



Iterative Phase Error Compensation Joint Channel Estimation in OFDM Systems

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Abstract. Orthogonal frequency division multiplexing (OFDM) system is very sensitive to the phase noise especially in high frequency since the orthogonality between sub-carriers is easily destroyed. It is very important to estimate and compensate the phase noise in the research of 5G systems. The influence of phase noise on OFDM systems is manifested in two aspects: introducing common phase error (CPE) and causing inter-carrier interference (ICI). In this paper, we propose a new joint channel and CPE estimation algorithm to obtain more accurate channel and CPE estimates through iterations. In each iteration, we update the channel and CPE estimates to make them closer to the true value. Besides, the performance improvement brought by the algorithm under the simplified system model is analyzed. Simulation results show that this algorithm has a great impact on improving the accuracy of channel and CPE estimation.

Keywords: Phase noise · CPE compensation · Channel estimation

1 Introduction

Higher demands will be put on wireless communication systems when it comes to the service requirements of mobile Internet and Internet of Things in the future. In the 5G (5th-Generation) wireless communication system, it is especially important to suppress the influence of phase noise. Therefore, phase noise research was included in the 3GPP work items in Ref. [1]. The phase noise is caused by the instability of the local RF circuit crystal oscillator. When the frequency is higher than 6 GHz, the influence of phase noise cannot be ignored [2]. The impact of phase noise on OFDM systems receivers is manifested in two aspects, namely CPE (Common Phase Error) and ICI (Inter-Carrier Interference) [3]. The CPE causes the rotation of the phase of the received signal, while the ICI increases the system noise that will lead system performance degradation. Especially when the OFDM system adopts a high modulation order

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(such as 64 QAM, etc.), the phase noise has an increasing impact on the performance of the system. Hence, at the 3GPP RAN1#87 conference, a reference signal for phase tracking is specified by PTRS (Phase Tracking Reference Signal) [4].

At present, there are a lot of literatures on phase noise estimation and compensation. In Ref. [5], the authors propose a joint channel and data symbol phase estimation algorithm that includes estimation and correction of CPE and ICI. And the exact data model of phase noise is given in the paper. However, a large number of pilot signals is required to be placed on the transmitting end in the algorithm, which will affect the system performance. In Ref. [6], the authors consider an iterative feedback correction method to estimate the phase noise. In the iterative process, the received signal is first corrected by the CPE estimated by the pilot signal, and then enters the feedback system to continuously correct by estimating the log likelihood ratio of the reliability. In Ref. [7], the author introduces the least-mean-square (LMS) adaptive filter to estimate CPE, calculate the receiver tap weight vector and correct CPE by minimizing the MSE. However, the performance of the algorithm depends on the system step size. To obtain a more accurate phase estimation, the system step size must be reduced, which will inevitably lead to slower system convergence. In Ref. [8], the author proposes a non-data-aided phase noise compensation algorithm which means there is no pilot needed, and shows that it works well for high-order constellation CO-OFDM systems.

In this paper, we propose a new joint channel and CPE estimation algorithm, and analyze the performance improvement brought by the algorithm under the simplified system model. In this algorithm, we use reference signals to perform channel estimation and CPE compensation and then correct the estimates iteratively. An iterative process with more prior information is also considered. It can be seen from the theoretical analysis and simulation results that the joint channel and CPE estimation algorithm can bring good performance improvement.

This paper is organized as follows. Section 2 introduces the system model with phase noise and the basic algorithm for CPE estimation. Section 3 introduces the joint channel and CPE estimation algorithm and gives the algorithm flows. Section 4 analyzes the performance of the algorithm under two simplified conditions. Simulation results are given in Sect. 5.

