An Integration-Enhanced ZNN Approach for Chaotic Combination Synchronization with External Disturbances*

Chenfu Yi(✉), Mingdong Zhu, Jingjing Chen, and Jinghui Peng

School of Cyber Security, Guangdong Polytechnic Normal University, 510635 Guangzhou, China
chenfuyi@gpnu.edu.cn

Abstract. Robust combination synchronization has garnered extensive attention within the domains of science and engineering, particularly in the realm of secure communication in recent years. In contrast to conventional single master-single slave system, the introduction of multiple variables and intricate combination methods in combination synchronization significantly increases the complexity of decryption, boosting the confidentiality and security of signal transmission. However, due to the ubiquity of time-varying external interference, the synchronization results of ordinary methods are not ideal or may even be divergent. In view of these challenges, this paper proposes the integration-enhanced zeroing neural network (IEZNN) model and its associated controller to achieve robust combination synchronization of chaotic systems. Theoretical research fully substantiate the effectiveness of the proposed IEZNN approach and its related controller. Additionally, the numerical findings show that, in comparison to the conventional zeroing neural network (CZNN) method, the controller designed by IEZNN model have remarkable anti-interference performance in the presence of external time-varying disturbances.

Keywords: Zeroing neural network · Chaotic combination synchronization · Chaotic systems · External disturbances · Robustness

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1 Introduction

At the end of the 20th century, Pecora and Carroll [12] accomplished a watershed moment when they realized the synchronization control of chaotic systems, kicking off chaotic synchronization research. The concept of chaotic synchronization refers to the process of achieving consistency between two or more chaotic systems by modifying specific motion characteristics while under the influence of coupling or external forces. It is extensively applied in a variety of domains, including secure communication [5,7], biomedicine [10], power electronics [4], signal processing [8], and economics [6]. For instance, Vaseghi et al. [17] used the adaptive sliding mode control method for chaos synchronization and applied it to secure communication in wireless sensor networks. Additionally, Lin et al. [11] have achieved significant breakthroughs in the field of secure communication, attaining chaos synchronization with the aid of controllers created utilizing brain-limited neural networks.

However, when employed for secure communication, the conventional single master-single salve system synchronous control architecture is relatively straightforward and easily cracked. Therefore, Sun et al. [16] achieved combination synchronization between two master systems and two slave systems based on the Lyapunov stability theorem and adaptive control. For the purpose of using combination synchronization in secure communication, the transmission signal can be split into several pieces and loaded into various master systems. Alternatively, time can be divided into various intervals, and signals from those intervals can be loaded into various master systems. [13]. In addition, chaotic combination synchronization has an advantage over the conventional single master-single slave system due to the use of numerous variables and sophisticated combination techniques. Decrypting becomes increasingly challenging because it requires access to essential state variables and their corresponding combination techniques. Therefore, compared to earlier transmission techniques, using combination synchronization for signal transmission improves confidentiality.

It is worth noting that there are numerous control strategies that can effectively address the issue of chaotic combination synchronization. Specifically, the lyapunov stability theorem and adaptive control technology [14,16], the active backstepping design method [13], and the sliding mode control technology [15]. Moreover, Li et al. [9] used the conventional zeroing neural network (CZNN) to construct a straightforward chaotic synchronization controller while taking parameter disturbances, model uncertainty, and outside disturbances into consideration.

While the aforementioned methods, such as the CZNN method, have found widely use in practical engineering, research on the robustness of the CZNN method in chaotic combination synchronization is still in its early stages. Proverbially, chaotic systems inevitably encounter some external disturbances, such as bias errors, interactions with environment, electromagnetic interference in circuit systems, and noise during signal transmission [3]. These time-varying external disturbances may affect the stability and accuracy of the relevant controllers for synchronizing chaotic systems, which can result in the failure of the synchronization process. [2]. To accomplish chaotic combination synchronization in practical applications, effective and reliable approaches to suppress time-varying external disturbances are therefore urgently needed.

