Spectra Efficient Space Time Coding

Rui Yin[1(✉)], Zhiqun Zou[1], Celimuge Wu[2], Hongjun Xu[3], and Chao Chen[4]

[1] School of Information and Electrical Engineering, Zhejiang University City College, Hangzhou 310015, China
yinrui@zucc.edu.cn
[2] Graduate School of Informatics and Engineering, The University of Electro-Communications, 1-5-1, Chofugaoka, Chofu-shi, Tokyo 182-8585, Japan
[3] School of Electrical, Electronic and Computer Engineering, KwaZulu-Natal University, Durban, South Africa
[4] School of Information Science and Electronic Engineering, Zhejiang Gongshang University, Hangzhou, China

Abstract. In this letter, a new *space time* (ST) coding scheme is proposed to improve the system *spectral efficiency* (SE). To improve the SE while achieving full diversity gain, the ST coded symbol transmission sequence (denoted as ST coding pattern in time domain) is exploited to improve the transmission rate. Since the orthogonal construction of the ST codes is preserved, the simple decoding scheme (linear maximum likelihood detector) is still applicable. Based on the analysis and Monte Carlo simulations, we demonstrate that the data rate increases by 25% and the its *bit error rate* (BER) is close to the conventional ST codes when 16-QAM or 16-PSK is used in the system.

Keywords: Spectrum efficiency · Space time coding · MIMO

1 Introduction

The rapid increase on personal intelligent terminals, such as tablets and smart phones, has led to the demand for high data rate with high reliability transmission in wireless communication systems [1]. To deal with such challenge, scholars and engineers have developed new technologies to improve the *spectrum efficiency* (SE) of the limited frequency resource. *Multiple input and multiple output* (MIMO) is one of them, which can improve the SE tremendously via introducing the spatial freedom. Specifically, the massive MIMO technique has become one of the key technologies for 5G system [2].

Theoretically, by using MIMO technique, both diversity and multiplex gain can be achieved to improve the transmit data rate and reliability. According to [3], there is a fundamental tradeoff between diversity and multiplex gain in MIMO system. To achieve these advantages in a practical MIMO system, Alamouti *space-time-block code* (STBC) has been proposed in [4]. In a conventional

S. W. Loke et al. (Eds.): MONAMI 2020, LNICST 338, pp. 213–225, 2020.
https://doi.org/10.1007/978-3-030-64002-6_14

Alamouti scheme [5], two antennas are deployed at a transmitter and two independent source symbols are transmitted via the antennas simultaneously over two consecutive time slots. The essence of the Alamouti scheme is that the encoding matrix maintains an orthogonal structure in two time slots, which allows a simple linear *maximum-likelihood* (ML) decoder in the quasi-static frequency flat Rayleigh block fading channels. Basically, Alamouti can achieve full diversity gain as demonstrated in [5]. But, it can not achieve full multiplex gain, which would reduce the transmission data rate.

To improve the SE of STBC, the efficiency of the STBC with *quadrature phase shift key* (QPSK) modulation has been investigated in [6]. Since the code efficiency is only 0.5, authors in [6] have used two QPSK constellations for STBC, which allows an additional bit to be sent. For QPSK based STBC, the SE increases by 0.5 bit/s/Hz. As an extension work of [7], the same strategy has been utilized when the number of transmission antenna is 4. In [8], a *space-time block coded spatial modulation* (STBC-SM) has been proposed, which combines *spatial modulation* (SM) and STBC to improve the SE. In the STBC-SM scheme, the transmitted symbols include not only the source information, but also the antenna indices information. As an extensional work of [8], a differential transmission scheme based on cyclic temporally and spatially modulated STBC has been developed in [9] to improve the SE. A high data rate STBC spatial modulation with cyclic structure has been proposed in [10]. In the scheme, a pair of transmit antennas is selected to send STBC symbols drawn from two different constellations, while the antenna pairs are moving cyclically along the total transmit antenna array. In [11], a constellation collaborated nonlinear orthogonal space-time block codes has been proposed to improve the BER performance. A corresponding fast maximum-likelihood detection scheme has been also proposed. Super-orthogonal space-time trellis codes have been applied in STBC-SM to further improve the SE in [12].

Most works mentioned above sacrifice either the diversity gain or the complexity of the detector at the receiver to achieve more SE. In this paper, we design a novel STBC scheme to improve the system SE while retaining the simple linear maximum likelihood detector and the diversity gain of the classic Alamouti scheme. The basic idea of the new scheme is to transmit more information by exploiting the sequence of transmitted symbols in time domain. As a result, additional bits can be transmitted.

