Distributed Cooperative Positioning Algorithm Based on Message Passing Using Box Particles in UAVs Networks

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Abstract. Distributed cooperative positioning has become more and more attractive for large-scale unmanned aerial vehicles (UAVs) networks. In this paper, inspired by the box particle filter which combines interval analysis and Monte Carlo methods, a novel distributed cooperative positioning algorithm named Box-Particles Message Passing (BPMP) is proposed. In BPMP, the expressions of messages cannot be obtained in a closed form by belief propagation (BP) algorithm due to the nonlinearity of models and the complexity of computation. Accordingly, we use non-parametric belief propagation (NBP) also known as message passing methodology with a set of box particles to solve the inference problem of cooperative positioning on factor graph (FG) model in a 3-dimensional UAVs network. The proposed BPMP algorithm can reduce the number of particles while maintaining high accuracy. Simulation results demonstrate the effectiveness of proposed BPMP algorithm.

Keywords: UAVs networks · Cooperative positioning · Factor graph · Belief propagation · Box particles · Interval analysis

1 Introduction

The unmanned aerial vehicles (UAVs) networks are increasingly covered in diverse applications such as entertainment, security, surveillance. In these applications, the position information plays a significant role. Traditionally, the position information is provided by Global Navigation Satellite System (GNSS) due to its high performance and global coverage. Currently, the navigation of UVA cluster network is mostly based on the GNSS. However, the performance of navigation can be significantly degraded or nulled in the cities, gorges or indoor scenes. Moreover, GNSS signals can be easily disturbed or deceived by electronic counter-measurement. Therefore, novel navigation technique free of GNSS is needed and the concept of cooperative navigation is proposed [1]. In addition, some UAVs cannot be equipped with GNSS module due to the limits of cost

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and energy. In a large-scale UAVs network, it is impossible to manually position and it may lead to a communication bottleneck by a central system. Therefore, distributed structure is preferred for the large-scale networks [2]. As a new and popular method to resolve these problems, distributed cooperative positioning, incorporating peer-to-peer range measurements and exchanging position information between neighboring nodes, has attracted more and more attentions due to its ability to enhance the position accuracy and to improve availability [3].

Various algorithms have been proposed for distributed cooperative positioning. Generally, current methods can be divided into deterministic and probabilistic algorithms. A well-known deterministic algorithm is the least-square (LS) method, considering positioning as an optimization problem. Unlike deterministic algorithm, the probabilistic methods assign probability distributions for the whole space and attempt to avoid the trap of local optima [4].

Probabilistic graphical models such as Bayesian network and Markov random fields provide flexible and appealing tools for solving estimates problems with un-certainty in terms of network structures [5]. There have been several related inference methods under probabilistic graphical models. An attractive approach is the belief propagation (BP), which is also known as message passing or sum–product algorithm. It can efficiently perform exact or approximate marginalization [6]. Currently, the most important branch of BP algorithms is based on parametric messages. The parametric messages exchanged between neighboring nodes are represented by their means and variances [7]. Another branch, using particles to approximate the position distribution, is named non-parametric message passing (NBP) [8]. When the expression of UAV state transition equation and measurement equation is complicated, it is difficult or impossible to use a specific parameter to describe the state information. In this case, the state information can be described by a nonparametric method. In the confidence propagation algorithm, non-parametric belief propagation (NBP), based on particle filter, uses a set of weighted particles to approximate the distribution of states. A distributed cooperative positioning algorithm based on non-parametric belief propagation (NBP) for static networks was proposed in [9] and [10].

Nevertheless, a large number of particles should be used to improve the accuracy of the NBP, which leads to a high computational complexity and introduces communication overhead. Hence, it is strongly demanded to find a method to reduce the number of particles while maintaining high accuracy. As an alternative to the traditional NBP approaches, a novel distributed BPMP algorithm for cooperation positioning is proposed in UAVs networks. With the approximations, all messages passed on factor graph (FG) can be represented in the non-parametric forms with box particles which can significantly reduce computational complexity and improve communication cost.