In order to mitigate external noise interference during the synchronization procedure, unlike the research based on the CZNN approach [9] or research foucusing on rate of convergence [1], this article uses the IEZNN model and devises a corresponding controller to achieve robust combination synchronization of chaotic systems in the presence of time-varying external disturbances. Moreover, the usefulness and superiority of the IEZNN model and its associated controller in practical applications are further confirmed by numerical experiments that include three examples of combination synchronization.

The remainder of this essay is organized as follows. In Sect. 2, the IEZNN model and its associated controller are described, together with a preliminary formula for the synchronization problem between two chaotic systems under the impact of time-variant external disturbances. Besides, some lemmas are presented in Sect. 3. In Sect. 4, simulation experiments were carried out using three examples of combination synchronization in the presence of time-varying external noise, and the IEZNN model was compared with the CZNN model. The complete material is summarized in Sect. 4.1. Before wrapping up this introduction, the letter's main contributions are given below.

i) Differing from traditional single master-single salve system synchronization, chaotic combination synchronization exhibits greater confidentiality in secure communication owing to its multiple variables and complex combination methods.
ii) This article proposes the IEZNN model and designs associated controller to synchronize chaotic systems in the presence of time-varying external disturbances, overcoming the shortcomings of traditional techniques that fall to attain synchronization.
iii) Under the influence of external disturbances, the effectiveness and superiority of the IEZNN model and its associated controller in practical applications are validated through numerical research that include three examples of chaotic combination synchronization.

2 Preliminary and Approaches

In this section,the expression of the synchronisation issue between master and slave chaotic systems with noise is introduced. Moreover, the relevant synchronous controller designed with IEZNN model is proposed.

2.1 Synchronization of Chaotic Systems

Chaos synchronization refers to the consistency and stability of the trajectories of the master chaotic system and the slave chaotic system. In the presence of external noise influences, the differential equation of the master chaotic system can be defined as:

$$\begin{cases} \dot{y}_{m1}(t) = z_{m1}(y_{m1}(t)) + \beta_1, \\ \dot{y}_{m2}(t) = z_{m2}(y_{m2}(t)) + \beta_2, \\ \dot{y}_{m3}(t) = z_{m3}(y_{m3}(t)) + \beta_3, \end{cases} \tag{1}$$

where $y_m(t) = [y_{m1}(t), y_{m2}(t), \cdots, y_{mn}(t)] \in \mathbb{R}^n$ is the state vector of the master chaotic system; $z_m(\cdot) : \mathbb{R}^n \to \mathbb{R}^m$ represents the nonlinear-mapping function and β stands for the external noise and acting during synchronization. Similarly, the general expression for the chaotic system with controller input can be expressed as follows:

$$\begin{cases} \dot{y}_{s1}(t) = z_{s1}(y_{s1}(t)) + u_1(t), \\ \dot{y}_{s2}(t) = z_{s2}(y_{s2}(t)) + u_2(t), \\ \dot{y}_{s3}(t) = z_{s3}(y_{s3}(t)) + u_3(t), \end{cases} \tag{2}$$

where $y_s(t) = [y_{s1}(t), y_{s2}(t), \cdots, y_{sn}(t)] \in \mathbb{R}^n$ denotes the state vector of the slave chaotic system; $z_s(\cdot) : \mathbb{R}^n \to \mathbb{R}^m$ stands for the nonlinear-mapping function and $u(t) = [u_1(t), u_2(t), \cdots, u_n(t)]^T \in \mathbb{R}^n$ represents controller and acts on the synchronization process.

Complete synchronization is the earliest proposed method in chaotic synchronization. Under the control of a controller, two chaotic systems with distinct initial conditions gradually converge in their trajectories amidst external noise interference. Therefore, the controller's design should guarantee that the slave system can follow the master system, ultimately achieving synchronization. In other words, let the error expression $\varepsilon(t) = y_m(t) - y_s(t)$ approaches to 0.

2.2 Controller Design

With the objective of achieving chaotic synchronization and accelerate convergence time, this paper presents an IEZNN-based controller to mitigate external noise during the synchronization process. The specific design steps are delineated as follows.