The rest of the paper is organized as follows. In Sect. 2, a MIMO system with two transmit antennas and N_r receive antennas using Alamouti scheme is introduced. Section 3 proposes a novel space time coding scheme. Following this, *bit error rate* (BER) performance of the new scheme is analyzed in Sect. 4. Simulation results are demonstrated in Sect. 5 and we conclude in Sect. 6.

Notation: $|\cdot|$ and $\|\cdot\|_F$ denote the Euclidean and Frobenius norm operations, respectively. $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent the complex conjugate, transpose and Hermitian operations, respectively.

2 Classic Alamouti Scheme

In the system, there are two transmit antennas and N_r antennas deployed at the receiver. The source bits are first divided into two streams. Each binary stream then goes through a modulator to map bits into symbols, x_1 and x_2. Each symbol contains $r = \log_2(M)$ bits, where M is the number of constellation points of QAM. Based on the Alamouti scheme, one transmission interval is divided into two consecutive time slots. Symbols x_1 and x_2 are transmitted via antenna 1 and antenna 2 at time slot one, and $-x_2^*$ and x_1^* are sent via two antennas at the consecutive time slot. Then, the received signals, $\mathbf{Y}^{(C)} = \left[\mathbf{y}_1^{(C)}, \mathbf{y}_2^{(C)}\right]$, are given by

$$\text{time slot 1}: \ \mathbf{y}_1^{(C)} = \sqrt{\frac{\rho}{2}}\left(\mathbf{h}_1 x_1 + \mathbf{h}_2 x_2\right) + \mathbf{N}_1 \tag{1}$$

and

$$\text{time slot 2}: \ \mathbf{y}_2{}^{(C)} = \sqrt{\frac{\rho}{2}}\left(-\mathbf{h}_1 x_2^* + \mathbf{h}_2 x_1^*\right) + \mathbf{N}_2, \tag{2}$$

respectively. Herein, $\mathbf{h}_1$ is an $N_r \times 1$ vector to denote the channel gain from the first transmit antenna to N_r receive antennas. Similarly, $\mathbf{h}_2$ has the same dimension as $\mathbf{h}_1$ to represent the channel gain from the second transmit antenna to receive antennas. The upper symbol, C denotes ‘classic’. In this paper, we assume that the channels are quasi-static frequency-flat Rayleigh fading channels. Therefore, $\mathbf{h}_1$ and $\mathbf{h}_2$ do not change in one transmission interval and the items in $\mathbf{h}_1$ and $\mathbf{h}_2$ are the complex Gaussian variables. $\mathbf{N}_1$ and $\mathbf{N}_2$ are $N_r \times 1$ noise vector with complex Gaussian distribution, $\mathcal{CN}(0, 1)$. $\sqrt{\frac{\rho}{2}}$ is the average *signal-to-noise ratio* (SNR) at each receive antenna.

When the receiver has the *perfect channel state information* (CSI), it manipulates the received signals to yield

$$\mathbf{Y}_r = \begin{bmatrix} \mathbf{y}_1^{(C)} \\ -\left(\mathbf{y}_2^{(C)}\right)^* \end{bmatrix} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 \\ -\mathbf{h}_2^* & \mathbf{h}_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \mathbf{N}_1 \\ -\mathbf{N}_2^* \end{bmatrix}. \tag{3}$$

Define

$$\mathbf{H} \triangleq \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 \\ -\mathbf{h}_2^* & \mathbf{h}_1^* \end{bmatrix}. \tag{4}$$

Then, the receiver computes the following metric to have

$$\mathbf{Z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \mathbf{H}^H \mathbf{Y}_r. \tag{5}$$

Due to the orthogonal structure of the Alamouti code, the equalized symbols are calculated as

$$\hat{x}_1 = \tfrac{z_1}{\|\mathbf{h}_1\|_F^2 + \|\mathbf{h}_2\|_F^2}, \qquad \hat{x}_2 = \tfrac{z_2}{\|\mathbf{h}_1\|_F^2 + \|\mathbf{h}_1\|_F^2}.$$

Then, the receiver can recover the transmitted symbols via demodulating $\widehat{x}_1$ and $\widehat{x}_2$. According to [6], the encoded symbols are not independent in Alamouti scheme. Taking (1) and (2) as examples, symbols x_1 and x_2 are transmitted in the first time slot. Redundancy related to x_1 and x_2 are transmitted in the second time slot to exploit the diversity gain in spatial domain. Since the same information is transmitted in two consecutive time slots, SE is sacrificed. To make up this shortcoming, in next section, we propose a new scheme to exploit the sequence of the transmitted information in time domain so that one more bit can be carried.

