Open-Closed-Loop Iterative Learning Control Based on Differential Evolution Algorithm for Nonlinear System

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Abstract. This paper proposes an iterative learning control problem based on the differential evolution algorithm for optimal control gains. The proposed framework for a nonlinear discrete-time system consists of open-loop ILC component and closed-loop control component, forming an open-closed-loop ILC structure. The inclusion of the open-loop component guarantees the convergence of the ILC tracking error in terms of mathematical expectation. Feedback control accelerates convergence with appropriate gain. The control gain is optimized by the differential evolution algorithm to achieve better system control and faster convergence. After conducting ILC convergence analysis and simulation, the tracking error tends to approach zero in mathematical expectation.

Keywords: Open-closed-loop · differential evolution algorithm · nonlinear discrete-time system · iterative learning control

1 Introduction

Iterative Learning Control (ILC) is a control methodology tailored for continuous systems, focusing on iterative task execution. It involves learning the system's dynamic characteristics through repeatedly performing the same task and optimizes the control algorithm based on the learned information, resulting in more precise control.

The advantages of ILC are that it can repeat learning and optimization for the same task, thus enhancing the precision and reliability of the control system. ILC has found extensive applications across diverse fields in recent years, including robot control and industrial automation. Moreover, ILC doesn't require an exact mathematical model of the system, only sampling of the system's output, making it highly adaptable. Because it doesn't require system to have a precise mathematical representation, it has become a typical control strategy for imprecise systems in [1, 2].

ILC is widely applied in achieving perfect tracking performance within a finite time interval [3–5] by generating control inputs from tracking information in the last iteration. However, relying solely on this approach for updating the current control input

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presents challenges in achieving a satisfactory convergence rate. To enhance convergence speed, this paper introduces a differential evolution algorithm to generate the current control input signal using tracking data from multiple front iterations [6, 7]. The tracking capability of the system is enhanced when utilizing information from multiple past iterations in the update process with the differential evolution algorithm compared to traditional ILC. This improvement is achieved through the skillful design of control gain.

Choosing the optimal control gain is an important issue in ILC design. Based on the above observations, this paper utilizes an differential evolution (DE) algorithm that mimics the biological evolution process to obtain the optimal solution, comparing with traditional ILC, the DE scheme based ILC (DE-ILC) [8, 9]. By designing the coding strategy, population initialization and fitness function of the DE algorithm, the optimal control gain of the dynamic system can be obtained, to minimize the number of iterations required by the algorithm and accelerating convergence, this approach has gained significant traction in addressing a wide range of optimization problems.

2 Problem Formulation

Examine the following discrete-time nonlinear system that performs repeated operations:

$$\begin{cases} x_k(t+1) = f(x_k(t), t) + B(t)u_k(t) \\ y_k(t) = C(t)x_k(t) \end{cases} \tag{1}$$

here, $k \in \{0, 1, 2, ...\}$ and $t \in \{0, 1, \cdots, T_d\}$ represent the iteration index and the time point. $x_k(t) \in R^w$, $u_k(t) \in R$ and $y_k(t) \in R$ denote the state, control input, and output of system (1), separately. $f(\cdot, \cdot) \in R^w$, $B(t) \in R^w$ and $C(t) \in R^{1\times w}$. The desired trajectory for system (1) is $y_d(t) = C(t)x_d(t)$, $t \in \{0, 1, \cdots, T_d + 1\}$, where $x_d(t)$ is the desired state. For the desired trajectory $y_d(t)$, assume that there exists an only desired input $u_d(t) \in R^w$, $t \in \{0, 1, \cdots, T_d\}$, such that:

$$\begin{cases} x_d(t+1) = f(x_d(t), t) + B(t)u_d(t) \\ y_d(t) = C(t)x_d(t) \end{cases} \tag{2}$$

For this discrete-time system (1), the tracking error can be outlined as $e_k(t) = y_d(t) - y_k(t)$, $t \in \{0, 1, \cdots, T_d + 1\}$.