2 System Model and CPE Estimation

In OFDM systems, the received signal after removing the CP (Cyclic Prefix) in the frequency domain can be expressed as

$$y_{k,n} = x_{k,n}h_{k,n}\varphi_{0,n} + \sum_{i=0, i \neq k}^{P-1} x_{i,n}h_{i,n}\varphi_{(k-i)_{[P]},n} + w_{k,n}, \quad (1)$$

where $x_{k,n}$ denotes the transmitted data in the frequency domain, $h_{k,n}$ denotes the frequency domain channel, $\varphi_{0,n}$ represents the CPE, k means the k -th sub-carrier where the reference signals are placed, n means the n -th OFDM symbol.

The second term in the formula denotes the ICI due to phase noise where P indicates the number of FFT points and subscript $[P]$ indicates modulo P operation. $w_{k,n}$ represents the additive white Gaussian noise.

It can be seen from (1) that the influence of phase noise on the received OFDM signals consists of two parts:

1. CPE, which scales and rotates the ideal received signal and can be expressed as

$$\varphi_{0,n} = \frac{1}{P} \sum_{i=0}^{P-1} e^{j\phi_i} \approx e^{j\theta_n}, \quad (2)$$

where ϕ_i indicates the phase noise. In general, the magnitude of the CPE is close to 1, mainly as a phase error.

2. ICI, which breaks the orthogonality between subcarriers in OFDM systems.

Since the common phase error is the main influencing factor, and it is same for all subcarriers on an OFDM symbol which can be estimated and eliminated, this article only analyzes the impact of CPE. Considering ICI as a part of Gaussian noise, (1) can be simplified to

$$y_{k,n} = x_{k,n} h_{k,n} e^{j\theta_n} + w_{k,n}'. \quad (3)$$

To estimate and eliminate the CPE, a new reference signal named PTRS (Phase Tracking Reference Signal) is proposed in the 5G systems. For the convenience of subsequent formula derivation, we assume that one DMRS symbol and $(N-1)$ PTRS symbols are placed on each sub-carrier. We also define n_d as the OFDM symbol index for placing DMRS, and $\{n_{p_i}\}, i = 0, 1, \dots, N-2$ is the set of OFDM symbols for placing PTRS.

Therefore, the received signals of the DMRS and PTRS can be expressed as

$$y_{k,n_d} = x_{k,n_d} h_{k,n_d} e^{j\theta_{n_d}} + w_{k,n_d}', \quad (4)$$

$$y_{k,n_p} = x_{k,n_p} h_{k,n_p} e^{j\theta_{n_p}} + w_{k,n_p}'. \quad (5)$$

Since the radio channel changes slowly, the channel at n_d and n_p OFDM symbol can be considered equal and we defined it as h_{k,n_d} . Divided by the known reference signal, Eqs. (4) and (5) becomes

$$h'_{k,n_d} = h_{k,n_d} e^{j\theta_{n_d}} + n_{k,n_d} = \hat{h}_{k,n_d} + n_{k,n_d}, \quad (6)$$

$$h'_{k,n_p} = h_{k,n_p} e^{j\theta_{n_p}} + n_{k,n_p} = \hat{h}_{k,n_p} + n_{k,n_p}, \quad (7)$$

where $\hat{h}_{k,n_d} = h_{k,n_d} e^{j\theta_{n_d}}$, $\hat{h}_{k,n_p} = h_{k,n_d} e^{j\theta_{n_p}} = \hat{h}_{k,n_d} e^{j\theta_\tau}$, $\theta_\tau = \theta_{n_p} - \theta_{n_d}$, here θ_τ is the difference of CPE between two OFDM symbols. When there are multiple sub-carriers in the frequency domain to place PTRS, θ_τ is usually estimated by [4].

$$\theta_\tau = \frac{1}{K} \sum_{k=1}^K \text{angle}\{y_{k,n_d}^* y_{k,n_p}\}. \quad (8)$$

Equation (8) estimates the difference of the CPE between the PTRS and DMRS. Therefore, the channel estimate of PTRS \hat{h}_{k,n_p} can be obtained by performing a phase rotation operation on the channel estimate of the DMRS \hat{h}_{k,n_d} .