2 Problem Formulation and System Model

A UAVs network can be divided into two types of UAVs. Those UAVs with known positions are anchor nodes A, while those with unknown positions are agent nodes B, S represents a collection of all UAVs, where $|S| = A + B$, $A = \{a_1, a_2, \cdots, a_n\}$ and $B = \{b_1, b_2, \cdots, b_m\}$. Let $\boldsymbol{X}_i^k = (\boldsymbol{x}_i^k, v_i^k)^T$, $i \in S$ denotes the state of the i - th UAV at time k comprising its position vector $\boldsymbol{x}_i^k \triangleq \left[x_i^k, y_i^k, z_i^k\right]^T$ and velocity vector v_i^k. An example scheme of partial network for UAVs can be seen in Fig. 1.

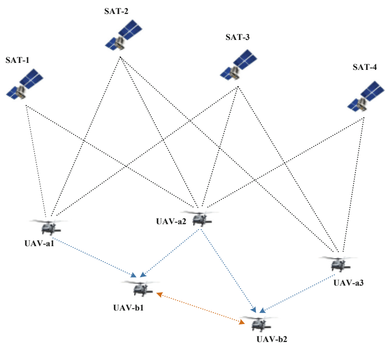

Fig. 1. An example scheme of partial network for UAVs.

In general, the nonlinear state-evolution model of UAV i can be given by

$$\boldsymbol{X}_i^k = \mathrm{F}_i^k(\boldsymbol{X}_i^{k-1}, \boldsymbol{Q}_i^k), i \in S \tag{1}$$

For anchor nodes at time step k, the GNSS observation model is modeled as

$$\boldsymbol{Y}_i^k = G_i^k(\boldsymbol{X}_i^k, b_i^k, \boldsymbol{P}_i^k), i \in A \tag{2}$$

Moreover, all agent nodes perform range measurements with its neighboring UAVs. We assume that the range measurement between node i and j is less than the communication radius R in line-of-sight environment. Consequently, peer-to-peer range measurement at the k th slot can be given as:

$$Z_{ij}^k = H(\boldsymbol{x}_i^k, \boldsymbol{x}_j^k, \boldsymbol{R}^k) = \|\boldsymbol{x}_i^k - \boldsymbol{x}_j^k\| + \boldsymbol{R}^k, i, j \in B \tag{3}$$

where $F(\bullet)$ is the motion state transition function, $G(\bullet)$ and $H(\bullet)$ are nonlinear observation function, $\boldsymbol{Q}_i^k$ represents the system noise, $\boldsymbol{P}^k$ and $\boldsymbol{R}^k$ are the additive white Gaussian noise and $\boldsymbol{R}_{ij}^k = \boldsymbol{R}_{ji}^k$, the $\|\bullet\|$ denotes the Euclidean distance. Within the Bayesian framework, the distributed cooperation positioning in UAVs networks can be substantially formulated as an inference problem on a graphical model.

3 Factor Graph Based and Belief Propagation

Generally, direct marginalization is intractable as the high dimensional integration. To solve this problem, we can employ FG model and BP algorithm for an efficient calculation [6]. Suppose that mobility is modeled as a Markov process, mutually independent for all UAVs, based on the Bayesian criterion and FG, a complex positioning problem in

UAVs networks at time step k can be break down into factors as

$$\begin{aligned} & p(\boldsymbol{X}_S^k | \boldsymbol{Z}^k) \\ & \propto p(\boldsymbol{Z}^k | \boldsymbol{X}_S^k) p(\boldsymbol{X}_S^k) \\ & \propto \prod_{i \in S} p(\boldsymbol{X}_i^k \Big| \boldsymbol{X}_i^{k-1}) \prod_{j \in A} p(y_j^k \Big| \boldsymbol{x}_j^k) \prod_{\alpha \in S, \beta \in n(\alpha)} p(d_{\alpha \to \beta}^k \Big| \boldsymbol{x}_\alpha^k, \boldsymbol{x}_\beta^k) \end{aligned} \tag{4}$$

A factor graph for distributed cooperative positioning in UAVs network as shown in Fig. 1 is depicted in Fig. 2, which is comprised of factor nodes f_i^k, g_i^k, h_{ij}^k (depicted by a squares) and variable node $\boldsymbol{x}_i^k$ (depicted by a circle). For simplicity, we define $f_i^k \triangleq p(\boldsymbol{X}_i^k \Big| \boldsymbol{X}_i^{k-1})$ represents the mobility of all nodes, $g_i^k \triangleq p(y_i^k | x_i^k)$ represents the pseudo range measurement likelihood for anchor nodes and $h_{ij}^k \triangleq p(d^k \Big| \boldsymbol{x}_i^k, \boldsymbol{x}_j^k)$ represents the peer-to-peer range measurement likelihood given by the positions of agent nodes i and j.