Assumption 1. The starting condition $x_k(0)$ for iterations is subject to random variability, yet its mathematical expectation complies with:

$$E\{\|x_d(0) - x_k(0)\|\} = 0 \tag{3}$$

Lemma 1. Give a difference inequality:

$$g(t+1) \leq h(t) + sg(t) \tag{4}$$

given that $g(t)$ and $h(t)$ are scalar functions dependent on $t \geq 0$, with s represents a non-negative value, the ensuing result for $t \geq 1$ can be inferred:

$$g(t) \leq \sum_{i=0}^{t-1} s^{t-i-1}h(i) + s^t g(0) \tag{5}$$

Assumption 2. Presuming the nonlinear function $f(\cdot,\cdot)$ in system (1) exhibits differentiability towards t and maintains global Lipschitz continuity in its first variable, applicable to all instances of t and $\forall \overline{x}(t), \overline{\overline{x}}(t) \in R^w$:

$$\left\| f(\overline{x}(t), t) - f\left(\overline{\overline{x}}(t), t\right) \right\| \leq k_f \left\| \overline{x}(t) - \overline{\overline{x}}(t) \right\| \tag{6}$$

where $k_f > 0$ is the Lipschitz constant.

3 ILC Design and Convergence Analysis

In this part, we consider the nonlinear discrete-time system (1) based on Assumptions 1–2, a P-type ILC control law is designed for $t \in \{0, 1, \cdots, T_d\}$.

$$u_{k+1}(t) = u_{f,k+1}(t) + u_{b,k+1}(t) \tag{7}$$

$$u_{f,k+1}(t) = u_k(t) + Pe_k(t+1) \tag{8}$$

$$u_{b,k+1}(t) = Le_{k+1}(t) \tag{9}$$

here, $u_{f,k+1}(t)$ denotes P-type ILC element incorporating a open-loop control gain of $P \in R$, while $u_{b,k+1}(t)$ refers to the closed-loop control element utilizing a closed-loop control gain of $L \in R$.

Theorem 1. If the control gains P in (11) satisfy:

$$\|1 - PC(t+1)B(t)\| < 1 \tag{10}$$

then $\lim_{k\to\infty} E\{e_k(t)\} = 0$ for $t \in \{0, 1, \cdots T_d + 1\}$.

Defined $\Delta u_{f,k}(t) = u_d(t) - u_{f,k}(t)$, $\Delta u_k(t) = u_d(t) - u_k(t)$, $\Delta x_k(t) = x_d(t) - x_k(t)$. Deducting expression (8) from $u_d(t)$ on both sides, we obtain

$$\Delta u_{f,k+1}(t) = \Delta u_k(t) - Pe_k(t+1) \tag{11}$$

From (1), (2) and (11), we have

$$\Delta u_{f,k+1}(t) = \Delta u_k(t) - PC(t+1)\Delta x_k(t+1)$$

$$= \Delta u_k(t) - PC(t+1)(f(x_d(t), t) - f(x_k(t), t))$$

$$-PC(t+1)B(t)\Delta u_k(t) \tag{12}$$

From (7), (8) and (9), we have

$$\Delta u_k(t) = \Delta u_{f,k}(t) - Le_k(t) \tag{13}$$

Replacing (12) with (13), we obtain

$$\Delta u_{f,k+1}(t) = [1 - PC(t+1)B(t)]\Delta u_k(t) - PC(t+1)(f(x_d(t),t) - f(x_k(t),t))$$

$$= [1 - PC(t+1)B(t)]\Delta u_{f,k}(t) - [1 - PC(t+1)B(t)]Le_k(t)$$

$$-PC(t+1)(f(x_d(t),t) - f(x_k(t),t)) \tag{14}$$

Calculating the norm of both ends of expression (14) results in

$$\left\|\Delta u_{f,k+1}(t)\right\| \le \|[1 - PC(t+1)B(t)]\| \cdot \left\|\Delta u_{f,k}(t)\right\| +$$

$$\|[1 - PC(t+1)B(t)]L\| \cdot \|e_k(t)\| + \|PC(t+1)\| \cdot \|(f(x_d(t),t) - f(x_k(t),t))\| \tag{15}$$

Thus, expression (15) can be transformed into

$$\left\|\Delta u_{f,k+1}(t)\right\| \le \phi\left\|\Delta u_{f,k}(t)\right\| + z_1\|e_k(t)\| + z_1\|\Delta x_k(t)\| \tag{16}$$

here, $\phi = \|1 - PC(t+1)B(t)\|$, $z_1 = \max\left\{\|[1 - PC(t+1)B(t)]L\|, \left\|k_f PC(t+1)\right\|\right\}$.
According to (1), (2), (4) and (6), we can obtain

$$\|\Delta x_k(t)\| \le k_f\|\Delta x_k(t-1)\| + \|B(t-1)\| \cdot \|\Delta u_k(t-1)\| \tag{17}$$