3 Proposed STBC with Additional Spectral Efficiency

To improve the SE, another Alamouti coding pattern is designed by switching the transmitted symbols in two consecutive time slots. Then, the received signal, $\mathbf{Y}^{(N)} = \left[\mathbf{y}_1^{(N)}, \mathbf{y}_2^{(N)}\right]$, at the receiver is given by

$$\text{time slot 1}: \ \mathbf{y}_1^{(N)} = \sqrt{\frac{\rho}{2}} \left(-\mathbf{h}_1 x_2^* + \mathbf{h}_2 x_1^*\right) + \mathbf{N}_1, \tag{6}$$

and

$$\text{time slot 2}: \ \mathbf{y}_2^{(N)} = \sqrt{\frac{\rho}{2}} \left(\mathbf{h}_1 x_1 + \mathbf{h}_2 x_2\right) + \mathbf{N}_2, \tag{7}$$

where the upper symbol, N, means 'new'. According to (1), (2), (6) and (7), it is observed that the only difference between the *new Alamouti coding pattern* (NAP) and the *classic Alamouti coding pattern* (CAP) is that $-x_2^*$ and x_1^* are transmitted in the first time slot, and x_1 and x_2 are transmitted in the second time slot. Therefore, the orthogonal structure is still preserved in NAP and the simple ML detector can be used to recover the transmitted symbols at the receiver.

In order to carry one more bit, transmitter can select either CAP or NAP scheme in each transmission interval based on the value of a variable, b. When b is equal to zero, CAP is selected. Otherwise, NAP scheme is chosen at the transmitter. The new transmission scheme is demonstrated in Fig. 1.

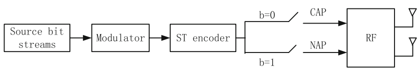

Fig. 1. Transmission with new STBC scheme

To recover the source symbols, the receiver applies the ML detection twice based on the NAP and CAP pattern, respectively. Assuming the symbols obtained based on NAP pattern are denoted as $\mathbf{x}^{(NAP)} = \left\{x_1^{(NAP)}, x_2^{(NAP)}\right\}$. On the other hand, the symbols recovered based on CAP are represented as

$\mathbf{x}^{(CAP)} = \left\{x_1^{(CAP)}, x_2^{(CAP)}\right\}$. Then, replace $\mathbf{a} = \{a_1, a_2\}$ of the following equation with $\mathbf{x}^{(NAP)}$ and $\mathbf{x}^{(CAP)}$,

$$\widehat{\mathbf{y}}_1 = \sqrt{\frac{\rho}{2}} \left(\mathbf{h}_1 a_1 + \mathbf{h}_2 a_2\right), \tag{8}$$

$$\widehat{\mathbf{y}}_2 = \sqrt{\frac{\rho}{2}} \left(-\mathbf{h}_1 a_2^* + \mathbf{h}_2 a_1^*\right), \tag{9}$$

respectively, to obtain $\mathbf{Y}^{(CAP)} = \left[\widehat{\mathbf{y}}_1^{(CAP)}, \widehat{\mathbf{y}}_2^{(CAP)}\right]$ and $\mathbf{Y}^{(NAP)} = \left[\widehat{\mathbf{y}}_1^{(NAP)}, \widehat{\mathbf{y}}_2^{(NAP)}\right]$. Since the received signal, $\mathbf{Y}$, at the receiver is either equal to $\mathbf{Y}^{(C)}$ or $\mathbf{Y}^{(N)}$, the receiver compares the Frobenius norm $d^{(CAP)} = \left\|\mathbf{Y} - \mathbf{Y}^{(CAP)}\right\|_F^2$ and $d^{(NAP)} = \left\|\mathbf{Y} - \mathbf{Y}^{(NAP)}\right\|_F^2$. If $d^{(CAP)} < d^{(NAP)}$, then $\mathbf{x}^{(CAP)}$ and $b = 0$ are the outputs of the decoder. Otherwise, the decoder outputs $\mathbf{x}^{(NAP)}$ and $b = 1$. According to the above description, the detection scheme at the receiver is concluded in Table 1. Since one more bit is transmitted in the proposed scheme, the SE is improved by $1/\log_2(M)$ when M-ary modulation is used in the system.

