Performance of Uplink Ultra Dense Network with Antenna Selection

Trung Ninh Bui[1], Sinh Cong Lam[1(✉)], Duc Tan Tran[2], and Nguyen Thi Dieu Linh[1,2,3]

[1] Faculty of Electronics and Telecommunications, VNU University of Engineering and Technology, Hanoi, Vietnam
{ninhbt,congls}@vnu.edu.vn
[2] Faculty of Electrical and Electronic Engineering, Phenikaa University, Ha Dong, Vietnam
[3] Faculty of Electronics Engineering Technology, Hanoi University of Industry, Hanoi, Vietnam

Abstract. Ultra Dense Network is promising as the potential topology of the 5G and Beyond network systems where various advanced techniques are being developed to improve the user performance. In this paper, the antenna selection technique is studied in terms of uplink coverage probability analysis. With assumption that the wireless transmission condition is under the effects of Nakagami-m as the fast fading and Stretched Path Loss Model (SPLM) as the slow fading, the paper derives the uplink coverage probability of the user in the Spatial Poisson Point Process network layout. Through the analytical results and Monte Carlo simulation, the paper illustrates that an increase in the number of antennas at the user can moderately reduce the its coverage probability.

Keywords: 5G · ultra dense network · antenna selection · Poisson point process

1 Introduction

Utilization of multi-antenna at both users and BSs is a potential technique to provide a high Quality of Service in the Ultra Dense network where BSs (Base Stations) are distributed with a very high density. In this technique, the users may use more than one antennas to convey its message to the serving BS. At the BSs, several copies of the transmitted messages are received but only the messages are carried by the strongest signal is processed. Conventionally, a higher number of antennas results in a better desired signal at the BS.

Research on antenna selection technique has been studied in the literature such as [6–9]. In [6], the multi-antenna is studied to assist the short packet communication in the IoT network. The analytical results proved that the packet loss can be improved by increasing the number of antennas. The network with limited - feedback and interference have been studied in [7,9] for device-to-device

N. Thi Dieu Linh et al. (Eds.): ADHOCNETS 2023, LNICST 558, pp. 196–204, 2024.
https://doi.org/10.1007/978-3-031-55993-8_15

network and moving interfers. In both cases, it was shown that the network performance in terms of outage probability and ergodic data capacity increases with the number of antennas. The benefits of utilizing multi-antenna were also presented in Reference [8] where the antennas are distributed as a Spatial Poisson Point Process (PPP). Therefore, it is clear that the utilization of multi-antenna, particularly antenna selection technique, is a possible solution to improve the 5G and Beyond cellular networks.

Although these papers illustrated the significant benefits of the antenna selection technique, main characteristics of the wireless transmission have not been well-captured. Particularly, these works either ignores the slow fading, or utilizes the regular path loss model, which is not well-captured the characteristics of millimeter wave transmission, as the slow fading, or model the fast fading as the Rayleigh random variable. This paper employs the Stretched Path Loss model (SPLM) and Nakagami-m as the slow fading and fast fading, respectively. While SPLM was recently developed as the most suitable to capture the complicated transmission phenomenon of millimeter wave, the Nakagami-m is considered the general case of fast fading. Thus, it can be said that the analysis of system model in this paper can reflect the practical network performance.

2 System Model

In this paper, an uplink millimeter wave cellular network where both the active users and the BSs are distributed according to independent Spatial Poisson Point Processes with a density of $\lambda^{(u)}$ $(user/m^2)$ and λ (BS/m^2), is studied. In an ultra dense network, the BSs are usually distributed at a very high of density. According to the authors in [2], the density of BSs may be upto $1-10$ stations in every m^2. Meanwhile, the number of users dynamically change by time. Therefore, some BSs do not have any associated user and turn into the idle mode to reduce the power consumption as well as intercell interference. As proved in the literature [3], the density of active BSs, which have at least an user, is

$$\overline{\lambda} = \lambda \left[1 - \frac{1}{\left(1 + \frac{\lambda^{(u)}}{q\lambda} \right)^q} \right] \tag{1}$$

where q is obtained from empirical activities, particularly $q = 3.5$.

To analyze the uplink network performance, both slow fading and fast fading are considered in this paper. To capture the slow fading, the Stretched Path Loss model (SPLM), which was recently developed to estimate the path loss in the ultra dense network with complicated transmission environments [4], is utilized. Particularly, the path loss over a distance of r (m) in the linear unit is computed by

$$L(r) = \exp\left(-\alpha r^{\beta}\right) \tag{2}$$

where β and α are empirical values which represent the density of obstacles and their signal resistance properties, respectively. In addition, the instantaneous value of fast fading is assumed to have a Nakagami-m distribution. Thus, the channel power is an normalized Gamma random variable $G(m, 1/m)$ with a cumulative density function of

$$F = \frac{\gamma(m, mx)}{\Gamma(m)} \tag{3}$$

where $\gamma(.,.)$ and $\Gamma(m)$ are the lower incomplete and complete Gamma functions.