According to (13) and (17), we obtain

$$\|\Delta x_k(t)\| \le k_f\|\Delta x_k(t-1)\| + \|B(t-1)\| \cdot \left\|\Delta u_{f,k}(t-1)\right\|$$

$$+\|B(t-1)\| \cdot \|L\| \cdot \|e_k(t-1)\| \tag{18}$$

According to (1), (2) and (18), we can obtain

$$\|e_k(t)\| \le \|C(t)\| \cdot \|\Delta x_k(t)\|$$

$$\le \|C(t)\|k_f\|\Delta x_k(t-1)\| + \|C(t)B(t-1)\| \cdot \left\|\Delta u_{f,k}(t-1)\right\|$$

$$+\|C(t)B(t-1)\| \cdot \|L\| \cdot \|e_k(t-1)\| \tag{19}$$

Adding both sides of (18) and (19), we get

$$\|\Delta x_k(t)\| + \|e_k(t)\| \le \left(k_f + k_f\|C(t)\|\right)\|\Delta x_k(t-1)\| + (1 + \|C(t)\|)\|B(t-1)\| \cdot \|L\| \cdot \|e_k(t-1)\|$$

$$+(\|B(t-1)\| + \|B(t-1)\| \cdot \|C(t)\|)\left\|\Delta u_{f,k}(t-1)\right\|$$

$$\le z_2(\|\Delta x_k(t-1)\| + \|e_k(t-1)\|) + \overline{b}\left\|\Delta u_{f,k}(t-1)\right\|$$

$$\leq z_2^t(\|\Delta x_k(0)\| + \|e_k(0)\|) + \sum_{s=0}^{t-1} z_2^{t-s-1}\overline{b}\|\Delta u_{f,k}(s)\|$$

$$\leq \sum_{s=0}^{t-1} z_2^{t-s-1}\overline{b}\|\Delta u_{f,k}(s)\| \tag{20}$$

where $z_2 = \max\{k_f + k_f\|C(t)\|, (1 + \|C(t)\|)\|B(t-1)\| \cdot \|L\|\}$ and $\overline{b} = \|B(t-1)\| + \|B(t-1)\| \cdot \|C(t)\|$.

Substituting (20) into expression (16) and taking the expectation simultaneously, we obtain

$$E\{\|\Delta u_{f,k+1}(t)\|\} \leq E\{\phi\}E\{\|\Delta u_{f,k}(t)\|\} + z_1 \sum_{s=0}^{t-1} z_2^{t-s-1}\overline{b}E\{\|\Delta u_{f,k}(s)\|\} \tag{21}$$

Let $z_3 = \left\|z_1 z_2^{t-s-1}\overline{b}\right\|$, expression (21) can be transformed into

$$E\{\|\Delta u_{f,k+1}(t)\|\} \leq E\{\phi\}E\{\|\Delta u_{f,k}(t)\|\} + z_3 \sum_{s=0}^{t-1} E\{\|\Delta u_{f,k}(s)\|\} \tag{22}$$

Using mathematical induction to prove as follows:

When $t = 0$, we can obtain from expression (22) and under the condition of Assumption 1 that

$$E\{\|\Delta u_{f,k+1}(0)\|\} \leq E\{\phi\}E\{\|\Delta u_{f,k}(0)\|\} \tag{23}$$

When $E\{\phi\} < 1$, as the iterations k tend toward an extremely high number

$$\lim_{k\to\infty} E\{\|\Delta u_{f,k+1}(0)\|\} = 0 \tag{24}$$

When $t = 1$, we can obtain from expression (22) and under the condition of Assumption 1 that

$$E\{\|\Delta u_{f,k+1}(1)\|\} \leq E\{\phi\}E\{\|\Delta u_{f,k}(1)\|\} + z_3 E\{\|\Delta u_{f,k}(0)\|\} \tag{25}$$

Considering expression (24) and Theorem 1, we obtain

$$\lim_{k\to\infty} E\{\|\Delta u_{f,k+1}(1)\|\} = 0 \tag{26}$$

Assuming $t = 2, ..., m-1$, we have

$$\lim_{k\to\infty} E\{\|\Delta u_{f,k}(t)\|\} = 0 \tag{27}$$

When $t = m$, we can obtain from expression (22) that

$$E\{\|\Delta u_{f,k+1}(m)\|\} \leq E\{\phi\}E\{\|\Delta u_{f,k}(m)\|\} + z_3 \sum_{s=0}^{m-1} E\{\|\Delta u_{f,k}(s)\|\} \tag{28}$$

As the iterations k approaches infinity, we can obtain $\lim_{k\to\infty} E\left\{\left\|\Delta u_{f,k+1}(t)\right\|\right\} = 0$, so

$$\lim_{k\to\infty} E\left\{\left\|\Delta u_{f,k}(t)\right\|\right\} = 0 \tag{29}$$

Evidently, this leads to.

$$\lim_{k\to\infty} E\{e_k(t)\} = 0,\ \ t \in \{0, 1, \cdots, T_d + 1\} \tag{30}$$

This concludes is proved.