Table 1. Detection scheme

Algorithm 1

Initialize two vectors, $\mathbf{x}^{(NAP)}$ and $\mathbf{x}^{(CAP)}$. Two matrices, $\mathbf{Y}^{(NAP)}$ and $\mathbf{Y}^{(CAP)}$;
Decode received signals based on CAP pattern and ML detection method and output $\mathbf{x}^{(CAP)} = \left\{x_1^{(CAP)}, x_2^{(CAP)}\right\}$. Then calculate $\mathbf{Y}^{(CAP)}$ based on (10) and (14);
Decode received signals based on NAP pattern and ML detection method and output $\mathbf{x}^{(NAP)} = \left\{x_1^{(NAP)}, x_2^{(NAP)}\right\}$. Then calculate $\mathbf{Y}^{(NAP)}$ based on (10) and (14);
Calculate $d^{(CAP)} = \left\|\mathbf{Y} - \mathbf{Y}^{(CAP)}\right\|_F^2$ and $d^{(NAP)} = \left\|\mathbf{Y} - \mathbf{Y}^{(NAP)}\right\|_F^2$;
if $d^{(CAP)} < d^{(NAP)}$ **then**
    $\mathbf{x}^{(CAP)}$ and $b = 0$;
else
    $\mathbf{x}^{(NAP)}$ and $b = 1$;
end if
Demodulate the output symbols based on the constellation of the modulation;

Comparing to the STBC-SM proposed in [8], the new scheme has the advantages on low detection complexity. At the transmitter, it only needs to swap the ST coded symbols of different slots, which is easy to be implemented in practice. At the receiver, the computational complexity is $O(4M)$ in Algorithm 1. On the other hand, the computational complexity at the receiver for STBC-SM is $O(cM^2)$, where c is the number of possible antenna combinations.

The proposed scheme can also be applied to the system with more than two transmit antennas. For example, the STBC $\mathcal{G}_4$ evaluated in [13] uses four transmit antennas and four consecutive time slots. Based on the algorithm 1, there exists 24 STBC patterns by swapping the STBC coded symbols of different time slots. Therefore, $1 \leq b \leq 4$ extra bits can be carried by the new scheme.

4 BER Performance of the Proposed Scheme

In this section, an asymptotic BER performance bound for the proposed scheme is analyzed when *M-ary phase shift keying* (MPSK) or *M-ary quadrature amplitude modulation* (MQAM) modulation are applied. The joint detection scheme represented in Table 1 includes estimation of two quantities: the index bit, b, and transmitted symbols. Therefore, the BER performance is decided by these two processes. Since it is hard to derive a closed-form expression for the joint detection, we assume that the index and symbol estimation processes are independent as [14] to derive a low bound on BER of the proposed scheme. Let p_i be the BER of index estimation given that the symbols are perfectly detected and p_s be the BER of symbol estimation given that the index is perfectly decoded. Then, the BER of the system is lower bounded by

$$p_b \geqslant 1 - (1 - p_i)\,(1 - p_s)\,. \tag{10}$$

It is noteworthy that Algorithm 1 detects the index bit, b, and transmitted symbols jointly. As a result, the assumption of independent estimation processes is not realistic and cannot reach an accurate BER performance. But with this assumption, we can derive a closed-form BER lower bound for the proposed scheme as presented in (10).

First, we derive the BER of index bit estimation given that symbols, x_1 and x_2, are detected correctly. According to the appendix A, it is low bounded by (32).

Then, we derive the BER of transmitted symbols. Due to the orthogonality of the Alamouti STBC, an Alamouti scheme with two transmit and N_r receive antennas is equivalent to an *maximum ratio combination* (MRC) scheme with one transmit and $2N_r$ receive antennas with 3 dB loss [5]. Under the assumption that the index bit can be recovered correctly, the detection for symbols in our scheme is equal to the detection process for the Alamouti scheme. Therefore, the SER over a Rayleigh fading channel is given by [15]

$$p_s^{(PSK)} \approx \frac{\alpha}{2c} \left\{ \begin{array}{l} (1 + s(0))^{-2N_r} + (1 + s(\alpha\pi))^{-2N_r} + \\ 2\sum_{k=1}^{c-1} \left(1 + s\left(\frac{k\alpha\pi}{c}\right)\right)^{-2N_r} \end{array} \right\} \tag{11}$$

where $\kappa = \sin^2\left(\frac{\pi}{L}\right)$, $\alpha = \frac{M-1}{M}$, $s(\omega) = \frac{-\kappa\rho}{\sin^2\omega}$, c is a constant chosen for convergence, when M-ary PSK is used.