However, the amplitude of $y_{k,n_d}^* y_{k,n_p}$ is ignored in (8) which means that the magnitude of each complex has the same effect during the merge. In fact, the larger the amplitude, the closer the estimated CPE is to the true value. Therefore, we introduce a better solution to obtain θ_τ [9]

$$\theta_\tau = \text{angle}\left\{\sum_{k=1}^K y_{k,n_d}^* y_{k,n_p}\right\}, \quad (9)$$

where $\text{angle}\{*\}$ represents the operation of taking the angle of the complex, y^* represents the conjugation of y . On this basis, continue to perform operations such as equalization and data demodulation, more accurate source data can be obtained.

3 Joint Estimation of CPE and Channel

When the Gaussian noise is large, the CPE estimated by (9) is not accurate enough. y_{k,n_d} in (9) can be replaced by \hat{h}_{k,n_d} if the channel estimate \hat{h}_{k,n_d} is accurate enough. Thus (9) becomes

$$\theta_\tau = \text{angle}\left\{\sum_{k=1}^K \hat{h}_{k,n_d}^* y_{k,n_p}\right\}. \quad (10)$$

In this way, the influence of Gaussian noise can be eliminated and the CPE estimate is more accurate. In order to estimate CPE by using PTRS, Fig. 1 gives an example of reference signals pattern.

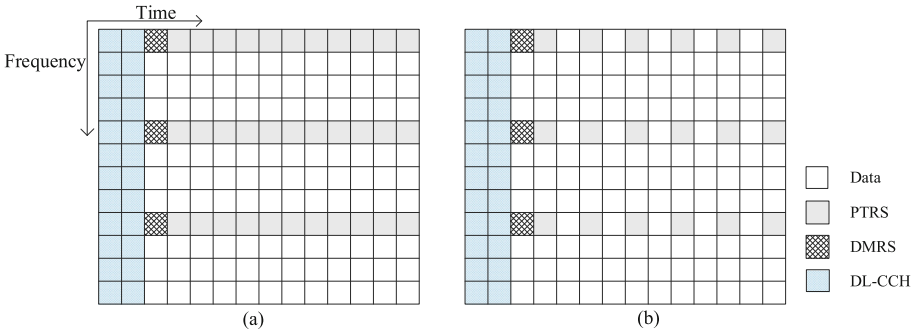


Fig. 1. Distribution of reference signal

In Fig. 1(a), the PTRS are located on the OFDM symbols where all datas are located, and cooperate with the DMRS on the same sub-carrier to estimate the phase error on each OFDM symbol. In Fig. 1(b), the PTRS is separated by

one OFDM symbol in the time domain. CPE at the position where the PTRS is not placed can be obtained by the interpolation of adjacent CPE estimates.

We assume that the pattern of the reference signals is the same as shown in Fig. 1, and there is 1 DMRS and $N-1$ PTRS on each sub-carrier. Since only DMRS and PTRS are considered, DMRS is defined as the starting OFDM symbol. For the sake of simplicity, we abbreviate \hat{h}_{k,n_d} as \hat{h}_k .

Since the distribution of channel and phase errors to be estimated is unknown, the optimal estimation criterion is ML (Maximum Likelihood) criterion. This problem can be expressed as

$$(\{\hat{h}_k\}, \{\hat{\theta}_n\}) = \arg \min_{\{h_k\}, \{\theta_n\}} \sum_k \sum_n |y_{k,n} - h_k e^{j\theta_n}|^2, \quad (11)$$

where $\{h_k\}$ is the channel to be sought and $\{\theta_n\}$ is the CPE to be sought, $y_{k,n}$ is the received signal on the n -th symbol of the k -th subcarrier.

For the optimization problem in (11), if $\{\theta_n\}$ is determined, we can get

$$\hat{h}_k = \frac{1}{N} \sum_{n=1}^N y_{k,n} e^{-j\theta_n}. \quad (12)$$

When $\{h_k\}$ is determined, we can get (9). Therefore, the optimization problem in (11) can be solved in an iterative manner. If there is more prior information about $\{h_k\}$, we will discuss it separately.