To attain the stated objective, the controller must fulfill the subsequent conditions:

$$\begin{cases} \lim_{t\to\infty} |y_{m1}(t) - y_{s1}(t)| = 0, \\ \lim_{t\to\infty} |y_{m2}(t) - y_{s2}(t)| = 0, \\ \lim_{t\to\infty} |y_{m3}(t) - y_{s3}(t)| = 0, \end{cases} \tag{3}$$

As was previously noted, the goal is to make the problem's error converge to 0 and to some extent muffle outside noise. Using the IEZNN design technique, we create the corresponding error function and design formula. Taking time-varying matrix inversion problem $A(t)X(t) = I$ as an example, the design procedure of IEZNN to solve this problem is outlined below:

1) To make the right side of the equation equal to 0, we can obtain the following equation: $A(t)X(t) - I = 0$.

2) An errot function $E(t) = A(t)X(t) - I$ can be constructed and it is the core design of the ZNN
3) To make the equation hold, the error function must tend to 0. One design formula $\dot{E}(t) = -\gamma E(t) - \lambda \int_0^t E(\delta)d\delta$ can be designed to ensure $E(t)$ converge to 0. Besides, $\gamma > 0$ and $\lambda > 0$ are employed to regulate the convergence rate of the IEZNN model.

Inspired by the aforementioned IEZNN design process, the error function for the complete synchronization problem can be conceived to

$$\begin{cases} \varepsilon_1(t) = y_{m1}(t) - y_{s1}(t), \\ \varepsilon_2(t) = y_{m2}(t) - y_{s2}(t), \\ \varepsilon_3(t) = y_{m3}(t) - y_{s3}(t), \end{cases} \tag{4}$$

For the purpose of achieving finite-time convergence for each component of the error function $\varepsilon(t)$, give the following IEZNN design formulation:

$$\dot{\varepsilon}_i(t) = -\gamma\varepsilon_i(t) - \lambda \int_0^t \varepsilon_i(\delta)d\delta \tag{5}$$

Considering the existence of external noise interference, the design of the IEZNN model is:

$$\dot{\varepsilon}_i(t) = -\gamma\varepsilon_i(t) - \lambda \int_0^t \varepsilon_i(\delta)d\delta + \beta_i \tag{6}$$

With the derivation of Eq.(4), we can obtain

$$\begin{cases} \dot{\varepsilon}_1(t) = \dot{y}_{m1}(t) - \dot{y}_{s1}(t), \\ \dot{\varepsilon}_2(t) = \dot{y}_{m2}(t) - \dot{y}_{s2}(t), \\ \dot{\varepsilon}_3(t) = \dot{y}_{m3}(t) - \dot{y}_{s3}(t), \end{cases} \tag{7}$$

then put Eqs.(1) and (2) into Eq.(7) and it will evolve into

$$\begin{cases} \dot{\varepsilon}_1(t) = z_{m1}(y_{m1}(t)) + \beta_1 - z_{s1}(y_{s1}(t)) - u_1(t), \\ \dot{\varepsilon}_2(t) = z_{m2}(y_{m2}(t)) + \beta_2 - z_{s2}(y_{s2}(t)) - u_2(t), \\ \dot{\varepsilon}_3(t) = z_{m3}(y_{m3}(t)) + \beta_3 - z_{s3}(y_{s3}(t)) - u_3(t). \end{cases} \tag{8}$$

Utilizing Eqs.(6) and (8), the controller for the IEZNN model can be derived, and its expression is:

$$u_i(t) = z_{mi}(y_{mi}(t)) - z_{si}(y_{si}(t)) + \gamma\varepsilon_i(t) + \lambda \int_0^t \varepsilon_i(\delta)d\delta \tag{9}$$

3 Performance Analysis

Two lemmas about the IEZNN technique are offered in this subsection, showing their ability to synchronize chaotic systems even in the presence of external time-varying disturbances.