On the other hand, when M-ary QAM is used, the SER is expressed as

$$p_s^{(QAM)} \approx \frac{a}{c} \left\{ \begin{array}{l} \left(\frac{2}{bp+2}\right)^{N_r} - \frac{a}{2}\left(\frac{1}{bp+1}\right)^{N_r} + \\ (1-a)\sum_{i=1}^{c-1} \left(\frac{S(\theta_i)}{bp+S_i}\right)^{N_r} + \sum_{i=c}^{2c-1} \left(\frac{S(\theta)}{bp+S_i}\right)^{N_r} \end{array} \right\} \tag{12}$$

where $a = 1 - \frac{1}{\sqrt{M}}$, $b = \frac{3}{M-1}$, $S(\theta_i) = 2\sin^2\theta_i$, $\theta_i = \frac{i\pi}{4n}$, and c is the number of summations. It is demonstrated that the difference between the simulated and theoretical SER for 4QAM configuration is only 0.0015 dB when c is greater than 10.

The BER, p_s, can be then derived from the approximate relationship as

$$p_s \approx \frac{p_s^{(PSK)} \text{ or } p_s^{(QAM)}}{\log_2(M)}. \tag{13}$$

Then, according to the (10), we can derive the BER of the proposed new scheme.

5 Numerical Results

In this section, the Monte Carlo simulation results are presented to demonstrate the performance of the proposed scheme. The 16-QAM and 16-PSK based STBC are used as references to validate that the BER of the proposed scheme is close to that of the conventional STBC. Assuming that the receiver has the accurate quasi-static frequency-flat Rayleigh fading channel information and the Gray coded M-QAM and M-PSK constellation points are used in the system. Figure 2 demonstrates the BER performance (SNR versus average BER) of the proposed scheme by the Monte Carlo simulation and analysis in (10) when 16AQM is used. It is evident that the formulated BER is the lower bound of the simulation results, which validate our analysis.

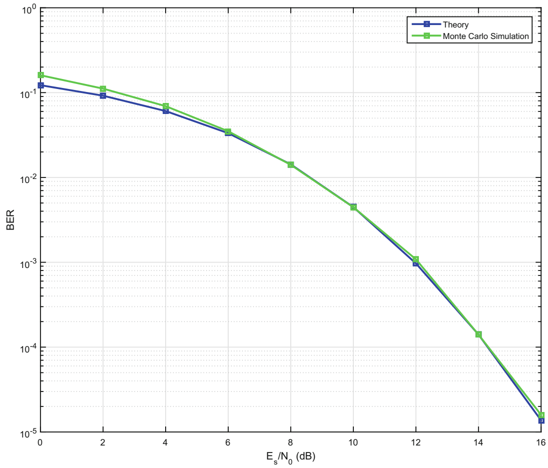

Fig. 2. Validation of analysis on BER for proposed scheme when 16-QAM is used

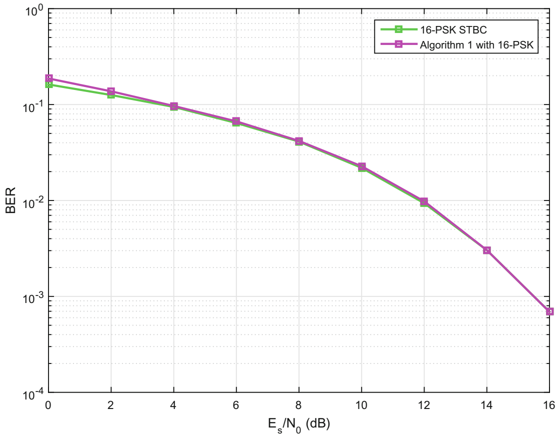

Fig. 3. Monte Carlo simulation results for the proposed and STBC schemes with 16-PSK

In Fig. 3 and 4, we compare the BER performance of the proposed scheme with the classic STBC scheme when 16-PSK and 16-QAM are used, respectively. We can observe that both schemes have the same slop which means our scheme can exploit the same diversity order as the classic STBC. Moreover, the BER performance of the new scheme is very close to that of the classic STBC. Therefore, without sacrificing the BER performance, the new scheme can improve the system SE by 25% when 16-QAM or 16-PSK is used in the system.