3.1 No Prior Information

In this part, we assume that there is no prior information about the channel. Through the above analysis, the iterative process can be given as

- 0: Initialize the estimated CPE: $\theta_1 = 0, \theta_n = \text{angle}\{\sum_{k=1}^K y_{k,1}^* y_{k,n}\}$;
- 1: Update $\{h_k\}$ according to (12);
- 2: Update $\{\theta_n\}$ according to (10);
- 3: Repeat step 1 and 2 until there is no significant improvement in the objective function $|y_{k,n} - h_k e^{j\theta_n}|^2$.

At first, an initial value is given to the CPE on each symbol where PTRS is placed, then we obtain the channel estimate through the CPE according to (12), and then update the CPE based on the channel estimate according to (10). Finally, by repeating the above steps, the gap between $y_{k,n}$ and $h_k e^{j\theta_n}$ is continuously narrowed, more accurate CPE and channel estimates can be obtained.

3.2 With Known Delay Power Spectrum

If the delay power spectrum is known at the receiver, the channel correlation matrix r_{hh} can be obtained. The channel estimation based on LMMSE is

$$\hat{h} = r_{hh} \left[r_{hh} + \sigma^2 (x * x)^{-1} \right]^{-1} h_{LS}, \quad (13)$$

where $r_{hh} = E(h \cdot h^*)$, $h = [h_1, h_2, \dots, h_K]^T$, x denotes the known reference signal, σ^2 indicates Gaussian noise variance, h_{LS} represents LS channel estimate which can be replaced during the iteration. Hence, the iterative process is given as

- 1: Through frequency domain filtering, we can get $\{h_k\}$ according to (13);
- 2: Update $\{\theta_n\}$ according to (10);
- 3: Update $\{h_k\}$ according to (12);
- 4: Repeat steps 1,2 and 3 until there is no significant improvement in the objective function $|y_{k,n} - h_k e^{j\theta_n}|^2$.

Different from scheme one, we first obtain the channel estimate by frequency domain filtering, then obtain the CPE through channel estimate, and then update the channel estimation based on the CPE.

At the first iteration, channel estimation is performed by LMMSE if the delay power spectrum is known, otherwise LS channel estimation is used. Also, when the channel delay power spectrum is known, the frequency domain filtering to update $\{h_k\}$ is added in the iterative process. If there is any other prior information about $\{h_k\}$, you can add it to the process.

4 MSE Analysis of Proposed Joint Estimation

Due to the complexity of the objective function, an analytical expression cannot be derived in theoretical analysis. Therefore, we will analyze the MSE of channel under the following two conditions.

4.1 $K=1, N=2$

Assuming that there is only one sub-carrier and the number of DMRS and PTRS is both 1, the optimization problem in (11) becomes

$$(\hat{h}, \hat{\theta}) = \arg \min_{h, \theta} (|y_1 - h|^2 + |y_2 - h e^{j\theta}|^2), \quad (14)$$

where \hat{h} , $\hat{\theta}$ are the channel to be sought and the CPE to be sought, y_1 and y_2 are the received signals of the DMRS and PTRS. The problem in (14) can be equivalently converted to another minimization problem:

$$(\hat{a}, \hat{\varphi}_1, \hat{\varphi}_2) = \arg \min_{a, \varphi_1, \varphi_2} (|y_1 - a e^{j\varphi_1}|^2 + |y_2 - a e^{j\varphi_2}|^2), \quad (15)$$

where $a = |h|$, $\varphi_1 = \text{angle}(h)$, $\varphi_2 = \varphi_1 + \theta$, Eq. (15) can be broken down into 3 separate problems for solving:

$$\begin{cases} \hat{\varphi}_1 = \arg \min_{\varphi_1} (|y_1 - a e^{j\varphi_1}|^2) = \text{angle}(y_1), \\ \hat{\varphi}_2 = \arg \min_{\varphi_2} (|y_2 - a e^{j\varphi_2}|^2) = \text{angle}(y_2), \\ \hat{a} = \arg \min_a [(|y_1| - a)^2 + (|y_2| - a)^2]. \end{cases} \quad (16)$$