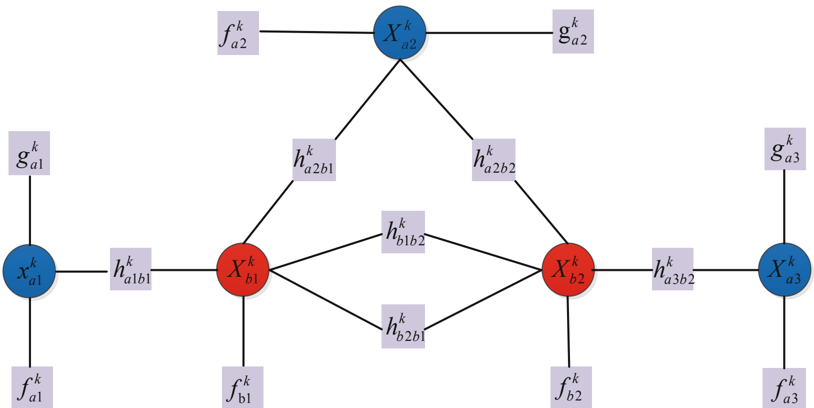

Fig. 2. Factor graph of partial UAVs network at time step k.

BP algorithm, passing messages along the edges between the nodes, is an appropriate tool with the advantage of convenient to get the posterior marginal probability on FG [11]. In BP, at time step k, the message from node i to j at time step k is denoted as

$$m_{ij}^k(x_j) = \int \psi(x_i, x_j) \psi(x_i) \prod_{h \in \mathrm{N}eb(i) \backslash j} m_{hi}(x_i) dx_i \tag{5}$$

The belief of node i can be approximated as the posterior marginal $p(x_i|Z)$.

4 Non-parametric Message Passing with Box Particle

Interval analysis is an effective tool to deal with bounded uncertainties [12]. In this section, basic notions of interval analysis and constraints satisfaction problems (CSP) which are also known as consistency techniques are presented firstly. Then, a box-particles message passing (BPMP) algorithm that uses interval analysis method is introduced.

4.1 Basic Notions of Interval Analysis

A real interval denoted $[x]$ in $\mathbb{R}$ is defined as

$$[x] = \left[\underline{x}, \bar{x}\right] = \left\{x \in \mathbb{R} \middle| \underline{x} \le x \le \bar{x}\right\} \tag{6}$$

A box $[\boldsymbol{x}]$ is defined as a Cartesian product of n-dimensional intervals can be expressed

$$[\boldsymbol{x}] = [x_1] \times [x_2] \times \cdots \times [x_n] \tag{7}$$

Where $\mathbb{R}$ is a closed and connected subset, $\underline{x}$ and $\bar{x}$ refer to the lower and upper bound of interval $[x]$ respectively. $|[x]| = \bar{x} - \underline{x}$ denotes the length of an interval $[x]$.

The usual arithmetic operations for intervals are given by [12]

$$[x] \pm \left[y\right] = \left[\underline{x} \pm \underline{y}, \bar{x} \pm \bar{y}\right] \tag{8}$$

$$[x] \times \left[y\right] = \left[\min(\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}), \max(\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y})\right] \tag{9}$$

If the interval $\left[y\right]$ does not include the 0 value,

$$[x]/\left[y\right] = [x] \times \left[1/\bar{y}, 1/\underline{y}\right] \tag{10}$$

Furthermore, elementary functions such as exp, ln, cos and sin, can be simply extended to intervals. All operations on intervals also can be extended to boxes.

In order to accelerate convergence velocity and reduce computational complexity, it is necessary to find inclusion functions $\left[f\right]$ approximating the image of $f(\boldsymbol{x})$ in a reasonable time.

