



Application of GNSS Virtual Reference Station in Poyang Lake Area

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Abstract. Virtual Reference Station (VRS) is a kind of network RTK technology. For solving the limitations of real-time standard dynamic (RTK) system, a new method for high-precision RTK positioning VRS is proposed. At the same time, aiming at the real-time solution of the ambiguity of VRS network, a new method based on the non-ionospheric combination equation to solve the ambiguity is proposed. The linear interpolation model is used to generate the comprehensive error correction number, and the virtual observation value is proposed. The method was generated and tested in Nanchang University and Poyang Lake area. The test was conducted at different locations around Poyang Lake to assess the accuracy of VRS RTK positioning; the results show that the use of multiple reference station networks increases the coverage of location service areas and enhances robustness over a single reference station system, got higher positioning accuracy. The VRS RTK is positioned in the experimental network with a horizontal accuracy of 1–2 cm and a vertical accuracy of 3–5 cm.

Keywords: RTK · VRS · Ambiguity

1 Introduction

RTK (Real-time kinematic) carrier phase difference technology is a differential method for real-time processing of carrier phase observations of two measurement stations. The carrier phase acquired by the base station is sent to the user receiver to perform the difference calculation coordinate. RTK positioning is typically implemented in a single reference station mode. However, the single reference frame lacks data redundancy and must be within 10–15 km of the user, as GPS errors become less spatially correlated over longer baselines, resulting in reduced positioning accuracy [1, 2]. Therefore, in order to achieve high-precision RTK positioning for longer distances, the network RTK method came into being. In order to overcome the limitations of conventional RTK positioning technology and achieve centimeter-level high-precision uniform real-time dynamic positioning in a region, this paper demonstrates a new method for creating VRS positioning VRS, for ambiguity resolution and VRS network in VRS algorithm. The generation of error correction numbers and the generation of VRS virtual observations were studied. The test results of VRS for RTK positioning in different locations of the Poyang Lake network were discussed.

2 Principle of LOAM Algorithm

2.1 VRS Network Full-Circumference Ambiguity Real-Time Solution

Real-time and correct fixation of the ambiguity of the whole week is a prerequisite for users to achieve precise positioning. In this paper, the wide lane ambiguity is first solved, and then the L1 ambiguity and the tropospheric zenith delay (RTZD) are estimated by the adaptive Kalman filter using the ionization-free layer observation.

According to the double-difference ionospheric observation ($\nabla \Delta \phi_{77,-60}$), the relationship between the double-difference L1 ambiguity ($\nabla \Delta N_1$) and the double-difference wide ambiguity ($\nabla \Delta N_{1,-1}$), this paper focuses on the modified adaptive Kalman filter for real-time estimation of RTZD. And the double difference L1 ambiguity, the observation equation is:

$$\lambda_{77,-66} \nabla \Delta \phi_{77,-60} = \nabla \Delta \rho + \nabla \Delta N_1 (17\lambda_{77,-66}) + \nabla \Delta N_{1,-1} (60\lambda_{77,-66}) + \nabla \Delta d_{trop} + \varepsilon (\nabla \Delta \phi_{77,-60}) \quad (1)$$

where: $\nabla \Delta \rho$ is the geometric distance of the double-difference station star, $\varepsilon (\nabla \Delta \phi_{77,-60})$ is the phase difference observation noise of the double-difference ionosphere, $\lambda_{77,-66}$ is the combined observation wavelength without ionosphere, and $\nabla \Delta d_{trop}$ is the double-difference tropospheric delay.

First, since the wavelength of the wide-circumference ambiguity is 86.4 cm, it is easy to accurately determine the ambiguity of the whole week. Using the search method or the sequence estimation method, it is possible to determine $\nabla \Delta N_{1,-1}$, in the Eq. (1), The ionospheric ambiguity is determined by the combination of wide lane all-round ambiguity $\nabla \Delta N_{1,-1}$ and L1 full-circumference ambiguity. Since the wide lane ambiguity is first determined, the above relationship is used to obtain L1 full-circumference ambiguity [3]. Using known reference station exact coordinates and wide lane ambiguity, L1 full-circumference ambiguity is only affected by tropospheric delay, multipath interference, and observed noise. The effect of multipath effects can be reduced by selecting an appropriate reference station and using a GPS receiver with multipath mitigation. The challenge therefore is how to mitigate the tropospheric double residual and reduce the observed noise. Since the influence of the troposphere is related to the satellite height angle, introducing a projection function $MF(\cdot)$, the double-difference tropospheric delay can be expressed as the product of the RTZD and the projection function [4]:

$$\nabla \Delta d_{trop}^{xy} = RTZD \cdot [MF(\varepsilon^x) - MF(\varepsilon^y)] \quad (2)$$

where: ε^x and ε^y represent the elevation angle of the satellite x, y, $MF(\cdot) = 1/\sin(\cdot)$, Therefore: (1) can be turned into:

$$\begin{aligned} &\lambda_{77,-66} \nabla \Delta \phi_{77,-60} - \nabla \Delta \rho - \nabla \Delta N_{1,-1} (60\lambda_{77,-66}) \\ &= \nabla \Delta N_1 (17\lambda_{77,-66}) + RTZD \cdot [MF(\varepsilon^x) - MF(\varepsilon^y)] + \varepsilon (\nabla \Delta \phi_{77,-60}) \end{aligned} \quad (3)$$

To estimate the RTZD and the double-difference ambiguity on L1 in real time, an adaptive Kalman filter is used to solve each baseline solution between the primary

reference station and other reference stations. The measurement equation and the state equation are:

$$Z_k = H_k X_k + V_k \quad , \quad V_k \sim N(0, R_k) \tag{4}$$

$$X_k = \Phi_{k,k-1} X_{k-1} + W_k \quad , \quad W_k \sim N(0, Q_k) \tag{5}$$

where: V_k is the observed noise, W_k representing the system noise, R_k is the covariance matrix of the observed noise, and the observation vector is:

$$Z_k = \begin{bmatrix} \lambda_{77,-66} \nabla \Delta \phi_{77,-66}^{12} - \nabla \Delta \rho^{12} - \nabla \Delta N_{1,-1}^{12} \cdot (60\lambda_{77,-66}) \\ \lambda_{77,-66} \nabla \Delta \phi_{77,-66}^{13} - \nabla \Delta \rho^{13} - \nabla \Delta N_{1,-1}^{13} \cdot (60\lambda_{77,-66}) \\ \vdots \\ \lambda_{77,-66} \nabla \Delta \phi_{77,-66}^{1n} - \nabla \Delta \rho^{1n} - \nabla \Delta N_{1,-1}^{1n} \cdot (60\lambda_{77,-66}) \end{bmatrix} \tag{6}$$

Among them: superscript 1 represents the reference satellite, and 2 to n represent other satellites. The state vector is defined as:

$$X_k = [RTZD \quad \nabla \Delta N_1^{12} \quad \nabla \Delta N_1^{13} \quad \dots \quad \nabla \Delta N_1^{1n}]^T \tag{7}$$

And the measurement matrix is:

$$H_k = \begin{bmatrix} MF(\varepsilon^1) - MF(\varepsilon^2) & 17\lambda_{77,-66} & 0 & \dots & 0 \\ MF(\varepsilon^1) - MF(\varepsilon^3) & 0 & 17\lambda_{77,-66} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ MF(\varepsilon^1) - MF(\varepsilon^n) & 0 & 0 & \dots & 17\lambda_{77,-66} \end{bmatrix} \tag{8}$$

where: RTZD is a first-order Gauss-Markov process and assumes that L1 ambiguity is very small white noise. Then define the state transition matrix and the corresponding covariance matrix as:

$$\Phi_{k,k-1} = \begin{bmatrix} e^{-\Delta t/\tau} & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \tag{9}$$

$$Q_k = \begin{bmatrix} \frac{\tau}{2}(1 - e^{-2\Delta t/\tau})q & 0 & \dots & 0 \\ 0 & 1e^{-16} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1e^{-16} \end{bmatrix} \tag{10}$$

where: q is the variance of the RTZD process noise at the relevant time τ , which is the sampling interval Δt .

Traditional Kalman filtering requires a priori information, namely noise statistics parameters R_k and Q_k . However, in practice, it is difficult to know the true statistical characteristics of measurement and system noise. Without prior information, it is

impossible to do a Kalman filter. Incorrect values will result in inaccurate results and even divergence. On the other hand, the network RTK method usually requires 6 or more satellites to be corrected and no interrupts are allowed [5]. To overcome this problem, the following residual-based adaptive Kalman filter is proposed [6]:

$$X_{k,k-1} = \Phi_{k,k-1} X_{k-1,k-1} \tag{11}$$

$$P_{k,k-1} = \Phi_{k,k-1} P_{k-1,k-1} \Phi_{k,k-1}^T + Q_{k-1} \tag{12}$$

$$J_k = P_{k,k-1} H_k^T (H_k P_{k,k-1} H_k^T + R_{k-1})^{-1} \tag{13}$$

$$X_{k,k} = X_{k,k-1} + J_k (Z_k - H_k X_{k,k-1}) \tag{14}$$

$$P_{k,k} = (I - J_k H_k) P_{k,k-1} \tag{15}$$

$$R_k = C_{v_k} + H_k P_{k,k} H_k^T \tag{16}$$

$$Q_k = J_k C_{v_k} J_k^T \tag{17}$$

among them:

$$C_{v_k} = \frac{1}{N} \sum_{i=i_0}^k v_i v_i^T \tag{18}$$

Residual sequence:

$$v_k = Z_k - H_k X_k \tag{19}$$

where $i_0 = k - N + 1$ is the first epoch in the moving window of estimated size N , $X_{k,k-1}$ and $P_{k,k-1}$ are system state prediction values and covariance matrices, respectively, $X_{k,k}$ and $P_{k,k}$ are updated system state prediction values and covariance matrices, J_k represents the gain matrix.

Here, the covariance matrices (R_k and Q_k) of the measurement and system noise are estimated in real time from the previous epoch. With the adaptive Kalman filter recursive formula, the corresponding covariance matrix $P_{k,k}$ can be used to estimate the RTZD sum and the double difference L1 floating point ambiguity. The existing method is then used to perform a double difference L1 integer ambiguity search [7], such as the LAMBDA method. Thus, the L2 full-circumference ambiguity is solved. Details are not included in this article.

2.2 VRS Network Error Correction Number Generation Method

When the spatial correlation error at the baseline of each reference station is determined, the spatial correlation correction at the virtual reference station can be calculated by a

specific interpolation algorithm [8, 9]. In this paper, a distance-based linear interpolation algorithm is adopted. The advantage of this real-time method is that it is simple to implement and has high efficiency. By using this method, track deviation and ionospheric delay can be eliminated, and in addition, tropospheric delay, multipath and observed noise can be reduced. In order to apply this method in real time, a modified linear interpolation model with user horizontal coordinates as a parameter is adopted. In the network coverage area, any user station ionospheric correction number can pass through the known coordinates of the reference station and the outline of the subscriber station. The coordinates are interpolated.

When there are at least 3 GPS reference stations around the subscriber station, the interpolation model can be described as follows:

$$\begin{aligned}
 \begin{bmatrix} V_{1,n} \\ V_{2,n} \\ \vdots \\ V_{n-1,n} \end{bmatrix} &= \begin{bmatrix} \lambda \nabla \Delta \phi_{1n} - \nabla \Delta \rho_{1n} - \lambda \nabla \Delta N_{1n} \\ \lambda \nabla \Delta \phi_{2n} - \nabla \Delta \rho_{2n} - \lambda \nabla \Delta N_{2n} \\ \vdots \\ \lambda \nabla \Delta \phi_{n-1n} - \nabla \Delta \rho_{n-1n} - \lambda \nabla \Delta N_{n-1n} \end{bmatrix} \\
 &= \begin{bmatrix} \Delta X_{1,n} & \Delta Y_{1,n} \\ \Delta X_{2,n} & \Delta Y_{2,n} \\ \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} \tag{20}
 \end{aligned}$$

where $V_{i,n}$ represents the correction of the deviation of the ionosphere; $\Delta X_{i,n}$ and $\Delta Y_{i,n}$ ($i = 1, 2, \dots, n-1$) represent the plane coordinate difference between the $n-1$ sub-reference stations and the n -th main station. The parameters a and b are the coefficients of $\Delta X_{i,n}$ and $\Delta Y_{i,n}$. In the case of more than 3 reference stations, the coefficients a and b can be obtained by the least squares method, which is obtained by Eq. (20):

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (A^T A)^{-1} A^T V \tag{21}$$

$$V = \begin{bmatrix} V_{1,n} \\ V_{2,n} \\ \vdots \\ V_{n-1,n} \end{bmatrix} = \begin{bmatrix} \lambda \nabla \Delta \phi_{1n} - \nabla \Delta \rho_{1n} - \lambda \nabla \Delta N_{1n} \\ \lambda \nabla \Delta \phi_{2n} - \nabla \Delta \rho_{2n} - \lambda \nabla \Delta N_{2n} \\ \vdots \\ \lambda \nabla \Delta \phi_{n-1n} - \nabla \Delta \rho_{n-1n} - \lambda \nabla \Delta N_{n-1n} \end{bmatrix} \tag{22}$$

$$A = \begin{bmatrix} \Delta X_{1,n} & \Delta Y_{1,n} \\ \Delta X_{2,n} & \Delta Y_{2,n} \\ \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} \end{bmatrix} \tag{23}$$

It is assumed that the correction numbers of each sub-reference station to the primary reference station are equal weights of the same precision, and their weight matrix $P =$