Then we can get

$$\begin{cases} \hat{a} = \frac{|y_1|+|y_2|}{2}, \\ \hat{\theta} = \hat{\varphi}_2 - \hat{\varphi}_1 = \text{angle}(y_2) - \text{angle}(y_1) = \text{angle}(y_1^* y_2). \end{cases} \quad (17)$$

It can be seen that according to the ML criterion, the optimal estimation result of the CPE in (14) is consistent with the existing method in (9). According to (17), the performance of the MSE can be analyzed:

$$\begin{aligned} MSE_{channel} &= \frac{1}{2}E|h - \hat{a}e^{j\text{angle}(y_1)}|^2 + \frac{1}{2}E|h e^{j\theta} - \hat{a}e^{j\text{angle}(y_2)}|^2 \\ &= \frac{1}{2}E\left|h - \frac{|y_1|+|y_2|}{2}e^{j\text{angle}(y_1)}\right|^2 + \frac{1}{2}E\left|h e^{j\theta} - \frac{|y_1|+|y_2|}{2}e^{j\text{angle}(y_2)}\right|^2. \end{aligned} \quad (18)$$

The two terms in (18) should be equal, so only the first item is analyzed. Assuming that $h = 1, \theta = 0$:

$$\begin{aligned} MSE_{channel} &= E\left|1 - \frac{|1+n_1|+|1+n_2|}{2}e^{j\text{angle}(1+n_1)}\right|^2 \\ &= \frac{1}{4}E|1 - |1+n_2|e^{j\text{angle}(1+n_1)} - n_1|^2, \end{aligned} \quad (19)$$

where $e^{j\text{angle}(1+n_1)} \approx 1 + j\text{Im}(n_1)$, $|1+n_2| \approx 1 + \text{Re}(n_2)$, then we have

$$\begin{aligned} MSE_{channel} &= \frac{1}{4}E|1 - [1 + \text{Re}(n_2)][1 + j\text{Im}(n_1)] - n_1|^2 \\ &= \frac{1}{4}E|\text{Re}(n_1) + \text{Re}(n_2) + j\text{Im}(n_1)[2 + \text{Re}(n_2)]|^2 \\ &\approx \frac{1}{4}E|\text{Re}(n_1) + \text{Re}(n_2) + j2\text{Im}(n_1)|^2 \\ &= \frac{3}{4}\sigma^2. \end{aligned} \quad (20)$$

It can be seen from (20) that the MSE of the joint CPE and channel estimation is reduced compared to the no joint situation. However, since the CPE estimate is not accurate enough due to the small number of sub-carriers, the effect of LS channel estimation combining the two reference signals is not achieved.

4.2 $N = 2$

Assuming that the number of DMRS and PTRS is both 1 on each sub-carrier, the optimization problem in (11) becomes

$$(\{\hat{h}_k\}, \hat{\theta}) = \arg \min_{\{h_k\}, \theta} \sum_k (|y_{k,1} - h_k|^2 + |y_{k,2} - h_k e^{j\theta}|^2), \quad (21)$$

where $y_{k,1}$ and $y_{k,2}$ are the received signals of the DMRS and PTRS on the k -th subcarrier. When θ is determined, each item in $\{h_k\}$ affects only one of the additions, so problem (21) can be broken down into K independent optimization problems:

$$\begin{aligned} \hat{h}_k &= \arg \min_{h_k} (|y_{k,1} - h_k|^2 + |y_{k,2} - h_k e^{j\theta}|^2) \\ &= \arg \min_{h_k} (|y_{k,1} - h_k|^2 + |y_{k,2} e^{-j\theta} - h_k|^2) \\ &= \frac{y_{k,1} + y_{k,2} e^{-j\theta}}{2}. \end{aligned} \quad (22)$$

