Neural Network Algorithm of Multi-team Game and Its Application in Parallel-Link Communication Networks Flow Control

Zixin Liu[1,2](✉), Huawei Yang[3], and Lianglin Xiong[4]

[1] School of Mathematics and Statistics, Guizhou University of Finance and Economics, Guiyang, People's Republic of China
[2] Guizhou Key Laboratory of Big Data Statistics Analysis, Guizhou University of Finance and Economics, Guiyang, People's Republic of China
[3] School of Big Data Application and Economics, Guizhou University of Finance and Economics, Guiyang, People's Republic of China
[4] School of Mathematics and Computer Science, Yunnan Minzu University, Kunming, People's Republic of China

Abstract. This paper investigates the approximate calculation problem of noninferior Nash equilibrium (NNE) in multi-team game. Combined with variational inequalities theory, Nash equilibrium theory, and dynamic system theory, a projection neural network (PNN) algorithm for computing NNE of multi-team game with smooth payoff functions is derived. Utilizing stable theory, stability criteria of NNE in multi-team game are further given. As an application, a flow control model of parallel-link communication networks based on multi-team game and neural network algorithm is elaborated. Finally, a simulation result for two teams, two communication links, and two users in each team parallel-linkcommunication network is also given to illustrate the effectiveness of the PNN algorithm proposed in this paper.

Keywords: Projection neural network · Multi-team game · Noninferior Nash equilibrium · Variational inequalities · Flow control · Parallel-link communication networks

This work is supported by National Nature Science Foundation of China under Grants 62062018, 61472093, 11461082, 11761018, Guizhou Province University Science and Technology Top Talents Project KY2018047, Guizhou University of Finance and Economics 2018XZD01, Science and Technology Planning Project of Guizhou Province of China under Grants [2017] 1016, the Innovation Exploration and Academic New Seedling Project of Guizhou University of Finance and Economics (No. Qian Ke He Ping Tai Ren Cai [2017] 5736-025), and Guizhou Key Laboratory of Big Data Statistics Analysis (No.: Guizhou Science and Technology Cooperation Platform Talent [2019] 5103).

H. Song and D. Jiang (Eds.): SIMUtools 2020, LNICST 369, pp. 781–795, 2021.
https://doi.org/10.1007/978-3-030-72792-5_62

1 Introduction

In essence, game theory is a mathematical model, which mainly studies the conflict and cooperation between players in a rational situation [1]. It is widely used in computer science, psychology, politics, economics, radio resource allocation and so on [2–7], and has been recognized as one of the most useful tool to deal with all kinds of science problems.

Recently, considering that the large collections of small agents make strategically interdependent decisions in many economic, social, and technological environments, convex static multi-team was first introduced in [8,9]. This work is very important because this game model can find specific application scenarios in economic management, biological evolution, wireless network and other real systems. Different from conventional noncooperative games and cooperative games, multi-team games not only have the characteristics of noncooperative games, but also the characteristics of cooperative games. In multi-team games, there is a non cooperative relationship among decision makers among groups, but it is a cooperative relationship among decision makers within a group. The characteristic of multi-team games lies in that they merge the concepts of team theory and game theory. And the equilibrium strategy introduced in [8] and [9] is called noninferior Nash strategy (NNS). This new equilibrium strategy provides a new framework for the analysis of the master-slave group game to solve the complex coordination and competeing problem between decision-makers. In [10], E. Ahmed et al. generalized the static multi-team game derived in [8,9] into dynamical case with bounded rationality. And they pointed out that the NNS is Pareto optimal if the players belong to different teams.

As is well known that, in game theory, computation problems, existence and stability analysis problems for Nash equilibrium are very important. In recent decades, there are many existence theorems and stability criteria for Nash equilibrium [11–13]. For existence problem, utilizing Brouwer fixed point theorem, Schauder fixed point theorem, and Ky fan inequality, J. Yu in [11] deeply researched the existence problems of all kinds of different game models, and derived many existence results. Additional, for set-valued case, applying Kukutani fixed point theorem, set-valued existence results are also given in [11]. For stability problem, by introducing a Hausdorff distance on a complete metric space, J. Yu also established some profound and significant stability results under Baire classification. It is worth pointing out that most of these previous existence and stability results are theoretical perfection but lack of practicality, since they are too abstract. They are theoretical significance instead of practice. In economic and engineering problems, people often care about the calculation of Nash equilibrium, it is a key step in the practical application of game theory. Recently, optimal algorithm, neural network algorithm, and experiment methods for Nash equilibrium have been derived in [14–16]. However, these previous work did not mention the noninferior Nash equilibrium computation problem on multi-team game. In [17] and [18], the authors only researched the noninferior Nash equilibrium computation problem on multi-team Cournot game in a simple case. For general case, how to establish a more practice algorithm is worth discussing, which motivates this study.

Combined with variational inequalities theory, Nash equilibrium theory, and dynamic system theory, a projection neural network (PNN) algorithm for computing NNE of multi-team game with smooth payoff functions is derived. Utilizing stable theory, stability criteria of NNE in multi-team game are further given. As an application, a flow control model of parallel-link communication networks based on multi-team game and neural network algorithm is elaborated. Finally, a simulation result for two teams, two communication links, and two users in each team parallel-link communication network is also given to illustrate the effectiveness of the PNN algorithm proposed in this paper.