4 DE-ILC Scheme Design

The proposed method's asymptotic convergence is presented in the third part. It is well known that the convergence performance is greatly influenced by the control gains. In this section, the integration of the DE algorithm into the proposed P-type open-closed-loop ILC law (7) aims to optimize the control gain, thereby minimizing the required iterations for system convergence as cited in [10].

DE refers to an intelligent optimization algorithm that mimics biological evolution to find the best solution. The core concept of DE-ILC can be outlined as follows:

(1) First, set the control gain coefficients in the ILC control law to by P and L, initialize them as the initial population $\lambda = \left[P\ L\right]$ and encode them.
(2) According to the convergence condition (10), we get the initial population, without losing the generality, we assume that the initial population is even, and the representation of the variable vector for each individual within population h-th is denoted by $\lambda_h = \left[P_h\ L_h\right] \in R^{1\times 2}$, in order to assess the individual dominance in the population, the fitness function $fit_h(\lambda_h) = A - E_h(\lambda_h)$ is introduced, where A is a sufficiently large number, $E_h(\lambda_h) = \sum_{t=0}^{29} |y_d(t) - Y_h(t, \lambda_h)|$, $(T_d = 29)$.
(3) Then, we adopt a random mutation strategy for the initial population, the mutagenic factor is $0 < F < 1$, generate a new offspring population, calculate its fitness value through the fitness function, screen out the excellent population individuals, and proceed to the next step.
(4) The excellent population individuals screened after mutation are cross-operated, the crossed factor is $0 < CR < 1$, and generate a random number within the range of 0 to 1, and when this number exceeds the crossed factor threshold, a new offspring population is generated.
(5) After the cross-operation is left, the fitness value is calculated through the fitness function, the best population individual is retained, and whether to proceed to the next operation is judged by the termination conditions.
(6) The termination condition within the algorithm relies on both fitness value and tracking error. The algorithm iterates through a hundred population generations, eventually deriving optimal control gains P and L from the most favorable individual generated by DE algorithm. Refer to Fig. 1 for the flowchart illustrating DE-ILC.

5 Illustrative Simulation

To assess the efficacy of the suggested DE-ILC approach, let's examine a DC motor drive system governed by the dynamics of the subsequent nonlinear control system:

$$\left(Q_m + \frac{Q_e}{n^2}\right)\ddot{\theta}_m + \left(C_m + \frac{C_e}{n^2}\right)\dot{\theta}_m + \frac{Mgl}{n}\sin\frac{\theta_m}{n} = u \tag{31}$$

here, θ_m represents motor angle, Q_m stands for motor inertia, Q_e denotes link inertia, C_m signifies motor damping coefficient, C_e represents link damping coefficient, n denotes gear ratio, u stands for motor torque, M represents lumped mass, g symbolizes gravitational acceleration, and l represents the center of mass distance from motion axis.

The system is discretized, and the discrete-time state-space expression of (31) incorporating $x_k^{(1)} = \theta_m$, $x_k^{(2)} = \dot{\theta}_m$ and $y_k = \frac{1}{n}\dot{\theta}_m$ is as follows:

$$\begin{cases} x_k^{(1)}(t+1) = x_k^{(1)}(t) + \Delta x_k^{(2)}(t) \\ x_k^{(2)}(t+1) = -\dfrac{\Delta Mgl}{n\left(Q_m + Q_e/n^2\right)}\sin\dfrac{x_k^{(1)}(t)}{n} + \left[1 - \dfrac{\Delta\left(C_m + C_e/n^2\right)}{Q_m + Q_e/n^2}\right]x_k^{(2)}(t) + \dfrac{\Delta}{Q_m + Q_e/n^2}u_k(t) \\ y_k(t) = \dfrac{x_k^{(2)}(t)}{n} \end{cases} \tag{32}$$

here, Δ is the sampling time. In simulation, let $\Delta = 0.05(s)$, $t \in \{0, 1, \ldots, T_d\}$. Other parameters are set as follows: $Q_m = 0.3$, $Q_e = 0.44$, $M = 0.5$, $g = 9.8$, $C_m = 0.3$, $C_e = 0.25$, $n = 1.6$ and $l = 0.15$ [11].