As can be seen from (22), \hat{h}_k can be represented by θ . Bringing (22) into (21), the problem is reduced to

$$\hat{\theta} = \arg \min_{\theta} \sum_k (|y_{k,1}e^{j\theta} - y_{k,2}|^2). \quad (23)$$

Equation (23) can be represented in vector form $\arg \min_{e^{j\theta}} \|\mathbf{y}_2 - \mathbf{y}_1 e^{j\theta}\|_F^2$, where

$$\begin{aligned} \mathbf{y}_1 &= [y_{1,1}, y_{2,1}, y_{3,1}, \dots, y_{K,1}]^T, \\ \mathbf{y}_2 &= [y_{1,2}, y_{2,2}, y_{3,2}, \dots, y_{K,2}]^T. \end{aligned} \quad (24)$$

Then, the problem in (23) can be transformed into a merger problem with one transmit and multiple receive, so we can get

$$\hat{\theta} = \text{angle}(\mathbf{y}_1^H \mathbf{y}_2) = \text{angle}\left(\sum_k y_{k,1}^* y_{k,2}\right). \quad (25)$$

When $K \gg 1$, the estimate of CPE is relatively accurate, and the combination in (22) can reduce the MSE to half of the noise power.

After the above theoretical analysis, we can see that this joint channel and CPE estimation algorithm can significantly improve system performance, and the performance is related to the number of sub-carriers. When the number of sub-carriers is large enough, the MSE can be reduced to the theoretical value.

5 Simulation Results and Analysis

In this section, we make simulations for comparison and analysis. Main assumptions are shown in Table 1.

Table 1. Simulation assumption

Parameters	Assumptions
Carrier frequency	30 GHz
Channel model	CDL-B
Subcarrier spacing	120 kHz
Allocated bandwidth	100 MHz
Number of RB	4
Coding scheme	Turbo
Channel estimation	LMMSE
SNR	13

5.1 No Prior Information

In this part, we compare the proposed joint estimation effects of different PTRS time domain densities. 1 DMRS and 3, 6 or 11 PTRS every sub-carrier in time domain are placed and three sub-carriers of the above pattern are placed in each RB. The simulation results are given in Fig. 2 where abscissa 0 represents without iteration. In Fig. 2, 1:3 represents the ratio of the number of DMRS and PTRS. It can be seen from the Fig. 2 that by the iteration, the MSE of the channel estimation is decreasing. However, what really works is the first iteration. That is, after the iterative process is called once, its MSE performance is basically stabilized. Also, we can see that the more the number of PTRS, the more the MSE declines since the number of PTRS will affect the accuracy of CPE estimate.

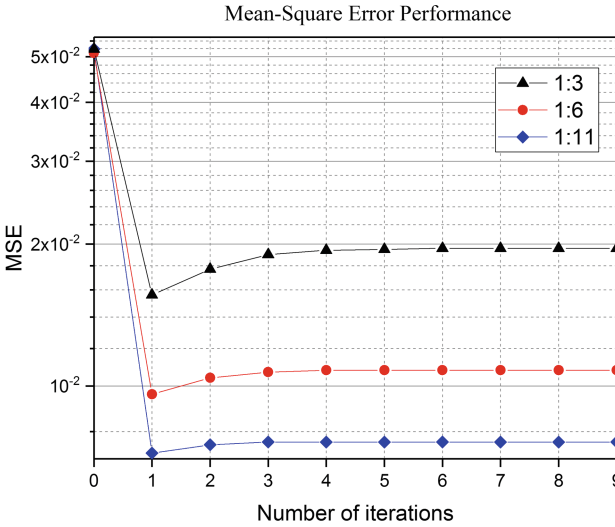


Fig. 2. Simulation result of joint estimation with no prior information

5.2 With Known Delay Power Spectrum

In this part, frequency domain filtering is added in the iterative process and the simulation results are given in Fig. 3. In order to get the system gain brought by each step in the iterative process, we do not use the iteration number as the abscissa, but the step in one iteration proposed in Sect. 3.2. It can be seen that every three steps is an iteration, step 1 is the beginning of the first iteration and step 3 is the end of the first iteration.