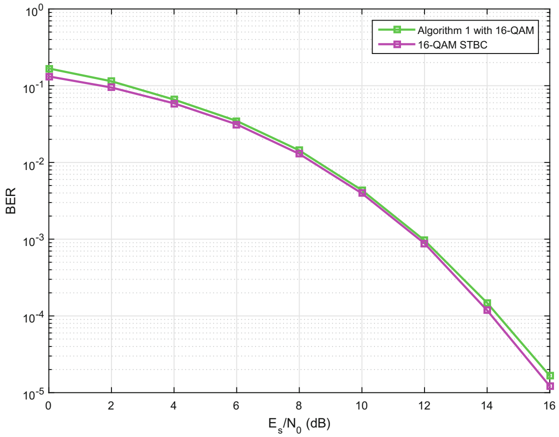

Fig. 4. Monte Carlo simulation results for the proposed and STBC schemes with 16-QAM

6 Conclusion

In this letter, a new STBC scheme is proposed to improve the transmission data rate while retaining the diversity gain of classic STBC scheme. Additional information bits are carried by using different STBC coding patterns in time domain. The efficiency of the scheme has been verified via both analysis and Monte Carlo simulation results. Its BER performance is close to the classic STBC scheme while 1 more bit can be transmitted in $2 \times N_r$ MIMO system. Moreover, the simple detection scheme at receiver is inherited from the classic STBC scheme. Therefore, the proposed scheme is an efficient and practical scheme for MIMO systems.

Acknowledgements. The authors would like to express our appreciation to the financial support of the National Natural Science Foundation of China (Grant No. 61771429), Zhejiang University City College Scientific Research Foundation (No. JZD18002).

A Bit error rate on index bit

The classic Alamouti scheme and new Alamouti scheme can be described as the following formulas

$$\mathbf{y}_1{}^{(C)} = \sqrt{\tfrac{\rho}{2}}\left(\mathbf{h}_1 x_1 + \mathbf{h}_2 x_2\right) + \mathbf{N}_1, \tag{14}$$

$$\mathbf{y}_2{}^{(C)} = \sqrt{\tfrac{\rho}{2}}\left(-\mathbf{h}_1 {x_2}^{*} + \mathbf{h}_2 {x_1}^{*}\right) + \mathbf{N}_2, \tag{15}$$

while

$$\mathbf{y}_1{}^{(N)} = \sqrt{\tfrac{\rho}{2}}\left(-\mathbf{h}_1 x_2{}^* + \mathbf{h}_2 x_1{}^*\right) + \mathbf{N}_1, \tag{16}$$

$$\mathbf{y}_2{}^{(N)} = \sqrt{\tfrac{\rho}{2}}\left(\mathbf{h}_1 x_1 + \mathbf{h}_2 x_2\right) + \mathbf{N}_2, \tag{17}$$

where each entry of both $\mathbf{h}_i$ and $\mathbf{N}_i$ is distributed as $CN(0,1)$, $\mathbb{E}\left[|x_i|^2\right] = 1$. Let $\mathbf{Y}^{(C)} = \left[\mathbf{y}_1^{(C)}, \mathbf{y}_2^{(C)}\right]$, $\mathbf{Y}^{(N)} = \left[\mathbf{y}_1^{(N)}, \mathbf{y}_2^{(N)}\right]$ and $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2]$. We derive the bit error rate of index bit estimation given that symbols, x_1 and x_2, are detected correctly. The *symbol error rate* (SER) of the index given the CSI can be expressed as

$$\begin{aligned} P_e\left(e \mid \mathbf{H}\right) &= P\left(\mathbf{Y}^{(C)} \rightarrow \mathbf{Y}^{(N)}\right) \\ &= P\left(\left\|\mathbf{Y} - \mathbf{Y}^{(N)}\right\|_F^2 < \left\|\mathbf{Y} - \mathbf{Y}^{(C)}\right\|_F^2\right) \end{aligned} \tag{18}$$

Further we have

$$P\left(A + B \leq \left(\|\mathbf{N}_1\|_F^2 + \|\mathbf{N}_1\|_F^2\right)\right), \tag{19}$$

where

$$A = \left\|\sqrt{\tfrac{\rho}{2}}\mathbf{h}_1\left(x_1 + x_2^*\right) + \sqrt{\tfrac{\rho}{2}}\mathbf{h}_2\left(x_2 - x_1^*\right) + \mathbf{N}_1\right\|_F^2, \tag{20}$$

$$B = \left\|\sqrt{\tfrac{\rho}{2}}\mathbf{h}_1\left(x_1 + x_2^*\right) + \sqrt{\tfrac{\rho}{2}}\mathbf{h}_2\left(x_2 - x_1^*\right) + \mathbf{N}_2\right\|_F^2. \tag{21}$$