This paper is organized as follows. The basic problem of multi-team game is given in Sect. 2. The equivalence between multi-team game with projection neural network is given in Sect. 3. Stability analysis is given in Sect. 4. In Sect. 5, an application of flow control model of wireless ad hoc network based on multi-team game is given. And its related simulation is also performed, while the conclusions are drawn in Sect. 6.

2 Continuous Static Multi-team Games

Considering a n-team continuous static multi-team games: Let team $N \in \mathbb{N}, \mathbb{N} = \{1, 2, \cdots, n\}$ have m_N members of decision makers, and let the control variable of the i^{th} member x_i^N be a vector of dimension m_i^N. Let $x^N = (x_1^N, x_2^N, \cdots, x_{m_N}^N)$ denote the overall control vector for team N; matric space X_i^N be the strategy set of the i^{th} player in team N, $X^N = \prod_{i=1}^{m_N} X_i^N$ denotes the strategy space for the overall control vector x^N of team N. For each $i \in \mathbb{N}$, denote $\hat{i} = \mathbb{N} \backslash \{i\}$, $X = \prod_{i=1}^{n} X^i$. $f_i^N : X \to R$ denotes the i^{th} player's payoff function in team N, respectively.

For n-team noncooperative game, if there exists $x_* = (x_*^1, x_*^2, \cdots, x_*^n) \in X$ such that

$$f^i(x_*^i, x_*^{\hat{i}}) = \max_{u^i \in X^i} \sum_{j=1}^{m_i} \lambda_j^i f_j^i(u^i, x_*^{\hat{i}}), \forall i \in \mathbb{N},$$

then x_* is called the noninferior Nash equilibrium (NNE), where $\lambda^i = (\lambda_1^i, \lambda_2^i, \cdots, \lambda_{m_i}^i) \in W^i, i \in \mathbb{N}$, and W^i is given by

$$W^i = \{\lambda^i \in R^{m_i} | \sum_{j=1}^{m_i} \lambda_j^i = 1, 0 \leq \lambda_j^i \leq 1\}, i \in \mathbb{N}.$$

Assumptions.

(1) $\forall j \in \{1, 2, \cdots, m_i\}, i \in \mathbb{N}$, strategy set $X_j^i \in R^{m_i}$ is nonempty, convex, and compact.

(2) $\forall j \in \{1, 2, \cdots, m_i\}, i \in \mathbb{N}$, payoff function $f^i : X \to R$ is continuously differentiable, and $\forall x^{\hat{i}} \in X^{\hat{i}}, u^i \to f_j^i(u^i, x^{\hat{i}})$ is concave on X^i.

Remark 1. If function f satisfies assumptions (1) and (2), then our concerned n-team continuous static multi-team game at least has one NNE. See Lemma 4 for details.

Lemma 1. *Set* $\nabla f(x) = (\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_n})$, *if payoff function* f *satisfies assumptions (1) and (2), then*

$$f(y) \leq f(x) + \langle \nabla f(x), y - x \rangle,$$

Remark 2. In [8–10], the authors deeply researched the noninferior Nash strategies problem for multi-team system. However, they only gave out the algorithm of the noninferior Nash equilibrium for two-team system instead of multi-team system. In this paper, we will give out a general neural network algorithm for the noninferior Nash equilibrium of multi-team system.

Remark 3. By utilizing Kakutani fixed point theorem, Y. Liu and M.A. Simaan in [9] gave out the noninferior Nash equilibrium's existence theorem for two-team system, when the payoff function f is jointly continuous and strictly convex. In this paper, we will show that for two-team or multi-team system, the existence result only requires the payoff function f satisfying assumption (2).

3 Equivalent Relation Between Variational Inequality and Multi-team Game

Under assumptions (1) and (2), we will prove that a continuous static multi-team game problem can also uniquely correspond to a variational inequality problem.

Lemma 2. *Let* $f : X \to R$ *be a continuous differentiable and concave function defined on a nonempty closed convex set* X*, then* $f(x^*) = \max_{x \in X} f(x)$ *if and only if*

$$\langle \nabla f(x^*), x - x^* \rangle \leq 0, \forall x \in X.$$

This result can be obviously obtained from Lemma 1, thus the proof is omitted here.

Remark 4. N-team non-cooperative game is called a concave one, if $\forall j \in m_i, i \in N$, payoff function f^i satisfies assumption (2). Reference [19] has proved that there is a special equivalence relationship between Nash equilibrium of non cooperative concave game and Brouwer fixed point theorem, as well as the solution of variational inequality. Moreover, the solution of variational inequality can be transformed into a dynamic system equilibrium point problem, which provides a solution for the neural network algorithm of Nash equilibrium.

Remark 5. In [8,9], and [17,18], the used opposite payoff functions can be regarded as special case of this paper. Thus the considered opposite payoff functions in this paper are more general.