Definition 1 (Inclusion function). Let f be a function from $\mathbb{R}^n$ to $\mathbb{R}^m$. The interval function $\left[f\right]$ is an inclusion function from $\mathbb{IR}^n$ to $\mathbb{IR}^m$ for f if

$$f([\boldsymbol{x}]) \subseteq \left[f\right]([\boldsymbol{x}]), \forall [\boldsymbol{x}] \in \mathbb{IR}^n \tag{11}$$

If f is continuous and monotonic, $\left[f\right]([\boldsymbol{x}])$ is equal to $f([\boldsymbol{x}])$. However, if f is a no monotonic continuous function, the computation of $\left[f\right]$ is usually not straight forward [18].

4.2 Constrains Satisfaction Problem

Another important concept in interval analysis is the constraints satisfaction problem (CSP). The goal of CSP is to find the smallest box $\left[\mathbf{x}'\right] \subseteq \left[\mathbf{x}\right]$ constraining the set of all $\boldsymbol{x}$ in the initial domain[x] which satisfies the constraints $\boldsymbol{f}$. The CSP can be denoted as $\mathbb{C}$ and formulated as follows [13]:

$$\mathbb{C} : (\boldsymbol{f}(\boldsymbol{x}) = 0, \boldsymbol{x} \in [\boldsymbol{x}]) \tag{12}$$

The solution set of the CSP $\mathbb{C}$ is given by

$$\mathbb{C} = \{\boldsymbol{x} \in [\boldsymbol{x}] | \boldsymbol{f}(\boldsymbol{x}) = 0\} \tag{13}$$

An contraction designates replacing [x] by a smaller domain $\left[\boldsymbol{x}'\right]$ such that

$$\boldsymbol{S} \subseteq \left[\boldsymbol{x}'\right] \subseteq [\boldsymbol{x}] \tag{14}$$

A simple and efficient contraction algorithm is constraints propagation (CP) method [14] which is independent of nonlinearities.

4.3 Box-Particle Message Passing

The key idea of BPMP is to use weighted box particles instead of weighted point samples representations to approximate the operations of non-Parametric Message on FG models. In BPMP, there are three main components, described as prediction message, update message and cooperative message respectively.

Prediction Message. Considering the box particles $\left\{\left[\boldsymbol{X}_{k-1}^{b}\right], \omega^{b}\right\}_{b=1}^{B}$ representing the state at time step $k-1$, the boxes particles at time step k can be passed through the state transition function.

$$\left[\boldsymbol{X}_{k}^{b}\right] = [\boldsymbol{F}]\left(\left[\boldsymbol{X}_{k-1}^{b}\right], \left[\boldsymbol{Q}_{k}\right]\right), b = 1, \cdots, B \tag{15}$$

Where $[\boldsymbol{F}]$ is an inclusion function for the state transition function $\boldsymbol{F}$, the system noise is enclosed in $\left[\boldsymbol{Q}_{k}\right]$.

At time step k, the prediction message $M_f(\boldsymbol{X}_i^k)$ is defined as

$$\begin{aligned} M_f(\boldsymbol{X}_i^k) &= \int p(\boldsymbol{X}_i^k \middle| \boldsymbol{X}_i^{k-1}) \sum_{b=1}^{B} w_{k-1}^{b} U_{\left[\boldsymbol{X}_{k-1}^{b}\right]}(\boldsymbol{X}_i^{k-1}) d\boldsymbol{X}_i^{k-1} \\ &= \sum_{b=1}^{B} w_{k-1}^{b} \int_{\left[\boldsymbol{X}_{k-1}^{b}\right]} p(\boldsymbol{X}_i^k \middle| \boldsymbol{X}_i^{k-1}) U_{\left[\boldsymbol{X}_{k-1}^{b}\right]}(\boldsymbol{X}_i^{k-1}) d\boldsymbol{X}_i^{k-1} \\ &= \sum_{b=1}^{B} w_{k-1}^{b} U_{[\boldsymbol{F}]\left(\left[\boldsymbol{X}_{k-1}^{b}\right], [\boldsymbol{Q}_k]\right)} \end{aligned} \tag{16}$$

Where $U_{[\boldsymbol{x}]}$ denotes the uniform distribution of the box $[\boldsymbol{X}]$.