Lemma 1. *In the context of multiple chaotic systems, starting from distinct initial state $y_m(0) \neq y_s(0)$, with initial error $\varepsilon(t) \neq 0$, the tajectory of the salve system's state vector $y_s(t)$ will synchronize with the trajectory of the master system's state vector $y_m(t)$ at time $t \geq 0$ with external noise if it satisfies*

$$\lim_{t \to \infty} |y_m^i(t) - y_s^i(t)| \leq \xi, i = 1, 2, \cdots, n, \tag{10}$$

for the arbitrary small $\xi > 0$.

Lemma 2. *Assuming that the chaotic system is affected by an additive disturbance β_i, and regulation parameters $\gamma > 0$ and $\lambda > 0$, the slave system starting from different initial position $y_m(0) \neq y_s(0)$, the IEZNN model is capable of achieving finite-time convergence when $y_m(t)$ converges to $y_s(t)$ within a finite-time interval T_f.*

4 Illustrative Verification

This chapter demonstrates the superiority of the IEZNN model through numerous simulation examples, including the one master chaotic system with three slave chaotic systems, two master chaotic systems with two slave chaotic systems, and three master chaotic systems with one slave chaotic system. To further illustrate the robustness of the IEZNN model, four types of noise are introduced as follows:

1) Constant noise, which is expressed as $\beta_i = c$.
2) Exponential noise, as a representative of dynamical bounded vanishing noise, which is formulated as $\beta_i = \beta'_m \exp(-(1+t)) + c$.
3) Random noise, which is an uniform random noise in $[-\beta_m, \beta_m]$.
4) Sinusoidal noise, as a representative of periodic noise, which is described as $\beta_i = \sin(\beta'_m + c)$, where c is a constant vector, β_m represents the upper bound of β_i, and β'_m denotes the upper bound of the gradient of β_i.

For the purpose of demonstrating the capability to resist external interference of the IEZNN model, we utilize the CZNN model for comparative analysis.Therefore, we can obtain the controller designed by the CZNN method is:

$$u_i(t) = z_{mi}(y_{mi}(t)) - z_{si}(y_{si}(t)) + \lambda \varepsilon_i(t) \tag{11}$$

with parameter $\lambda \geq 0$.

4.1 Synchronisation Examples

Considering the following four chaotic systems, the expression for the Chen chaotic system is:

$$\begin{cases} \dot{x}_1(t) = a_1(y_1(t) - x_1(t)), \\ \dot{y}_1(t) = -a_2 x_1(t) + a_3 y_1(t) - x_1(t) z_1(t), \\ \dot{z}_1(t) = -a_4 z_1(t) + x_1(t) y_1(t), \end{cases} \tag{12}$$

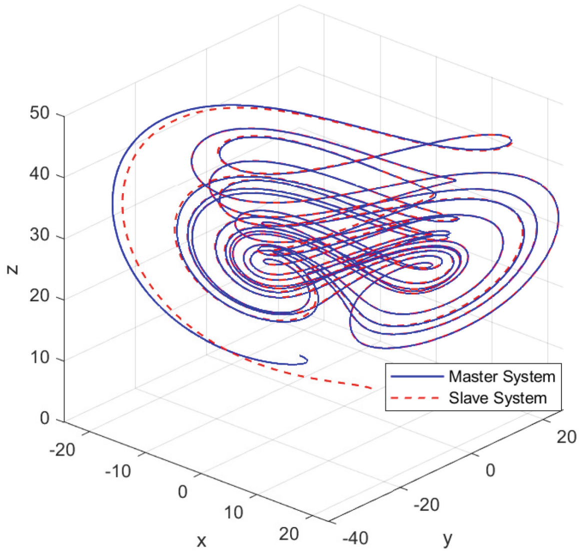


Fig. 1. Three-dimensional synchronization trajectory synthesized by IEZNN model (6) for systems (16) and (17) without noise.

with $a_1 = 35, a_2 = 7, a_3 = 28, a_4 = 3$. The improved Lü chaotic system is expressed as:

$$\begin{cases} \dot{x}_2(t) = b_1(y_2(t) - x_2(t) + y_2(t)z_2(t)), \\ \dot{y}_2(t) = b_2y_2(t) - x_2(t)z_2(t), \\ \dot{z}_2(t) = -b_3z_2(t) + x_2(t)y_2(t), \end{cases} \tag{13}$$

with $b_1 = 35, b_2 = 14, b_3 = 5$. The Lü chaotic system is expressed as:

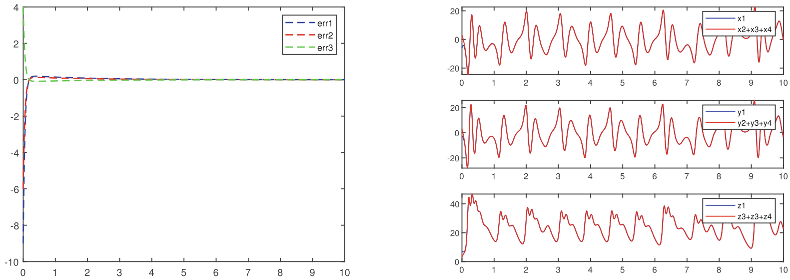
$$\begin{cases} \dot{x}_3(t) = c_1(y_3(t) - x_3(t)), \\ \dot{y}_3(t) = c_2y_3(t) - x_3(t)z_3(t), \\ \dot{z}_3(t) = -c_3z_3(t) + x_3(t)y_3(t), \end{cases} \tag{14}$$

with $c_1 = 36, c_2 = 20, c_3 = 3$. The Zhou chaotic system is expressed as:

$$\begin{cases} \dot{x}_4(t) = d_1(y_4(t) - x_4(t)), \\ \dot{y}_4(t) = d_2x_4(t) - x_4(t)z_4(t), \\ \dot{z}_4(t) = -d_3z_4(t) + x_4(t)y_4(t), \end{cases} \tag{15}$$

with $d_1 = 10, d_2 = 16, d_3 = 1$.

In this section, we have successively selected the synchronization between one master three salve chaotic systems,the synchronization between two master two salve chaotic systems,and the synchronization between three master one salve chaotic systems. Set the synchronization time to 10 s. In addition, adjust the parameters $\lambda = 5$ and $\gamma = 5$.



(a) Error curve (b) Time series diagram of state variables

Fig. 2. The error curve and time series diagram of state variables during the combination synchronization process between systems (16) and (17) without noise.

For this type of synchronization, we first set the initial values of these four systems as (-6, 3, 7), (1, 1, 1), (1, 1, 1), and (1, 1, 1) respectively. The system (12) is designated as the master chaotic system. Under the conditions of additional disturbances, its expression can be represented as:

$$\begin{cases} \dot{x}_1(t) = a_1(y_1(t) - x_1(t)) + \beta_1, \\ \dot{y}_1(t) = -a_2x_1(t) + a_3y_1(t) - x_1(t)z_1(t) + \beta_2, \\ \dot{z}_1(t) = -a_4z_1(t) + x_1(t)y_1(t) + \beta_3, \end{cases} \tag{16}$$

at the same time, the sum of the remaining three chaotic systems is considered as the slave system. Therefore, the expression for the slave chaotic system with controller input is:

$$\begin{cases} \dot{x}_2(t) =& b_1(y_2(t) - x_2(t) + y_2(t)z_2(t) + c_1(y_3(t) \\ & - x_3(t)) + d_1(y_4(t) - x_4(t)) + u_1, \\ \dot{y}_2(t) =& b_2y_2(t) - x_2(t)z_2(t) + c_2y_3(t) - x_3(t) \\ & z_3(t) + d_2x_4(t) - x_4(t)z_4(t) + u_2, \\ \dot{z}_2(t) =& - b_3z_2(t) + x_2(t)y_2(t) + -c_3z_3(t) + \\ & x_3(t)y_3(t) + -d_3z_4(t) + x_4(t)y_4(t) + u_3, \end{cases} \tag{17}$$

The corresponding synchronous simulation results for two nonidentical chaotic systems (16) and (17) using the proposed IEZNN model are depicted in the figure. Specifically, Fig. 1 illustrates the synchronization between systems (16) and (17) in a three-dimensional space without noise in real-time. It can be observed that the two systems quickly synchronize and achieve consistent trajectories at different initial values. Moreover, as shown in Fig. 2(a), the synchronization error between systems (16) and (17) approaches zero. Figure 2(b) displays the state variable curves of the two systems over time, demonstrating a rapid convergence and overlap of their state variable trajectories.