The equivalent model is given by

$$\begin{gathered} P_e\left(e \mid \mathbf{H}\right) = \\ P\left(\left\|\sqrt{\frac{\rho}{2}}\mathbf{h}_1 s_1 + \sqrt{\frac{\rho}{2}}\mathbf{h}_1 s_1 + \mathbf{N}_1\right\|_F^2 \leq \|\mathbf{N}_1\|_F^2\right), \end{gathered} \tag{22}$$

where $s_1 = x_1 + x_2{}^*$, $s_2 = x_2 - x_1{}^*$.
Then we can get the following formulas,

$$\begin{gathered} P_e\left(e \mid \mathbf{H}\right) = \\ P\left(\begin{gathered} \frac{\rho}{2}\|\mathbf{h}_1 s_1 + \mathbf{h}_2 s_2\|_F^2 + \\ 2\sqrt{\frac{\rho}{2}}\mathbb{R}\left\{\left(\mathbf{h}_1 s_1 + \mathbf{h}_2 s_2\right)\mathbf{N}_1{}^*\right\} \leq 0 \end{gathered}\right), \end{gathered} \tag{23}$$

$$P_e\left(e\mid \mathbf{H}\right) = P\left(\mathbb{R}\left\{\left(\mathbf{h}_1+\mathbf{h}_2\right)\mathbf{N}_1{}^*\right\} \geq \sqrt{\frac{\rho}{8}}\|\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\|_F^2\right). \tag{24}$$

According to the previous formulas, we can get the conclusion that $\mathbb{R}\left\{\left(\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\right)\mathbf{N}_1^*\right\}$ is distributed as $CN\left(0, \frac{1}{2}\left\|\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\right\|_F^2\right)$.
In order to normalize $\mathbb{R}\left\{\left(\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\right)\mathbf{N}_1^*\right\}$ and convert $P_e\left(e\mid \mathbf{H}\right)$ to Q function,

$$P_e\left(e\mid \mathbf{H}\right) = P\left(\frac{\mathbb{R}\left\{\left(\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\right)\mathbf{N}_1{}^*\right\}}{\sqrt{\frac{1}{2}\|\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\|_F^2}} \geq \frac{\sqrt{\frac{\rho}{8}}\|\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\|_F^2}{\sqrt{\frac{1}{2}\|\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\|_F^2}}\right), \tag{25}$$

$$P_e\left(e\mid \mathbf{H}\right) = Q\left(\sqrt{\frac{\rho}{4}\|\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\|_F^2}\right). \tag{26}$$

Based on $\|A+B\|^2 \leq \|A\|^2+\|B\|^2$,

$$\begin{aligned} P_e\left(e\mid \mathbf{H}\right) &= Q\left(\sqrt{\frac{\rho}{4}\|\mathbf{h}_1 s_1+\mathbf{h}_2 s_2\|_F^2}\right) \\ &\geq Q\left(\sqrt{\frac{\rho}{4}\left(\|\mathbf{h}_1 s_1\|_F^2+\|\mathbf{h}_2 s_2\|_F^2\right)}\right). \end{aligned} \tag{27}$$

Based on Trapezoidal rule we have

$$P_e\left(e\mid \mathbf{H}\right) \geq \frac{1}{2n}\left[\begin{array}{l} \frac{1}{2}\exp\left(-\frac{\rho\left(\|\mathbf{h}_1 s_1\|_F^2+\|\mathbf{h}_2 s_2\|_F^2\right)}{8}\right)+ \\ \sum_{k=1}^{n-1}\exp\left(-\frac{\rho\left(\|\mathbf{h}_1 s_1\|_F^2+\|\mathbf{h}_2 s_2\|_F^2\right)}{8\sin^2\left(\frac{k\pi}{2n}\right)}\right) \end{array}\right]. \tag{28}$$

We assume that $\gamma_1 = \rho\|\mathbf{h}_1\|_F^2$ and $\gamma_2 = \rho\|\mathbf{h}_2\|_F^2$,

$$P_e\left(e\mid \mathbf{H}\right) \geq \frac{1}{2n}\left[\begin{array}{l} \frac{1}{2}\exp\left(-\frac{\gamma_1\|s_1\|_F^2+\gamma_2\|s_2\|_F^2}{2}\right)+ \\ \sum_{k=1}^{n-1}\exp\left(-\frac{\gamma_1\|s_1\|_F^2+\gamma_2\|s_2\|_F^2}{2\sin^2\left(\frac{k\pi}{2n}\right)}\right) \end{array}\right]. \tag{29}$$