Theorem 1. *$\forall j \in m_i, i \in \mathbb{N}$, strategy set $X_j^i \in R^{m_i}$ satisfying assumption (1); payoff function $f^i : X \to R$ satisfying assumption (2); then, $x_* = (x_*^1, x_*^2, \cdots, x_*^n) \in X$ is a noninferior Nash strategies of N-team continuous static multi-team concave game if and only if x_* satisfies*

$$\langle \nabla f(x_*), x - x_* \rangle \leq 0, \forall x \in X,$$

where

$$\nabla f(x_*) = (\nabla_{x_*^1} f^1, \nabla_{x_*^2} f^2, \nabla_{x_*^n} f^n) \in R^m, m = \sum_{i=1}^{n} m_{m^i}.$$

$$m_{m^l} = \sum_{k=1}^{m^l} m_k^l, l \in \mathbb{N}.$$

Proof: $\forall i \in \mathbb{N}, \forall x^i \in X^i$, Set $\overline{x} = (x^i, x_*^{\hat{i}})$,then $\overline{x} \in X$, since $\forall x \in X$, $\langle \nabla f(x_*), x - x_* \rangle \leq 0$, from Lemma 2, we have $f^i(x_*^i, x_*^{\hat{i}}) = \max_{u^i \in X^i} f^i(u^i, x_*^{\hat{i}})$. Thus, $x_* = (x_*^1, x_*^2, \cdots, x_*^n) \in X$ is a noninferior Nash strategies of N-team concave game.

Conversely, if $x_* = (x_*^1, x_*^2, \cdots, x_*^n) \in X$ is a noninferior Nash strategies of N-team concave game, then $\forall i \in \mathbb{N}$, $f^i(x_*^i, x_*^{\hat{i}}) = \max_{u^i \in X^i} f^i(u^i, x_*^{\hat{i}})$, from Lemma 2, $\forall x \in X, \langle \nabla f(x_*), x - x_* \rangle \leq 0$. This completes the proof.

Remark 6. It is difficult to calculate the NNEs of N-team continuous static multi-team game directly, especially when there are many players in the game. However, the conclusion of Theorem 1 provides an indirect calculation method of NNEs. The approximate calculation of NNEs can be realized by solving a special variational inequality.

4 Neural Network Model for Multi-team Non-cooperative Concave Game

4.1 Neural Network Model Construction

To give out neural network algorithm for above N-team non-cooperative concave game, an important lemma is needed

Lemma 3. *For arbitrary $\alpha > 0$, x_* is a fixed point of equation $x = P_\Omega(x - \alpha f(x))$, if and only if it satisfies $\langle f(x_*), x - x_* \rangle \geq 0$ for all $x \in R^n$, where $f : R^n \to R^n$ is continuous on R^n, Ω is a subset of R^n, $P_\Omega(x - \alpha f(x))$ is a conventional projection operator.*

By Theorem 1 and Lemma 3, NNE's computation problem of N-team continuous static multi-team game can be further transformed into an equilibrium point's approximate calculation problem of the following neural network model.

$$\frac{dx(t)}{dt} = -x(t) + P_X(x(t) + \alpha \nabla f(x(t))), \tag{1}$$

where

$$P_X(y) = arg\min_{x \in X} \|y - x\|.$$

$\nabla f(x(t)) = (\nabla_{x^1(t)} f^1(x(t)),\ \nabla_{x^2(t)} f^2(x(t)), \cdots, \nabla_{x^n(t)} f^n(x(t)))$, $X = \Pi_{i=1}^n X^i$, $f^i : X \to R$ is the payoff function of the i-th team.

4.2 Equilibrium's Existence Analysis

From Theorem 1 and Lemma 3, through a series of equivalent changes, NNE x_* of a N-team multi-game is finally proved to be equivalent to the equilibrium point of a special neural network and the solution of a special variational inequality. Therefore, to give out the existence conditions of NNE and the equilibrium of neural network, we only need to research the solution's existence conditions of concerned variational inequalities.

Lemma 4. *[19] There exists at least one $x_* \in X$ such that*

$$\langle \beta(x_*), y - x_* \rangle \leq 0, \forall y \in X.$$

where $\beta(.)$ is continuous, and X is a subset of R^n satisfying nonempty bounded closed convex property.

From Lemma 4, and assumptions (1) and (2), the following follows existence theorem is obviously.

Theorem 2. *The neural network model (1) established in this paper has at least one equilibrium point.*

Proof: By assumption (1), $\forall j \in m_i, i \in \mathbb{N}$, strategy set $X_j^i \in R^{m_i}$ is nonempty, convex, and compact, thus X is a nonempty, convex, bounded, and closed set. By assumption (2), since $\forall j \in m_i, i \in \mathbb{N}$, payoff function $f^i : X \to R$ is continuously differentiable, thus $\nabla f(x)$ is continuous on X. From Lemma 4, variational inequality $\langle \nabla f(x_*), x - x_* \rangle \leq 0$ at least exists one solution x_*. Namely, The neural network model (1) established in this paper has at least one equilibrium point, which completes the proof.

Remark 7. If assumptions (1) and (2) hold, and $\forall x^{\hat{i}} \in X^{\hat{i}}, u^i \to f_j^i(u^i, x^{\hat{i}})$ is strong concave on X^i, similar to the proof in [13], the uniqueness property of the equilibrium point of system (1) can also be obtained.

4.3 Stability Analysis

let $y(t) = x(t) - x_*$, where x_* is a NNE of n-team multi-game, substitute $y(t)$ into system (1), through simple equivalent deformation operation, system (1) is rewritten as

$$\begin{aligned} \frac{dy(t)}{dt} &= -y(t) + P_X(y(t) + x_* + \alpha \nabla f(y(t) + x_*)) \\ &\quad - P_X(x_* + \alpha \nabla f(x_*)), t > 0. \end{aligned} \tag{2}$$

Lemma 5. *[20] For arbitrary closed convex set $\Theta \subseteq R^n$, $\xi, \eta, x \in R^n$, $y \in \Theta$, projection operator satisfies*

$$\|P_\Theta(\xi) - P_\Theta(\eta)\| \leq \|\xi - \eta\|,$$

$$(x - P_\Theta(x))^T (P_\Theta(x) - y) \geq 0, x \in R^n, y \in \Theta,$$

where $\|\cdot\|$ denotes $L^2(R^n)$ norm.