Within the experimental setup, let $u_0(t) = u_1(t) = 0$, $(0 \le t \le T_d)$ with $T_d = 29$. The designated reference path is denoted as:

$$y_d(t) = \sin\left(\frac{\pi t}{15}\right),\ (0 \le t \le T_d + 1) \tag{33}$$

The DE-ILC algorithm determines control gains P and L through the DE method, Considering mutagenic factor $F = 0.3$ and crossed factor $CR = 0.7$. Evaluating tracking performance involves defining two tracking indexes, namely the sum absolute error SE_k and total square error TE_k as follows:

$$SE_k = \sum_{t=0}^{30} |e_k(t)| \tag{34}$$

$$TE_k = \sum_{t=0}^{30} [e_k(t)]^2 \tag{35}$$

The DE-ILC is executed in simulation for 10 iterations, and the resultant optimized control gains are tabulated in Table 1.

To assess the convergence speed disparity between DE-ILC and conventional ILC with varying parameters, two distinct cases were chosen for comparison. Case 1: $P = 0.5$, $L = 0.3$. Case 2: $P = 0.6$, $L = 0.2$. Figure 2 displays the corresponding sum absolute error SE_k, while Fig. 3 illustrates the total square error TE_k for tracking. Observing these figures reveals that higher control gains P and L are contribute to faster convergence in conventional ILC. Additionally, the proposed DE-ILC method distinctly demonstrates fewer convergence iterations compared to conventional ILC of similar order.

Table 1. Optimized control gains achieved by DE-ILC vary at different instances.

Times	*P*	*L*
1	0.8036	0.3032
2	0.8577	0.3516
3	0.8655	0.3950
4	0.8656	0.3706
5	0.8842	0.3533
6	0.8679	0.3097
7	0.8280	0.3105
8	0.8136	0.3132
9	0.8455	0.3850
10	0.8758	0.3823
Average	0.8507	0.3475

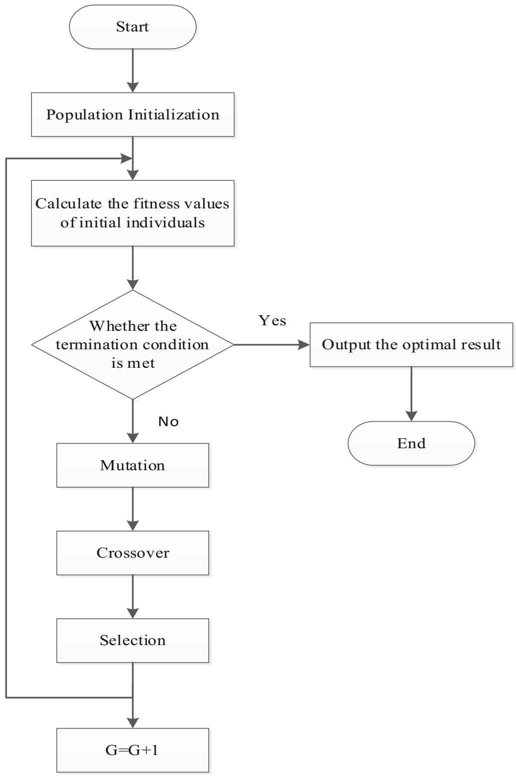

Fig. 1. The flowchart of the proposed DE-ILC.

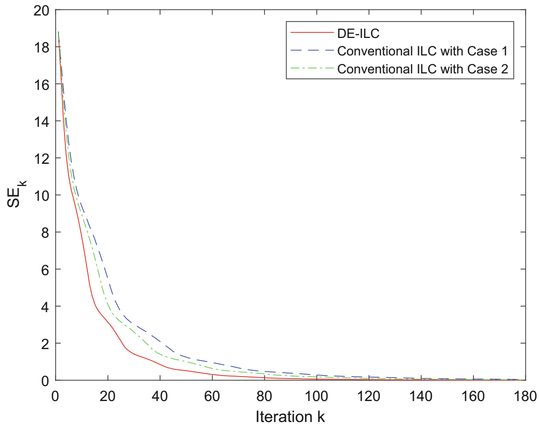

Fig. 2. The tracking indexes SE_k at various iterations obtained through DE-ILC and conventional ILC in two distinct cases.