Synchronization Between Two Master Two Salve Chaotic Systems. For this particular form of synchronization, we first set the initial values of these four systems as (8, 5, -7), (1, 1, 1), (1, 1, 1), and (1, 1, 1) respectively. The summation of the two systems (12) and (13) is considered as the master chaotic system. With the existence of additional disturbances, it can be articulated as:

$$\begin{cases} \dot{x}_1(t) = a_1(y_1(t) - x_1(t)) + b_1(y_2(t) - x_2(t) + y_2(t)z_2(t)) + \beta_1, \\ \dot{y}_1(t) = -a_2x_1(t) + a_3y_1(t) - x_1(t)z_1(t) + b_2y_2(t) - x_2(t)z_2(t) + \beta_2, \\ \dot{z}_1(t) = -a_4z_1(t) + x_1(t)y_1(t) + -b_3z_2(t) + x_2(t)y_2(t) + \beta_3, \end{cases} \tag{18}$$

after adding the controller input into the sum of systems (14) and (15), the slave chaotic system can be formulated as follows:

$$\begin{cases} \dot{x}_2(t) = c_1(y_3(t) - x_3(t)) + d_1(y_4(t) - x_4(t)) + u_1, \\ \dot{y}_2(t) = c_2y_3(t) - x_3(t)z_3(t) + d_2x_4(t) - x_4(t)z_4(t) + u_2, \\ \dot{z}_2(t) = -c_3z_3(t) + x_3(t)y_3(t) + -d_3z_4(t) + x_4(t)y_4(t) + u_3, \end{cases} \tag{19}$$

In the absence of noise interference, Fig. 3 depicts the synchronization between two distinct chaotic systems (18) and (19) using the proposed IEZNN model in three-dimensional space. It is obvious that even if the initial values are different, these two systems can quickly synchronize and achieve consistent trajectories. Furthermore, as shown in Fig. 4(a), the synchronization error between systems (18) and (19) converges to zero. Figure 4(b) displays the state variable curves for the two systems over time, and their state variable change curves quickly overlap.

Synchronization Between Three Master One Salve Chaotic Systems. For this type of synchronization, we first set the initial values of these four systems as (9, 7, 5), (1, 1, 1), (1, 1, 1), and (1, 1, 1) respectively. The aggregate of the three systems (12), (13), and (14) is considered the master chaotic system. When subjected to extra disturbances, its representation can be articulated as follows:

$$\begin{cases} \dot{x}_1(t) = a_1(y_1(t) - x_1(t)) + b_1(y_2(t) - x_2(t) \\ \qquad + y_2(t)z_2(t)) + c_1(y_3(t) - x_3(t)) + \beta_1, \\ \dot{y}_1(t) = -a_2x_1(t) + a_3y_1(t) - x_1(t)z_1(t) \\ \qquad + b_2y_2(t) - x_2(t)z_2(t) + c_2y_3(t) \\ \qquad - x_3(t)z_3(t) + \beta_2, \\ \dot{z}_1(t) = -a_4z_1(t) + x_1(t)y_1(t) + -b_3z_2(t) + \\ \qquad x_2(t)y_2(t) + -c_3z_3(t) + x_3(t)y_3(t) + \beta_3, \end{cases} \tag{20}$$

simultaneously, upon incorporating the controller input into the system described by equation (15), the formulation for the chaotic behavior of the slave system can be exhibited as:

$$\begin{cases} \dot{x}_2(t) = d_1(y_4(t) - x_4(t)) + u_1, \\ \dot{y}_2(t) = d_2x_4(t) - x_4(t)z_4(t) + u_2, \\ \dot{z}_2(t) = -d_3z_4(t) + x_4(t)y_4(t) + u_3, \end{cases} \tag{21}$$