Lemma 6. *[21] If $\forall x \in R^n$, $F(x) : R^n \to R^n$ is continuous, and there exist nonnegative functions $M(t), N(t)$ such that*

$$\|F(x)\| \leq M(t)\|x\| + N(t),$$

then the solution existence interval of $\dot{x}(t) = F(x)$ with initial value $x(t_0)$ is $[t_0, +\infty)$.

Lemma 7. *[22] Let function $V : R^n \to R$ be positive definite local Lipschitz normal satisfying*

$$\frac{dV(x(t))}{dt} \leq 0.$$

If there exists a constant $l > 0$ such that $L_l = \{x \in R^n | V(x) \leq l\}$ is bounded, then for every solution $x(t)$ of $\dot{x}(t) = F(x)$ with initial value x_0, when $t \to +\infty$, we have

$$\mathtt{dist}(x(t), M) \to 0,$$

here M is the maximum invariant set of $\bar{Z}_V \cap L_l$,

$$\bar{Z}_V = \{x \in R^n | 0 = \dot{V}(x)\}.$$

By using Lemma 5 and Lemma 6, the solution's existence result for system (1) can be derived as follows

Theorem 3. *For pre given initial value $x(t_0)$, the solution's existence interval of system (1) is $[t_0, +\infty)$.*

Proof: Set $F(x(t)) = -x(t) + P_X(x(t) + \alpha\nabla f(x(t)))$. Notice that if $f(x(t))$ is a convex function, then $f(x(t))$ is local Lipschitz continuous with Lipschitz constant $L(t)$, namely, $\|\nabla f(x(t))\| \leq L(t)$. From assumptions (1)–(2), and Lemma 5, we have

$$\begin{aligned} \|F(x(t))\| &= \| - x(t) + P_X(x(t) + \alpha\nabla f(x(t)))\| \\ &\leq \|P_X(x(t) + \alpha\nabla f(x(t))) - P_X(x(t))\| \\ &\quad + \|P_X(x(t)) - P_X(x_*)\| + \|P_X(x_*) - x(t)\|, \\ &\leq \|\alpha\nabla f(x(t))\| + 2\|x(t)\| + \|x_*\| + \|P_X(x_*)\| \\ &\leq 2\|x(t)\| + \alpha L(t) + \|x_*\| + \|P_X(x_*)\|. \end{aligned} \tag{3}$$

From Lemma 6, it follows that, for pre given initial value $x(t_0)$, the solution's existence interval of system (1) is $[t_0, +\infty)$.

Using the proof method in reference [23], for system (1), the following invariant-set property can be obtained.

Theorem 4. *For any initial value $x_0 \in X$, the state vector $x(t)$ of system (1) is still in X, namely X is a invariant set of the solutions of system (1).*

Theorem 5. *Under hypothesis (1), (2), state vector $x(t)$ of neural network (1) with initial value $x(t_0) \in X$ converges to the NNE x_* of the multi-team noncooperative concave game asymptotically, if*

$$\langle \nabla_{y^i} f^i(y^i(t) + x_*^i, x_*^{\hat{i}}), \dot{y}^i(t)\rangle \leq \langle \nabla_{y^i} f^i(y^i(t) + x_*^i, x^{\hat{i}}(t)), \dot{y}^i(t)\rangle, i = 1, 2, \cdots, n,$$

where $y^i(t) + x_*^i = x^i(t)$.

Proof: From Theorem 4, since X is an invariable set, thus $\forall x(t_0) \in X$, $x(t) \in X$. Let x^* denote an equilibrium point of system (1), here $x^* \in X$. From Theorem 1 and Lemma 3, it follows that x^* is a NNE of multi-game. By the definition of non-inferior Nash equilibrium, it follows that $f^i(x_*^i, x_*^{\hat{i}}) = \max_{x^i(t)\in X^i} f^i(x^i(t), x_*^{\hat{i}})$. Using this relationship, a simple Lyapunov function can be constructed.

$$V(y(t)) = \alpha \sum_{i=1}^{n} [f^i(x_*^i, x_*^{\hat{i}}) - f^i(y^i(t) + x_*^i, x_*^{\hat{i}})], \tag{4}$$

where $y(t) + x_* = x(t) \in X$. It follows that $V(0) = 0$ and $V(y(t)) \geq 0$. By direct derivation operation, one can obtain

$$\dot{V}(y(t)) = \langle \nabla V(y(t)), \dot{y}(t)\rangle. \tag{5}$$

where

$$\nabla V(y(t)) = (-\alpha \nabla_{y^1} f^1(y^1(t) + x_*^1, x_*^{\hat{1}}), \cdots, -\alpha \nabla_{y^n} f^n(y^n(t) + x_*^n, x_*^{\hat{n}})).$$

Since

$$\langle \nabla_{y^i} f^i(y^i(t) + x_*^i, x_*^{\hat{i}}), \dot{y}^i(t)\rangle \leq \langle \nabla_{y^i} f^i(y^i(t) + x_*^i, x^{\hat{i}}(t)), \dot{y}^i(t)\rangle,$$

one can obtain that

$$\langle \nabla V(y(t)), \dot{y}(t)\rangle \leq \langle \nabla \overline{V}(y(t)), \dot{y}(t)\rangle,$$

where

$$\overline{V}(y(t)) = (-\alpha \nabla_{y^1} f^1(y^1(t) + x_*^1, x^{\hat{1}}(t)), \cdots, -\alpha \nabla_{y^n} f^n(y^n(t) + x_*^n, x^{\hat{n}}(t))).$$