Update Message
The main purpose of this step is to update the predicted box-particles message using the contraction described above at time step k. For anchor node, the contraction can be represented using the intersection between the predicted box $[z_k] = [\boldsymbol{x}_{ki}]$ and the observation box $\left[\hat{z}_k\right]$, where $\hat{z}_k$ is the GPS information of anchor nodes at time k.

For agent nodes, let $\left[\hat{z}_k\right]$ denote the range measurement between anchor node j and agent node i, the observation is the range measurement, then the predict box updated by

$$\left[\hat{\boldsymbol{Z}}_k\right] = \sqrt{(\left[\boldsymbol{X}_{ki}^{b}(1)\right] - \boldsymbol{X}_j(1))^2 + (\left[\boldsymbol{X}_{ki}^{b}(2)\right] - \boldsymbol{X}_j(2))^2 + (\left[\boldsymbol{X}_{ki}^{b}(3)\right] - \boldsymbol{X}_j(3))^2} \tag{17}$$

Therefore, the updated box can be expressed as

$$[\boldsymbol{R}_k] = \left[\hat{z}_k\right] \cap [z_k] \tag{18}$$

If $\boldsymbol{R}_k \notin \emptyset$, we can contract $[\boldsymbol{x}_{ki}]$ with $[\boldsymbol{R}_k]$ by CP algorithm [14].

Cooperative Message. The cooperative message $M_{ji}^k(\boldsymbol{x}_i^k)$ can be expressed as

$$M_{ji}^k(\boldsymbol{x}_i^k) = \int \psi(\boldsymbol{x}_j^k, \boldsymbol{x}_i^k) \prod_{h \in \mathrm{N}eb(j)\backslash i} m_{hj}(\boldsymbol{x}_j^k) d\boldsymbol{x}_j^k \tag{19}$$

Where $Neb(j)\backslash i$ denotes the set of neighbors of agent node j except node i.

The message $m_{hj}(\boldsymbol{x}_j^k)$ from the node h to the agent node j can be represented using N weighted boxes $\left\{\left[\boldsymbol{x}_{h_l j}^{nl}\right], \omega_{h_l j}^{nl}\right\}_{nl=1}^{N}$.

$$m_{h_l}(\boldsymbol{x}_j^k) = \sum_{nl=1}^{N} \omega_{h_l j}^{nl} U_{\left[\boldsymbol{x}_{h_l j}^{nl}\right]}(\boldsymbol{x}_j^k), l = 1, \cdots, L \tag{20}$$

Where L denoted the number of the neighborhood nodes.

Replacing the expression above in that of the message product according algorithm1, we can obtain $M_{ji}(x_j^k)$ as

$$\begin{aligned} M_{ji}(x_j^k) &= \prod_{h \in \mathrm{N}eb(j)\backslash i} m_{hj}(\boldsymbol{x}_j^k) \\ &\propto \prod_{l=1}^{L} (\sum_{nl=1}^{N} \omega_{h_l j}^{nl} U_{\left[x_{h_l j}^{nl}\right]}(x_j^k)) \\ &\propto \sum_{n=1}^{N} \omega_{h_1 j}^{n1} \cdots \omega_{h_L j}^{nL} U_{\left[x_{h_1 j}^{n1}\right]} \cdots U_{\left[x_{h_L j}^{nL}\right]}(x_j^k) \end{aligned} \tag{21}$$

Table 1 describes the algorithm of combining two messages [14, 15].

Then the message $M_{ji}(\boldsymbol{x}_i^k)$ can be given as

$$M_{ji}(\boldsymbol{x}_i^k) = \int \psi(\boldsymbol{x}_i^k, \boldsymbol{x}_j^k) \sum_{N} \omega^N U_{[x^N]}(\boldsymbol{x}_i^k) d\boldsymbol{x}_j^k \tag{22}$$

For ease of calculation, the $\psi(\boldsymbol{x}_i^k, \boldsymbol{x}_j^k)$ can be transformed into a form through $\boldsymbol{x}_i^k = f(\boldsymbol{x}_j^k, d, \varepsilon)$ according Fig. 3. Then an inclusion function $[f]$ allowing for converting to the expression of tangent square and inscribed square of the annular.

In Fig. 3, the smallest rectangular region represents the position distribution of cooperative node i, the position of node j should fall on the annular. Assume that the ranging measurement between nodes is d, the error is ε, Then the circular region represents the

Table 1. Message combination algorithm.