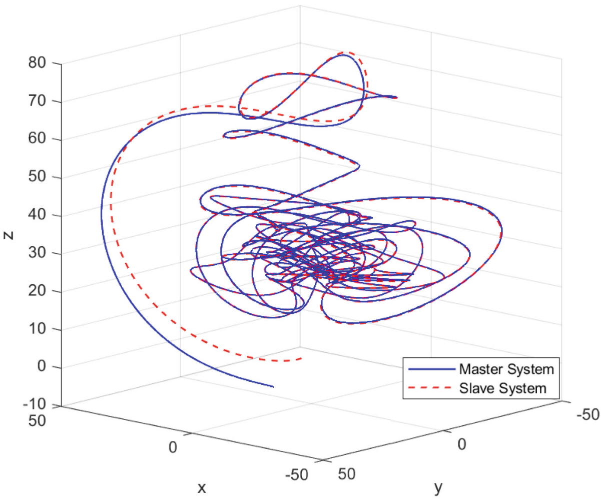
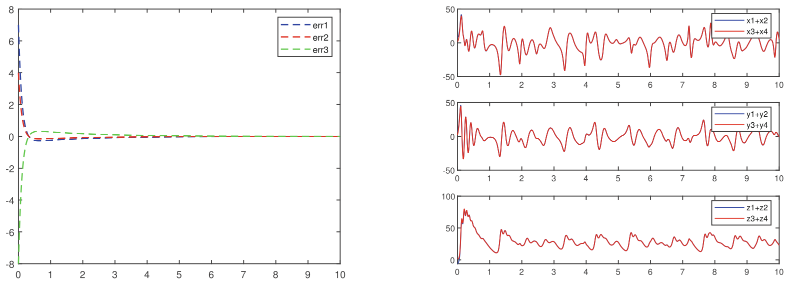


Fig. 3. Three-dimensional synchronization trajectory synthesized by IEZNN model (6) for systems (18) and (19) without noise.



(a) Error curve

(b) Time series diagram of state variables

Fig. 4. The error curve and time series diagram of state variables during the combined synchronization process between systems (18) and (19) without noise.

Excluding external perturbations, the real-time synchronous simulation results for two nonidentical chaotic systems (20) and (21) in a three-dimensional space, employing the proposed IEZNN model, are depicted in Fig. 5. As shown, the two systems quickly synchronize and achieve consistent trajectories at different initial values. Furthermore, as seen in Fig. 6(a), the synchronization error between systems (20) and (21) tends to 0, achieving convergence. Figure 6(b) displays the curves of each state variable of the two systems over time, with their state variable change curves rapidly overlapping.

The above experimental results show that the IEZNN model is effective in solving combination synchronization problems.

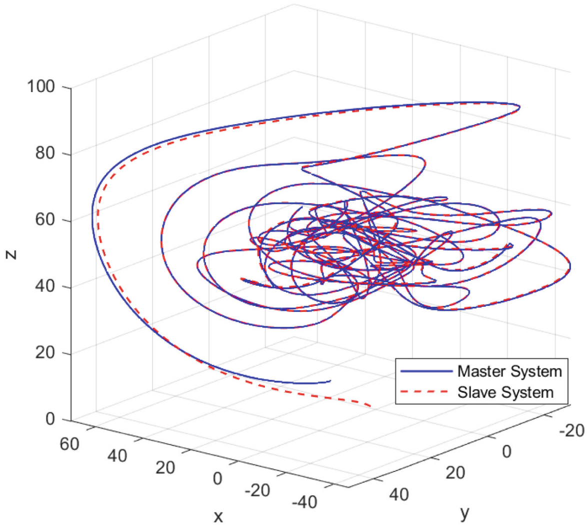
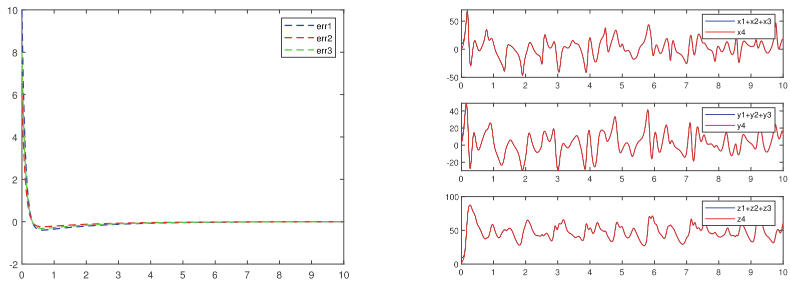


Fig. 5. The three-dimensional synchronization trajectory generated by the IEZNN model (6) for systems (20) and (21) is illustrated in the absence of noise.