E is taken. The network coverage user station deviation can be interpolated using the following two-dimensional model:

$$Corr_{u,n} = \hat{a} \cdot \Delta X_{u,n} + \hat{b} \cdot \Delta Y_{u,n} \quad (24)$$

$$Corr_{u,n} = [\Delta X_{u,n} \ \Delta Y_{u,n}] [\hat{a} \ \hat{b}]^T = [\Delta X_{u,n} \ \Delta Y_{u,n}] (A^T A)^{-1} A^T V \quad (25)$$

where $Corr_{u,n}$ represents the correction of the ionospheric deviation between the subscriber station and the primary reference station; $\Delta X_{u,n}$ and $\Delta Y_{u,n}$ respectively represent the plane coordinate difference between the subscriber station and the primary reference station.

2.3 VRS Observation Generation

Let x^s be the satellite position vector, x^r is the main reference station position vector, and x^v is the VRS position vector. At epoch time t , the geometric distance between the satellite and the primary reference station receiver is

$$\rho_r^s(t) = \|x^s - x^r\| \quad (26)$$

The geometric distance between the satellite and the VRS is

$$\rho_v^s(t) = \|x^s - x^v\| \quad (27)$$

The geometric correction amount $\Delta\rho^s = \rho_v^s(t) - \rho_r^s(t)$ can be applied to the calculation of satellite s observation data on the primary reference station to the VRS. After applying the geometric correction number to the primary reference station raw data, the data correction number generated according to Sect. 2.2 is used to correct the VRS data, and then the virtual observation value and the reference station coordinates are RTCM encoded and sent to the user station [10, 11], The user station can be decoded to perform differential positioning.

3 VRS/RTK Positioning Performance and Accuracy Analysis

In order to evaluate the basic implementation of the VRS RTK network, Nanchang University GPS observation network project includes four consecutive operation reference stations: Dean (DEAN), Poyang (POYA), Jinxian (JINX), and Duchang (DUCH). Each reference station is equipped. Trimble NetR9 reference station receiver and Zephyr Geodetic 2 antenna. The experiment uses dual-frequency phase observation data from 6:00:00 to 20:00:00 on December 30, 2017. The sampling time is 1S and the satellite cut-off angle is set to 10 degrees. The average baseline of the test network is (96) km. The layout of the system is shown in the figure. Select DEAN as the primary reference station and DUCH as the mobile station.

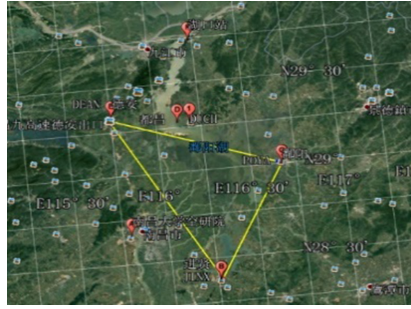


Fig. 1. Test network of Poyang Lake

The above network correction algorithm is integrated into the self-developed virtual reference station software platform SKY system, and applied to the network experiment of Poyang Lake virtual reference station in Jiangxi Province, as shown in Fig. 1. The results are shown in Fig. 2 and Table 1.



Fig. 2. Distribution of the RTK positioning Result at DUCH

Table 1. Accuracy of Network RTK position

Baseline vector	Network RTK positioning result /mm		
	MEAN	STD	RMS
Eastward	0.4	6.3	6.3
North	13.0	9.0	9.1
Vertical direction	13.0	35.6	35.6

It can be obtained from the RTK experimental results. The linear interpolation method is used to correct the number model. Using a sparse reference station network (3 reference stations with an average baseline length of 96 km), a centimeter-level RTK result of 1–2 cm in the horizontal direction and 3–5 cm in the elevation direction can be achieved.

4 Conclusion

In order to apply the network RTK technology to the medium and long-distance reference station network and achieve centimeter-level high-precision real-time dynamic positioning accuracy, the network correction algorithm model and the VRS observation value model must be established based on the existing error modeling technology. Based on the VRS observation value calculation model, based on the GPS continuous observation network in the Poyang Lake area of Jiangxi Province and the SKY system of the virtual reference station software platform independently developed, the real-time dynamic difference of the VRS centimeter network based on the Internet for the medium and long-distance reference station network over 96km is realized. The positioning service has a horizontal accuracy of 1–2 cm and a vertical accuracy of 3–5 cm in the experimental network. In the future, experiments outside the network and joint positioning of multiple satellites may be carried out.

Acknowledgments. The paper was supported by the projects of the National Natural Science Foundation of China (No. 41764002).

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