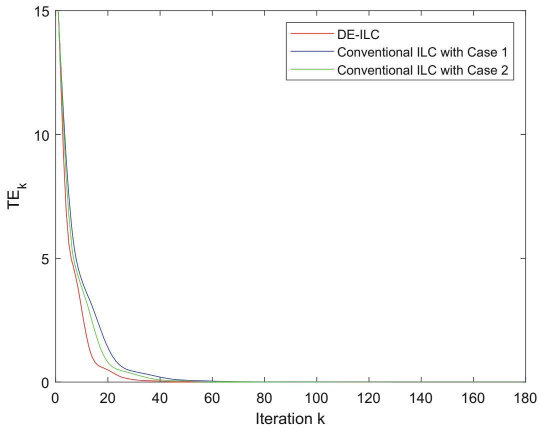

Fig. 3. The tracking indexes TE_k at various iterations obtained through DE-ILC and conventional ILC in two distinct cases.

6 Conclusion

This study introduces a differential evolution algorithm-based open-closed-loop iterative learning control (ILC) approach. The proposed ILC method utilizes feedback information to adapt the system, ensures stability control through closed-loop regulation, and continuously enhances the control algorithm for error correction during tracking. The convergence process of ILC can be expedited through the adjustment of feedback control gains, guaranteeing system reliability and stability. To achieve the suitable closed-loop control gain, the feedforward and feedback ILC law is enhanced using the differential evolution algorithm. This guarantees rapid convergence while upholding system stability. After conducting theoretical analysis and simulation, the tracking error tends to approach zero in mathematical expectation [12, 13].

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References

1. Shen, D., Wang, Y.: Survey on stochastic iterative learning control. J. Process. Control. **24**(12), 64–77 (2014)
2. Dong, J., He, B., Ming, M., Zhang, C., Li, G.: Design of open-closed-loop iterative learning control with variable stiffness for multiple flexible manipulator robot systems. IEEE Access **7**, 23163–23168 (2019)
3. Li, X., Shen, D., Ding, B.: Iterative learning control for output tracking of nonlinear systems with unavailable state information. IEEE Trans. Neural Netw. Learn. Syst. **33**(9), 5085–5092 (2021)
4. Wang, L., Yu, J., Zhang, R., Li, P., Gao, F.: Iterative learning control for multiphase batch processes with asynchronous switching. IEEE Trans. Syst. Man Cybern. Syst. **51**(4), 2536–2549 (2019)
5. Zhang, J., Meng, D.: Convergence analysis of saturated iterative learning control systems with locally Lipschitz nonlinearities. IEEE Trans. Neural Netw. Learn. Syst. **31**(10), 4025–4035 (2019)
6. Wei, Y.S., Li, X.D.: Robust higher-order ILC for non-linear discrete-time systems with varying trail lengths and random initial state shifts. IET Control Theory Appl. **11**(15), 2440–2447 (2017)
7. Meng, D., Zhang, J.: Robust tracking of nonrepetitive learning control systems with iteration-dependent references. IEEE Trans. Syst. Man Cybern. Syst. **51**(2), 842–852 (2018)
8. Yalçin, O., Canli, A., Yilmaz, A.R., Erkmen, B.: Robust tuning of PID controller using differential evolution algorithm based on FPGA. In: 2022 9th International Conference on Electrical and Electronics Engineering (ICEEE), pp. 180–184. IEEE, March 2022
9. Zhang, J., Wu, P., Wang, X., Yu, X., Duan, S.: PID parameter tuning of combined heat and power generation unit based on differential evolution algorithm. In: 2022 7th International Conference on Power and Renewable Energy (ICPRE). pp. 1186–1190. IEEE, September 2022
10. Wei, Y.S., Yang, X., Shang, W., Chen, Y.Y.: Higher-order iterative learning control with optimal control gains based on evolutionary algorithm for nonlinear system. Complexity **2021**, 1–9 (2021)
11. Zhang, Y., Liu, J., Ruan, X.: Iterative learning control for uncertain nonlinear networked control systems with random packet dropout. Int. J. Robust Nonlinear Control **29**(11), 3529–3546 (2019)
12. Zhao, Z., Liu, Z.: Finite-time convergence disturbance rejection control for a flexible Timoshenko manipulator. IEEE/CAA Journal of Automatica Sinica **8**(1), 157–168 (2020)
13. Wan, K., Li, X.D.: Robust iterative learning control of 2-D linear discrete FMMII systems subject to iteration-dependent uncertainties. IEEE Trans. Syst. Man Cybern. Syst. **51**(10), 5949–5961 (2019)