1. Set $\mathbf{x}$ and ω empty, $N = 0$
2. Message 1 and message 2 are represented by $\left\{\left[\mathbf{x}_1^l\right], \omega_1^l\right\}_{l=1}^{L}$ and $\left\{\left[\mathbf{x}_2^u\right], \omega_2^u\right\}_{u=1}^{U}$
3. For $l = 1 : L$

   For $u = 1 : U$

   If, $\left[\mathbf{x}_1^l\right] \cap \left[\mathbf{x}_2^u\right] \neq \varnothing$,

   -- $k = k + 1$

   -- $\left[\mathbf{x}^N\right] = \left[\mathbf{x}_1^l\right] \cap \left[\mathbf{x}_2^u\right]$

   -- $\omega^N = \omega^l \times \omega^u \times \dfrac{\left[\mathbf{x}_1^l\right] \cap \left[\mathbf{x}_2^u\right]}{\left[\mathbf{x}_1^l\right] \bullet \left[\mathbf{x}_2^u\right]}$

   End for

   End for
4. Normalize the weights, $\omega^n = \omega^n / \sum_{n=1}^{N} \omega^n$

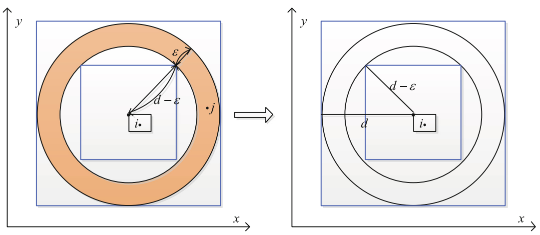

Fig. 3. Projection of ranging model on the XY axis.

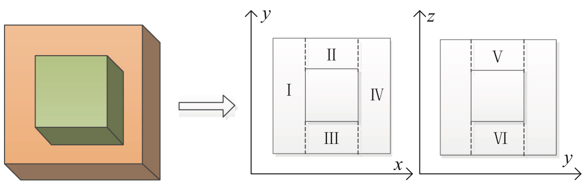

Fig. 4. The diagram of interval partition.

position distribution of the node *j*. In order to represent the annular region as a box particle, the region is divided into six intervals (Fig. 4).

Let the interval of cooperative node *i* be expressed as $[\mathbf{x}_i] = [\underline{x}_i, \overline{x}] \times [\underline{y}_i, \overline{y}_i] \times [\underline{z}_i, \overline{z}_i]$ and $[d] = [d - \varepsilon, d], \theta \in [0, \pi]$, then the outer cube area can be expressed as follows

$$\begin{cases} [x_b] = [\underline{x}_b, \overline{x}_b] = [x_i] + d \bullet \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \\ [y_b] = [\underline{y}_b, \overline{y}_b] = [y_i] + d \bullet \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \\ [z_b] = [\underline{z}_b, \overline{z}_b] = [z_i] + d \bullet \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \end{cases} \tag{23}$$

The inner cube area can be expressed as

$$\begin{cases} [x_s] = \left[\underline{x}_i - \sqrt{2}/2(d - \varepsilon), \bar{x}_i + \sqrt{2}/2(d - \varepsilon)\right] \\ [y_s] = \left[\underline{y}_i - \sqrt{2}/2(d - \varepsilon), \bar{y}_i + \sqrt{2}/2(d - \varepsilon)\right] \\ [z_s] = \left[\underline{z}_i - \sqrt{2}/2(d - \varepsilon), \bar{z}_i + \sqrt{2}/2(d - \varepsilon)\right] \end{cases} \tag{24}$$

