNR (and therefore) throughput field is know, decentralized

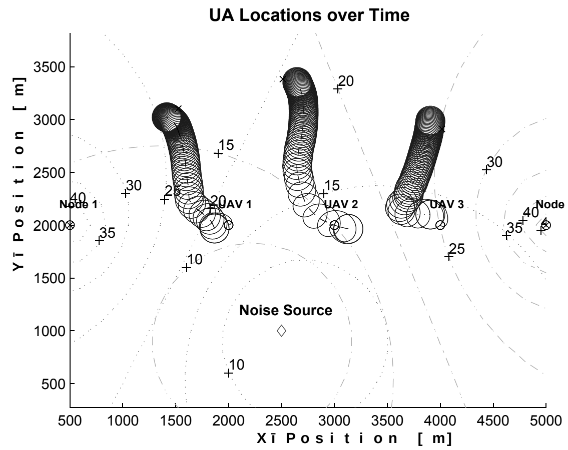

Fig. 8. Link throughput versus time

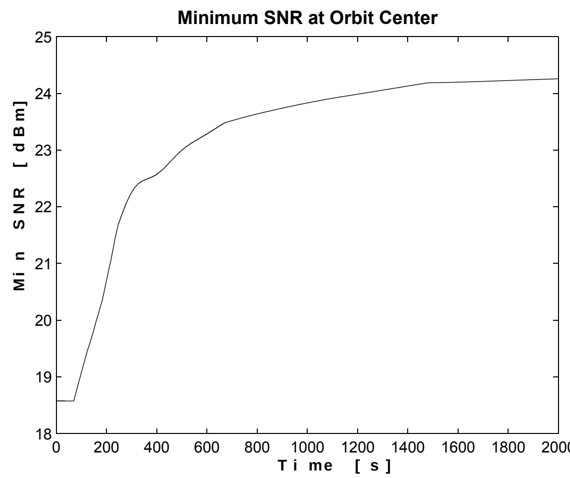

Fig. 9. Separation distance versus time

gradient ascent can be used to optimize the EIR. However, in general the RF environment and hence SNR field are unknown. In this case, concepts from extremum seeking (ES) control can be used to adaptively optimize the EIR for the chain.

Figure 8 and Figure 9 show results from an example using unmanned aircraft (UA) to provide an optimal communication chain in the presence of radio noise [6]. In this example, the UA have maximum and minimum velocity bounds that prevent them from stopping at a single location. Each UA is equipped with a transceiver that measures the SNR at a single point, requiring estimation of the SNR field gradient and the noise source is a malfunctioning node that continually broadcasts, causing position-dependent interference. The orbital motion of the UA is used to estimate the gradient of the local node throughput and drive the motion of each individual UA. Figure 8 shows the positions of the UA relative to the source, destination, and interfering nodes. At the beginning of the simulation the UA are aligned along the straight line connecting the source and destination nodes. The presence of the noise source causes the chain to bow away from the interference. Figure 9 shows how the estimate of the global throughput improves as a function of time using this approach. More details of the decentralized ES controller applied here can be found in [6].

V. Conclusion

This paper presented an information-theoretic framework to integrate sensing and communication for planning of robot sensor networks. Example problems showed how inclusion of communication performance into active sensing problems changes the optimal configuration relative to the case where it is not included. The expected information rate (EIR) metric was applied to a variety of problems, demonstrating its effectiveness for different problems.

Future work will address the deployment of robots into optimal configurations and use of the expected information rate as the basis of motion control strategies. The current work presented here only considered static placement in the sensing applications. Additional future work will address multi-hop communication within the robot sensor network, seeking to minimize information flow through the network.

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