This means that

$$\begin{aligned} \dot{V}(y(t)) &= \langle \dot{y}(t), \nabla V(y(t))\rangle \\ &= \langle -\alpha\gamma, \psi\rangle, \end{aligned} \tag{6}$$

where $\psi = -y(t) - x_* + P_X(y(t) + x_* + \alpha\gamma)$, $\gamma = \nabla f(y(t) + x_*)$. Denote $\omega = y(t) + x_* + \alpha\gamma$, notice that $x(t) = y(t) + x_*$, from (6), we have

$$\begin{aligned} \dot{V}(y(t)) &= \langle \dot{y}(t), \nabla V(y(t))\rangle \\ &= \langle -\alpha\gamma, \psi\rangle \\ &= \langle P_X(\omega) - x(t), -\omega + x(t), \rangle, \\ &\leq -\langle P_X(\omega) - x(t), \omega - P_X(\omega), \rangle \\ &- \langle P_X(\omega) - x(t), P_X(\omega) - x(t)\rangle. \end{aligned} \tag{7}$$

From Lemma 5, one can get $\dot{V}(y(t)) \leq 0$. Denote $L_l = \{y(t) \in R^n | V(y(t)) \leq l\}$, where $l = \max_{y(t)+x_* \in X} f(y(t) + x_*)$. Next, we will show that L_l is bounded. Denote $\gamma^* = \nabla f(x_*)$, by system (2), it follows that

$$\begin{aligned} y(t) = e^{-(t-t_0)} y(t_0) - \textstyle\int_{t_0}^{t} e^{-(t-s)} [P_X(x^* + \alpha\gamma^*) \\ -P_X(y(s) + x^* + \alpha\nabla f(y(s) + x^*))] ds. \end{aligned} \tag{8}$$

Then

$$\begin{aligned} \|y(t)\| \leq\, & e^{-(t-t_0)} \|y(t_0)\| \\ & + \int_{t_0}^{t} e^{-(t-s)} \|P_X(x^* + \alpha\nabla f(x(s))) \\ & - P_X(x(s) + \alpha\nabla f(x(s)))\| ds \\ & + \int_{t_0}^{t} e^{-(t-s)} \|P_X(x^* + \alpha\nabla f(x(s))) \\ & - P_X(x^* + \alpha\gamma^*)\| ds \\ & \leq e^{-(t-t_0)} \|y(t_0)\| + \alpha(L + \|\gamma^*\|) \\ & + \int_{t_0}^{t} e^{-(t-s)} \|y(s)\| ds. \end{aligned} \tag{9}$$

where $L = \max_{x(t) \in X} \nabla f(x(t))$. Notice that $X \in R^m$ is an invariant set for arbitrary $x(t_0) \in X$, $\nabla f(x(t))$ is continuous on X, and $X \in R^m$ is convex and compact, thus L exists. By Gronwall-Bellman inequality, one can obtain

$$\begin{aligned} \|y(t)\| & \leq e^{-(t-t_0)} \|y(t_0)\| e + e\alpha(L + \|\gamma^*\|) \\ & \leq e[\|y(t_0)\| + \alpha(L + \|\gamma^*\|)]. \end{aligned} \tag{10}$$

This means that L_l is bounded. Denote $M = \{x^*\}$, from Lemma 7, when $t \to +\infty$, it follows $\text{dist}(x(t), M) \to 0$, which complete the proof.

Remark 8. From Theorem 5, it follows that if the initial value $x(t_0)$ is in strategy set X, and $\langle \nabla_{y^i} f^i(y^i(t) + x_*^i, x_*^{\hat{i}}), \dot{y}^i(t) \rangle \leq \langle \nabla_{y^i} f^i(y^i(t) + x_*^i, x^{\hat{i}}(t)), \dot{y}^i(t) \rangle$, then the state vector of neural network (1) converges asymptotically to the equilibrium state. In order to get the convergence property and the convergence rate when $x(t_0)$ is not in X, we need more stronger condition for payoff function f.

Theorem 6. *Under assumptions (1) and (2), if there exists $\beta > 0$ such that $\|\nabla f(x(t)) - \nabla f(y(t))\| \leq \beta \|x(t) - y(t)\|$, then, the state vector $x(t)$ of system (1) with any initial value $x(t_0)$ exponentially globally converges to the Nash equilibrium point of system (1).*

Proof: If $\|\nabla f(x(t)) - \nabla f(y(t))\| \leq \beta \|x(t) - y(t)\|$, where constant $\beta > 0$, from (8), it follows that

$$
\begin{aligned}
\|y(t)\| \leq\; & e^{-(t-t_0)}\|y(t_0)\| \\
& + \int_{t_0}^{t} e^{-(t-s)}\|P_X(x^* + \alpha\nabla f(x(s))) \\
& - P_X(x(s) + \alpha\nabla f(x(s)))\|ds \\
& + \int_{t_0}^{t} e^{-(t-s)}\|P_X(x^* + \alpha\nabla f(x(s))) \\
& - P_X(x^* + \alpha\gamma^*)\|ds \\
& \leq e^{-(t-t_0)}\|y(t_0)\| \\
& + \int_{t_0}^{t} e^{-(t-s)}\|P_X(x^* + \alpha\nabla f(x(s))) \\
& - P_X(x(s) + \alpha\nabla f(x(s)))\|ds \\
& + \int_{t_0}^{t} e^{-(t-s)}\|P_X(x^* + \alpha\nabla f(x(s))) \\
& - P_X(x^* + \alpha\nabla f(x^*)\|ds \\
& \leq e^{-(t-t_0)}\|y(t_0)\| \\
& + \int_{t_0}^{t} e^{-(t-s)}(1 + \alpha\beta)\|y(s)\|ds.
\end{aligned}
\tag{11}
$$