That is, the six intervals divided can be expressed as

$$\begin{aligned} &\mathrm{I}: [\underline{x_b}, \underline{x_s}] \times [\underline{y_b}, \overline{y_b}] \times [\underline{z_b}, \overline{z_b}] \\ &\mathrm{II}: [\underline{x_s}, \overline{x_s}] \times [\overline{y_s}, \overline{y_b}] \times [\underline{z_b}, \overline{z_b}] \\ &\mathrm{III}: [\underline{x_s}, \overline{x_s}] \times [\underline{y_b}, \underline{y_s}] \times [\underline{z_b}, \overline{z_b}] \\ &\mathrm{IV}: [\overline{x_s}, \overline{x_b}] \times [\underline{y_b}, \overline{y_b}] \times [\underline{z_b}, \overline{z_b}] \\ &\mathrm{V}: [\underline{x_s}, \overline{x_s}] \times [\underline{y_s}, \overline{y_s}] \times [\overline{z_s}, \overline{z_b}] \\ &\mathrm{VI}: [\underline{x_s}, \overline{x_s}] \times [\underline{y_s}, \overline{y_s}] \times [\underline{z_b}, \underline{z_s}] \end{aligned} \tag{25}$$

According to the above formula, the box of node *j* can be calculated according to the interval of cooperative node *i* (Table 2).

Table 2. A summary of proposed BPMP algorithm.

Initialize: all UAVs $i \in S$ **in parallel**

---Generate B boxes with the same width and weights $\{[\mathbf{X}_0^b], \omega_0^b\}, \omega_0^b = 1/N$ for each node

Output: position estimate $\mathbf{X}_i^k$

1. **For time** $k = 1$ **to** K **do**

   $\forall i \in S$ **in parallel do**

2. **Message prediction**

   ---According to prediction message $M_f(x_i^k)$ using (15), get $\{[\mathbf{X}_k^b], \omega_k^b\}_{b=1}^B$

3. **Message update**

   ---Update the prediction message box $\{[\mathbf{X}_k^b], \omega_k^b\}_{b=1}^B$ through the contractions and CP

algorithm using observation measurement $[y_k]$, get $\{[\mathbf{X}_k^{b\_new}], \omega_k^{b\_new}\}$.

4. **for** iteration $t = 1 : T$

for $\forall i \in S$

5. **Cooperative message calculation**

   ---calculate the cooperative message $M_{ji}(\mathbf{x}_i^k)$ boxes according to Algorithm 1and inclusion

function $[f]$ using(23) and (24)

6. **end for**
7. **end for**
8. **Belief calculation**

   ---Calculate the belief $b^{(t)}(\mathbf{X}_i^k)$ according to (4) and Algorithm 1

9. **Estimate UAVs' positions using** $\hat{x} = \sum \omega_{k\_bel}^l [x_k^l]_{bel}$ **;**

5 Simulation Results

We evaluate the performance of the proposed BPMP algorithm using Monte Carlo simulations. An 100 m × 100 m × 100 m simulation scenario with 5 anchors (M1, M2,…, M5) and 10 agents (N1, N2,…, N10) is considered. In the simulation scenario, all nodes are randomly scattered and can move independently. For simplicity, ranging measurements are assumed to be performed in line-of-sight environment with 100 m communication range. All results are averaged from 20 independent Monte Carlo simulation runs. We also set the numbers of initial boxes is 10, and the width is 2.

As can be shown in Fig. 5, the true motion trajectories and estimate trajectories using BPMP algorithm of nodes (incluing3 agents and 2 anchors) are convergent.

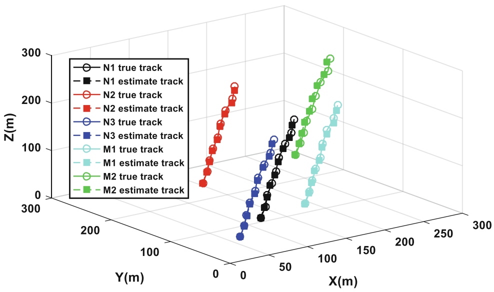

Fig. 5. The true track and estimate track of UAVs.