(a) Error curve (b) Time series diagram of state variables

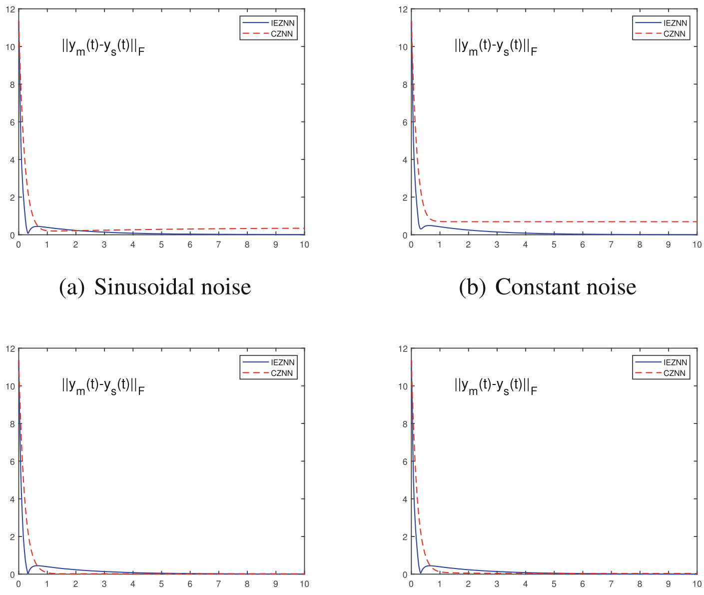
Fig. 6. The error curve and time series diagram of state variables during the combined synchronization process between systems (20) and (21) without noise.

Comparsion with CZNN Model

Without sacrificing generality, take the combination synchronization of two master two salve systems as an example. In this section, we substantiate that the IEZNN model has better performance than the previous CZNN model. Under the same external interference, we conducted combination synchronization experiments employing distinct controllers generated by these two divergent models. And the experimental results are compared and further elaborated. As delineated in Table 1, both the IEZNN and CZNN models exhibit the capability of achieving finite-time convergence without noise. However, when there exist external disturbances, the IEZNN model accomplishes finite-time convergence,

Table 1. Convergence performance of IEZNN model and CZNN model under different noise disturbances.

noise	Non-noise	Constant noise	Random noise	Exponential noise	Sinusoidal noise
IEZNN	3.418×10^{-3}	3.678×10^{-3}	3.237×10^{-3}	3.477×10^{-3}	9.435×10^{-3}
CZNN	8.165×10^{-15}	0.6928	0.0185	0.0346	0.3449



(a) Sinusoidal noise (b) Constant noise

(c) Random noise (d) Exponential noise

Fig. 7. Variations of error norm during the combination synchronization process with different noise using IEZNN and CZNN models.

while the CZNN model is not particularly ideal. This discrepancy signifies the CZNN model's inability to attain combination synchronization under such circumstances. Moreover, as shown in Fig. 7, the IEZNN model exhibits better convergence performance when exposed to various external noise compared with the CZNN model.

5 Conclusion

With the aim of enhancing signal confidentiality during transmission, this paper proposes the IEZNN model and its associated controller for achieving chaotic combination synchronization under the influence of external time-varying noise interference. Theoretical scrutiny reveals the IEZNN model's efficiency in mitigating the impact of external time-varying interference, leading to synchronization errors tend to zero. Contrast this with the CZNN technique, which is prone to external noise interruptions, leading to unsuccessful convergence to zero or even divergence in synchronization results. Furthermore, numerical results show that the IEZNN model outperforms the CZNN approach in terms of anti-interference performance when faced with external time-varying noise.

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