By Gronwall-Bellman inequality, one can obtain

$$\|y(t)\| \leq e^{-(t-t_0)}\|y(t_0)\|e^{(1+\alpha\beta)} \tag{12}$$

This means that, for any initial value $x(t_0)$, the state vector $x(t)$ of system (1) exponentially globally converges to the NNE. From (12), one can see the exponential convergence rate is equal to 1.

Remark 9. Since the stable state x_* of system (1) is equivalence to the NNE of mutil-team noncooperative concave game, thus Theorem 5 and Theorem 6 give out a neural network algorithm to approximatly calculate NNE of mutil-team game.

Remark 10. In [8–10], the payoff functions are all quadratic functions, obviously, any quadratic functions satisfy the constraint condition in Theorem 6, from Theorem 6, our neural network algorithm converges at the exponential convergence rate 1. Thus it is more advantageous that the methods used in [8–10].

5 Application of Multi-team Game in Flow Control of Parallel-Link Communication Networks

5.1 Model Analysis

Let $\mathbb{N} = \{1, 2, \cdots, n\}$ denote the set of team leaders. In each team, there exists several types of users. All users in the same team share a set $\mathbb{L} = \{1, 2, \cdots, L\}$ of communication links. And all users in the same team are connected to a common destination node and source node. If the capacity of communication link l is denoted by c_l, then $c = (c_1, c_2, \cdots, c_L)$ means the capacity configuration. Let M_N denote team leader $N \in \mathbb{N}$ who servers m_N users. We assume that the i^{th} user's throughput demand in team N is a typical Poisson process, and the average rate is denoted as $x_i^N = (x_{i,1}^N, x_{i,2}^N, \cdots, x_{i,L}^N)$, where $x_{i,l}^N, l \in \mathbb{L}$ denotes average rate of the i^{th} user in team N across link l. Namely, x_i^N is the strategy of the i^{th} user in team N. Assumption the queuing model of the users' flow across every link l is $M/M/1$, $u_i^N(X)$ demote the utility function of the i^{th} user in team N, similar to the analysis in [24], $u_i^N(X)$ can be defined as

$$u_i^N(X) = \alpha_i^N \sum_{l \in \mathbb{L}} \log(x_{i,l}^N + 1) - \beta_i^N \sum_{l \in \mathbb{L}} \frac{1}{c_l - \overline{x}_l}, N \in \mathbb{N},$$

$$\overline{x}_l = \sum_{i \in \mathbb{N}} \sum_{j \in M} x_{j,l}^i, M = \{1, 2, \cdots, \sum_{k=1}^{N} m_k\},$$

where $\alpha_i^N (0 < \alpha_i^N < 1)$ is sensitive parameter of the i^{th} user in team N to the flow average rate. $\sum_{l \in \mathbb{L}} \log(x_{i,l}^N + 1)$ denotes link cumulative gain of the i^{th} user in team N with strategy x_i^N. $0 < \beta_i^N < 1$ is sensitive parameter of the i^{th} user in team N to the network delay, $\sum_{l \in \mathbb{L}} \frac{1}{c_l - \overline{x}_l}$ denotes link cumulative delay of the i^{th} user in team N with strategy x_i^N.

For the N^{th}-team, team utility function is defined as all users' utility convex combination in N^{th}-team

$$f^N(X) = \sum_{j=1}^{m_N} \lambda_j^N u_j^N(X), \forall N \in \mathbb{N},$$

where $0 \le \lambda_j^N \le 1, \sum_{j=1}^{m_N} \lambda_j^N = 1$. The aim of every user in the same team is to select their flow average rate strategies such that the team utility to reach the maximum which they are belonged to. Obviously, this problem is a typical multi-team game model, and the noninferior Nash equilibrium strategy is $x_* = (x_*^1, x_*^2, \cdots, x_*^n) \in X$ satisfying

$$f^N(x_*^N, x_*^{\hat{N}}) = \max_{y^N \in X^N} \sum_{j=1}^{m_N} \lambda_j^N u_j^N(y^N, x_*^{\hat{N}}), \forall N \in \mathbb{N}.$$

The topological structure of parallel-link communication networks with multi-team is shown in Fig. 1.

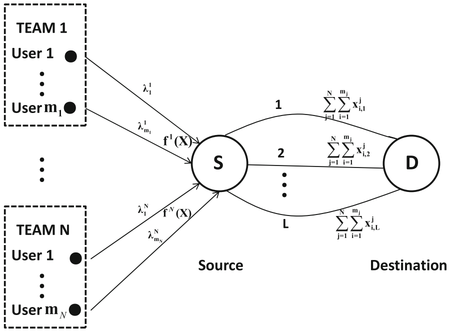

Fig. 1. The topological structure of parallel-link communication networks with multi-teams.