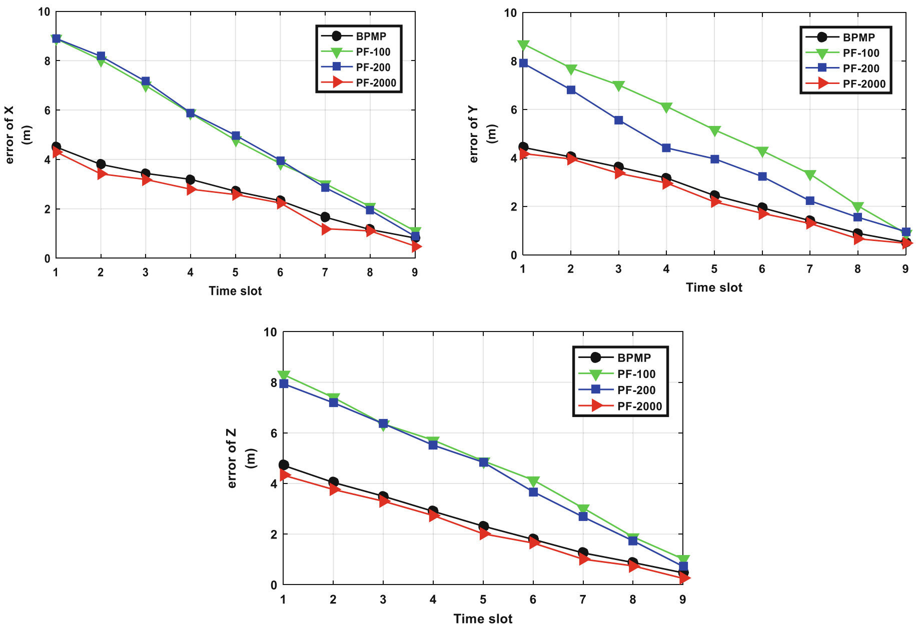

Fig. 6. The estimated error for x, y and z of BPMP and PF.

The Fig. 6 shows the interval error of x, y and z estimated by BPMP, PF-100, PF-200 and PF-2000 respectively. It can be obviously observed that the positioning accuracy of PF and the proposed BPMP algorithm tend to converge with timeslot increasing, and compared with PF, BPMP needs fewer boxes to achieve approximate positioning accuracy, which means the BPMP algorithm need simpler and faster computations and thus more time saving.

According to the Fig. 7, we can see that the positioning accuracy of different positioning algorithms in the UAVs network. The positioning accuracy of BPMP algorithm is better than the particle filter algorithm with fewer particles, and it is close to particle filter algorithm with large number of particles, however, the number of required box particles and resource consumption are significantly reduced.

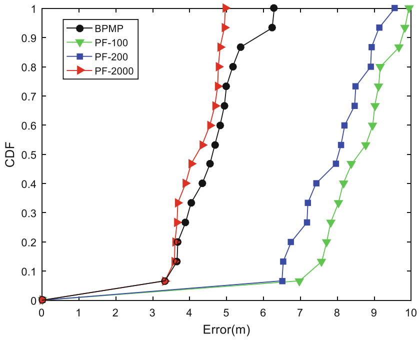

Fig. 7. Cumulative distribution function (CDF).

The Fig. 8 shows the effects of the box-particle numbers and interval width on the positioning accuracy. It can be seen that the more box-particle numbers, the smaller the interval width and the positioning error. When both of the box-particle numbers and the interval width are different, the interval width will have a greater impact on the positioning accuracy. The selection of the interval width and box-particle numbers are to be studied further.

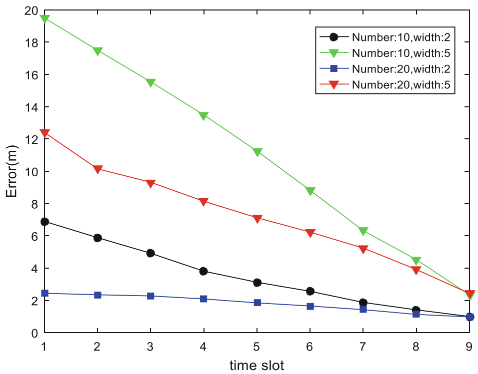

Fig. 8. The effect of the box-particle numbers and the interval width on the positioning accuracy.

6 Conclusion

In this paper, we propose BPMP method based on interval analysis and particles filter to improve the performance of the NBP algorithm. The proposed BPMP algorithm is achieved by box-particles representations of probability quantities to infer on factor graph model for distributed cooperative positioning. Simulation results show that the proposed BPMP algorithm has superior performance compared to PF method BPMP achieves better positioning accuracy with low communication overhead and computational complexity. In the process of message combination, the overlap remains as an issue. Therefore, resampling should be considered to prevent the degradation of box particles. Hence, further research on these problems will be carried out in follow-up work.

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