In Fig. 3, it can be seen that in each iteration, MSE is significantly reduced after frequency domain filtering, and MSE does not change much after other steps which shows the importance of frequency domain filtering. After the second iteration, the MSE is basically stabilized. We can see that after adding the

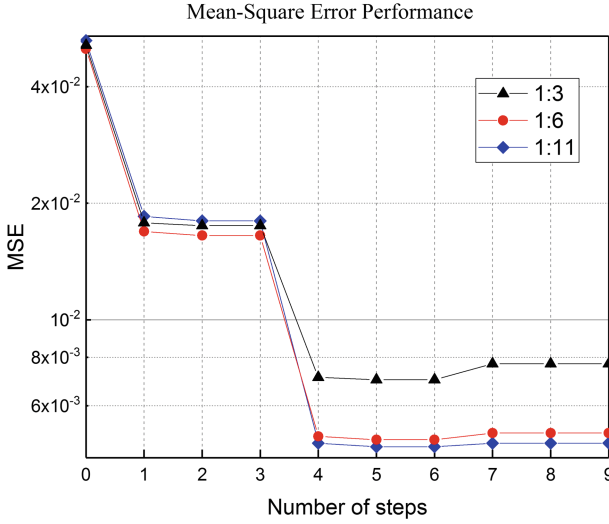


Fig. 3. Simulation result of joint estimation with known delay power spectrum

frequency domain filtering, the difference in system gain between the three cases is not as obvious as before. This is because after the first frequency domain filtering, we can get a more accurate channel estimate. Then we obtain CPE based on channel estimate instead of random assignment in Sect. 3.1. Therefore, the impact of the number of PTRS will be smaller.

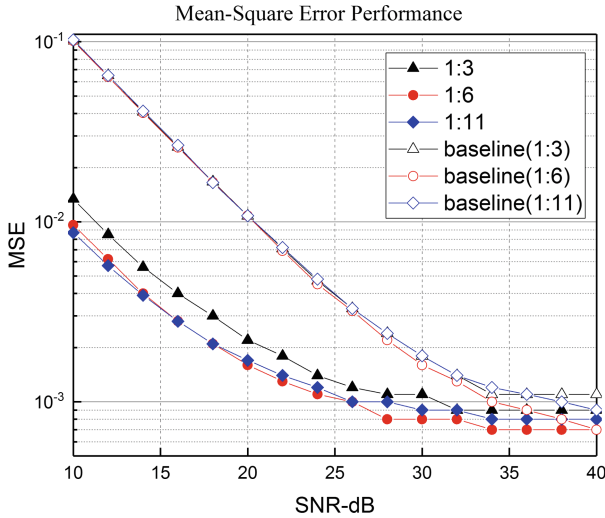


Fig. 4. Simulation result of the comparison between the joint estimation and the baseline

Then a set of baselines are added to make comparison, that LS was used for channel estimation and Eq. (8) for CPE estimation. As can be seen from the Fig. 4, the MSE decreases with increasing SNR under both algorithms. And under different SNR, the performance of the joint estimation algorithm is better than the traditional algorithm, especially when the SNR is small.

6 Conclusion

In this paper, an iterative algorithm is proposed to estimate CPE and channel. In this algorithm, a simplified system model which treats ICI as part of Gaussian noise is used to iteratively obtain more accurate channel and CPE estimates. We present the algorithm flow in two cases, one with no prior information and the other with known delay power spectrum. In addition, the system gain brought by the algorithm under two simplified conditions is analyzed. One condition is that the number of PTRS and DMRS is both 1, and the other condition is that the number of PTRS, DMRS and sub-carriers is all 1. The simulation results show that when there is more prior information about the channel, such as the delay power spectrum, the joint estimation of CPE and channel algorithm will perform much better. Meanwhile, the simulation results show that the proposed algorithm performs better than the traditional algorithm, especially when the SNR is small. Therefore, it can be concluded that this algorithm has a great impact on improving the accuracy of the estimation of channel and CPE.

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