Theorem 7. *$\forall j \in m_N, N \in \mathbb{N}$, $\forall x_{\hat{i}}^{\hat{N}} \in X_{\hat{i}}^{\hat{N}}, x_i^N \rightarrow f^N(x_i^N, x_{\hat{i}}^{\hat{N}})$ is concave on X_i^N, and f^N is continuously and differentiable.*

Proof. Since $\forall x_{i,l}^N$, $\frac{\partial^2 f^N(X)}{\partial (x_{i,l}^N)^2} = -\frac{\lambda_i^N \alpha_i^N}{(x_{i,l}^N+1)^2} - \sum_{i=1}^{m_N} \frac{2\lambda_i^N \beta_i^N}{(c_l - \overline{x}_l)^3}$, notice that $0 \leq \lambda_j^N \leq 1, \sum_{j=1}^{m_N} \lambda_j^N = 1$, $0 < \alpha_i^N < 1$, $0 < \alpha_i^N < 1$, thus, $\forall x_{i,l}^N$, $\frac{\partial^2 f^N(X)}{\partial (x_{i,l}^N)^2} < 0$, which means that $f^N : X \rightarrow R$ is continuously differentiable, and $\forall x_{\hat{i}}^{\hat{N}} \in X_{\hat{i}}^{\hat{N}}, x_i^N \rightarrow f^N(x_i^N, x_{\hat{i}}^{\hat{N}})$ is concave on X_i^N. This completes the proof.

Remark 11. Since the i^{th} user's average rate $x_{i,l}^N$ in team N across link l is not exceed the minimum capacity configuration $\min\{c_l, l \in \mathbb{L}\}$ of all links. Thus, X is a convex compact set. By Theorem 7, one can get that flow control of parallel-link communication networks based on multi-team game is a typical concave game, which means that there exists at least one NNE of this multi-team game.

By Theorem 1 and Lemma 3, following neural network model can be constructed to numerically calculate the NNE of our concerned multi-team concave game.

$$\frac{dx(t)}{dt} = -x(t) + P_X(x(t) + \alpha \nabla f(x(t))), \tag{13}$$

5.2 Numerical Simulation

In this subsection, a simulation example will be given to illustrate the validity of neural network algorithm derived in subsection A of Sect. 5. For the convenience of simulation, suppose there are two team leaders, each team leader includes two user, they all share two communication links. The topological structure of communication network with $N = 2, m^N = 2, L = 2$ is shown in Fig. 2. Let $\lambda_j^N = 0.5, j, N = 1, 2$; $\alpha_i^N = 0.6$; $\beta_i^N = 0.1, i, N = 1, 2$; $c_1 = 5, c_2 = 9$. In this case, since $0 \leq x_{i,l}^N \leq \min\{c_l, l \in \mathbb{L}\}$, it yields that $0 \leq x_{i,l}^N \leq 5$. Denote $\alpha > 0$

to be an arbitrary positive constant, from (13), by simple calculation, it follows that $x(t) = (x^1_{1,1}, x^1_{1,2}, x^1_{2,1}, x^1_{2,2}, x^2_{1,1}, x^2_{1,2}, x^2_{2,1}, x^2_{2,2})$,

$$\nabla_{x^1_{i,l}} f^1(x(t)) = \frac{1}{2}\sum_{i=1}^{2}[\sum_{l=1}^{2}\frac{3}{5(x^1_{i,l}+1)} - \sum_{l=1}^{2}\frac{1}{10(5-\overline{x}_l)^2}],$$

$$\nabla_{x^2_{i,l}} f^2(x(t)) = \frac{1}{2}\sum_{i=1}^{2}[\sum_{l=1}^{2}\frac{3}{5(x^2_{i,l}+1)} - \sum_{l=1}^{2}\frac{1}{10(9-\overline{x}_l)^2}],$$

$$P_X(x^N_{i,l}) = \frac{|x^N_{i,l}| + |x^N_{i,l} - 5|}{2} + \frac{5}{2}, i, j, n = 1, 2.$$

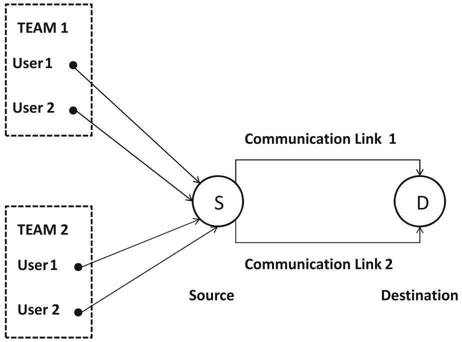

Fig. 2. The topological structure of parallel-link communication network with $N = 2, m^N = 2, L = 2$.

Utilizing MATLAB Simulink tool box, when $\alpha = 0.01, N = 2, m^N = 2, L = 2$, the simulation result with initial value $x(t) = (0.5, 1.6, 1.4, 2.3, 1.8, 0.9, 3.2, 2.7)$ for system (13) is shown in Fig. 3. From Fig. 3, one can see that the state vector of system (13) is asymptotically convergent to NNE x^*. From the result obtained in Theorem 5, this equilibrium point x^* is just a noninferior Nash equilibrium point of the multi-team noncooperative concave game.