Through the above formula, $P_e(e)$ can be computed as

$$P_e(e) \geq \int_0^\infty \int_0^\infty \frac{1}{2n} \left[\begin{array}{l} \frac{1}{2}\exp\left(-\frac{\gamma_1 \|s_1\|_F^2 + \gamma_2 \|s_2\|_F^2}{2}\right) \\ + \sum_{k=1}^{n-1} \exp\left(-\frac{\gamma_1 \|s_1\|_F^2 + \gamma_2 \|s_2\|_F^2}{2\sin^2\left(\frac{k\pi}{2n}\right)}\right) \end{array} \right] f(\gamma_1) f(\gamma_2)\, d\gamma_1\, d\gamma_2. \tag{30}$$

Define the moment generating function $M_s(s)$ as

$$M_s(s) = \int_0^\infty e^{-s\gamma} f_\gamma(\gamma)\, d\gamma = \left(\frac{1}{1+s\gamma}\right)^{N_R}. \tag{31}$$

Put $P_e(e)$ into function $M_s(s)$, we finally have

$$P_e(e) \geq \frac{1}{2n} \left[\begin{array}{c} \frac{1}{2} M_s\left(\frac{1}{2}|s_1|^2\right) M_s\left(\frac{1}{2}|s_2|^2\right) + \\ \sum_{k=1}^{n-1} M_s\left(\frac{1}{2\sin^2\left(\frac{k\pi}{2n}\right)}|S_1|^2\right) M_s\left(\frac{1}{2\sin^2\left(\frac{k\pi}{2n}\right)}|S_2|^2\right) \end{array} \right]. \tag{32}$$

References

1. Andrews, J.G., et al.: What will 5G be? IEEE J. Selected Areas in Commun. **32**(6), 1065–1082 (2014)
2. Larsson, E., Edfors, O., Tufvesson, F., Marzetta, T.: Massive MIMO for next generation wireless systems. IEEE Commun. Magaz. **52**(2), 186–195 (2014)
3. Gesbert, D., Shafi, M., Shiu, D.: From theory to practice: an overview of MIMO space-time coded wireless systems. IEEE J. Sel. Area Commun. **21**(3), 281–302 (2003)
4. H. Jafarkhani, Space-time coding: theory and practice[M]. *Cambridge university press* (2005)
5. Alamouti, S.: A simple transmit diversity technique for wireless communications. IEEE J. Sel. Area Commun. **16**(8), 1451–1458 (1998)
6. Ling, Q., Li, T.: Efficiency improvment for Alamouti Codes. In: Proceedings of 40th IEEE CISS, Princeton. pp. 569–572 (2006)
7. Baloch, Z.A., Baloch, M.U., Hussain, N.: Efficiency improvement of space time block codes. Int. J Commun. Network Syst. Sci. **3**, 507–510 (2010)
8. Basar, E., Aygolu, U., Panayirci, E., Poor, H.V.: Space-time block coded spatial modulation. IEEE Trans. Commun. **59**(3), 823–832 (2011)
9. Helmy, A.G., Di Renzo, M., Al-Dhahir, N.: Differential spatially modulated space-time block codes with temporal permutations. IEEE Trans. Veh. Technol. **66**(8), 7548–7552 (2017)
10. Li, X., Wang, L.: High rate space-time block coded spatial modulation with cyclic structure. IEEE Commun. Letters **18**(4), 532–535 (2014)

11. Zhu, Y., Wang, W., Zhang, J., Zhang, Y.: Constellation collaborated nonlinear orthogonal space-time block codes with fast maximum likelihood detection. IEEE Trans. Vehic. Techn. **66**(1), 513–528 (2016)
12. Hua, Y., Zhao, G., Zhao, W., Jin, M.: Modified codewords design for space-time block coded spatial modulation. IET Commun. **11**(2), 249–257 (2017)
13. Tarokh, V., Jafarkhani, H., Calderbank, A.: Space-time block coding for wireless communications: performance results. IEEE J. Selected Areas in Commun. **17**(3), 451–460 (2017)
14. Mesleh, R., Haas, H., Sinanovic, S., Chang, A., Sangboh, Y.: Spatial modulation. IEEE Trans. Veh. Technol. **57**(4), 2228–2241 (2008)
15. Jafarkhani, H.: Space-Time Coding. Cambidge University Press, Theory and Practice (2005)