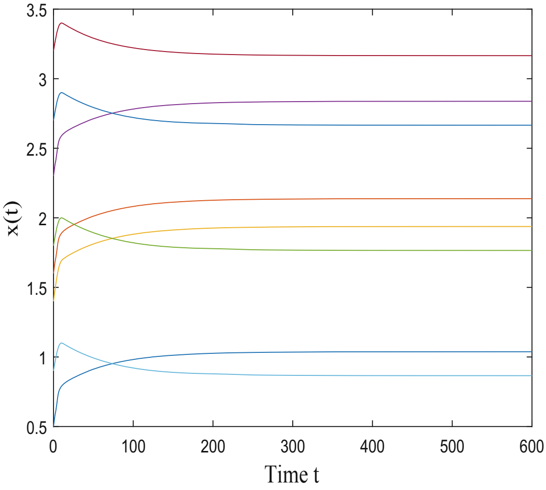

Fig. 3. State vector $x(t)$ of system (13) with $N = 2, m^N = 2, L = 2, \alpha = 0.01$.

6 Conclusion

In this paper, the computation problem of NNE of multi-team noncooperative game is investigated. Utilizing the relationship among variational inequalities, noninferior Nash equilibrium, and projection equation, the approximate calculation problem of NEE in general multi team game model with smooth and concave payoff function is transformed into a stable state numerical calculation problem by using projection neural network method. Finally, as an application, a flow control model of parallel-link communication networks based on multi-team game and neural network algorithm is also given. Simulation result shows that neural network algorithm for solving noninferior Nash equilibrium of multi-team noncooperative concave game is valid. In most cases, the payoff functions of communication network only satisfy concave condition, they are frequently nonsmooth. Thus, neural network algorithm for solving noninferior Nash equilibrium of multi-team with nonsmooth payoff functions is worth researching, and this will be our further work.

References

1. Myerson, R.B.: Game Theory: Analysis of Conflict. Harvard University Press, London (1991)
2. Aubin, J.P.: Mathematical methods of game and economic theory. Stud. Math. Appl. **235**(1), 19–30 (2007)
3. Glötzl, E.: Continuous time, continuous decision space prisoner's dilemma- a bridge between game theory and economic GCD-models. MPRA Paper (2016)
4. Morrow, J.D.: Game Theory for Political Scientists. Princeton University Press, New Jersey (1994)

5. Shoham, Y.: Computer science and game theory. Commun. ACM **51**(8), 74–79 (2008)
6. Abramsky, S., Mavronicolas, M.: Game theory meets theoretical computer science. Theoret. Comput. Sci. **343**(1–2), 1–3 (2005)
7. Salem, A.A., et al.: Profit of price with supermodular game for spectrum sharing in cognitive radio using genetic algorithm. Wireless Pers. Commun. **82**(4), 2601–2609 (2015)
8. Liu, Y., Simaan, M.: Noninferior Nash strategies for multi-team systems. J. Optim. Theory Appl. **120**(1), 29–51 (2004)
9. Liu, Y., Simaan, M.: Non-inferior Nash strategies for routing control in parallel-link communication networks. Int. J. Commun Syst **18**(4), 347–361 (2005)
10. Ahmed, E., et al.: On multi-team games. Physica A Stat. Mech. Appl. **369**(2), 809–816 (2006)
11. Yu, J.: Game Theory and Nonlinear Analysis Ekthesis. Science Press, Beijing (2011)
12. Yu, J.: Slightly altruistic equilibria of N-Person non-cooperative games. J. Syst. Sci. Math. Sci. **31**(5), 534–539 (2011)
13. Yu, J., Wang, N.-F., Yang, Z.: Equivalence results between Nash equilibrium theorem and some fixed point theorems. Fixed Point Theory Appl. **2016**(1), 1–10 (2016). https://doi.org/10.1186/s13663-016-0562-z
14. He, B.S., Yuan, X.M.: Convergence analysis of primal-dual algorithms for a saddle-point problem: from contraction perspective. SIAM J. Imag. Sci. **5**, 119–149 (2012)
15. Liu, Z.X., Wang, N.F.: Neural network to solve concave games. Int. J. Comput. Games Technol. **2014**, 1–10 (2014)
16. Herings, P.J.J., van den Elzen, A.: Computation of the Nash equilibrium selected by the tracing procedure in-person games. Games Econ. Behav. **38**(1), 89–117 (1998)
17. Askar, S.S., et al.: Dynamic Cournot duopoly games with nonlinear demand function. Appl. Math. Comput. **259**, 427–437 (2015)
18. Askar, S.S.: Complex dynamic properties of Cournot duopoly games with convex and log-concave demand function. Oper. Res. Lett. **42**(1), 85–90 (2014)
19. Yu, J.: Game Theory and Nonlinear Analysis. Science Press, Beijing (2008)
20. Li, J.H., Michel, A.N., Porod, W.: Analysis and synthesis of a class of neural networks-Linear systems operating on a closed hypercube. IEEE Trans. Circ. Syst. **36**, 1405–1422 (1989)
21. Naito, T., et al.: Differential Equations with Time Lag-Introduction to Functional Differential Equations. Makino Shoten, Tokyo (2002)
22. Liao, X.X.: Theory Methods and Application of Stability. Huazhong University of Science and Technology Press, Wuhan (2005)
23. Liu, Q.S., Wang, J.: A projection neural network for constrained quadratic Minimax optimization. IEEE Trans. Neural Netw. Learn. Syst. **26**(11), 2891–2900 (2015)
24. Feng, H.B., et al.: Wireless Ad hoc network flow control model based on non-coopertative game theory. J. Electron. Inf. Technol. **31**(44), 925–928 (2009)