To enhance the network performance, the users and BSs are usually equipped with multi-antennas. It is assumed that each user simultaneously uses N to transmit the signal to its serving BS, i.e. the nearest BS. Thus, each BS receives N copies of the desired signal from its associated user at distance r, where the power of the signal from k^{th} antenna is

$$S_k = P g_k L(r) \tag{4}$$

where P is the transmission power of the user; g_k and $L(r_k)$ are respectively the instantaneous power gain and the path loss.

With assumption that all antennas of the user transmit on the same sub-band, all N signals suffer the same intercell interference that originated from the user at adjacent cells. Since each user transmits on N antennas at the same time, the total intercell interference at the BS is given by

$$I = \sum_{n=1}^{N} \sum_{h \in \theta} P g_{n,h} L(r_h) = \sum_{h \in \theta} P L(r_h) \sum_{n=1}^{N} g_{n,h} \tag{5}$$

where θ is the set of interfering users. Since each user is only served by an unique BS, the density of interfering users is determined in Eq. 1. Since $g_{n,h}$ is the normalized Gamma random variable $G(m, 1/m)$, $g_h = \sum_{n=1}^{N} g_{n,h}$ is a Gamma random variable $G(Nm, 1/m)$.

Consequently, the uplink SINR that the BS receives from the k^{th} antenna of the user is

$$SINR_k = \frac{S_k}{I} \tag{6}$$

where the Gaussian noise power σ^2.

To reduce the processing complexity, the BS only process the signal with the highest quality. In other words, the received SINR at the BS is

$$SINR = \max_{1 \le k \le N} SINR_k = \frac{PL(r)}{I} \max_{1 \le k \le N} g_k \tag{7}$$

Let $g = \max_{1 \le k \le N} g_k$, the cumulative density function of g is determined as the following step

$$F_G(g) = P(G < g) = P\left(\max_{1 \le k \le N} g_k < g\right) \tag{8}$$

Since g_k $(1 \leq k \leq N)$ are random variables,

$$F_G(g) = \prod_{1 \leq k \leq N} P\left(g_k < T\right) = \left[\frac{\gamma(m, mx)}{\Gamma(m)}\right]^N \tag{9}$$

According to the result in Reference [5], the cumulative density function $F_G(g)$ is approximated by

$$\begin{aligned} F_G(g) &= \left[1 - \exp\left(-\tau g\right)\right]^{mN} \\ &= \sum_{u=0}^{mN} (-1)^u \mathbf{C}_{mN}^k \exp\left(-\tau u g\right) \end{aligned} \tag{10}$$

where $\tau = \frac{m}{[\Gamma(m+1)]^{1/m}}$ and $\mathbf{C}_n^k = \frac{n!}{k!(n-k)!}$.

3 Coverage Probability

In this section, we derive the coverage probability expression of the user in the network with multi-antenna selection technique. Theoretically, the uplink coverage probability is defined by

$$\mathcal{P}(T) = \mathbf{P}(SINR > T) \tag{11}$$

where T is the minimum required SINR of the BS to successfully decode the received signal from the user.

Theorem 1. *The uplink coverage probability of the user in the system - enabled transmit antenna selection technique under the affects of Nakagami-m and general path loss model $L(r)$ is given by*

$$\mathcal{P}(T) = \sum_{u=1}^{mN} (-1)^{u+1} \mathbf{E}\left[\prod_{k=0}^{mN-1} \exp\left(-\pi\lambda \int_0^\infty \frac{\frac{\tau u T}{m}\frac{L(r_h)}{L(r)} r_h dr_h}{\left(1 + \frac{\tau u T}{m}\frac{L(r_h)}{L(r)}\right)^{k+1}}\right)\right] \tag{12}$$

Proof. Substituting the formulation of $SINR$ in Eq. 7 into the coverage probability definition in Eq. 11, we obtain

$$\begin{aligned} \mathcal{P}(T) =& \mathbf{P}\left(\frac{PL(r)}{I} g > T\right) \\ =& \mathbf{P}\left(g > T\frac{I}{PL(r)}\right) \end{aligned} \tag{13}$$

Utilizing the approximated form of $F_G(g)$ in Eq. 10, we get

$$\mathcal{P}(T) = 1 - \sum_{u=0}^{mN} (-1)^u \exp\left(-\tau u T \frac{I}{PL(r)}\right) \tag{14}$$

Substituting the definition of I in Eq. 5 with notice that $g_h = \sum_{n=1}^{N} g_{n,h}$, the coverage probability is given by

$$\mathcal{P}(T) = 1 - \sum_{u=0}^{mN} (-1)^u \mathbf{E}\left[\prod_{h\in\theta} \exp\left(-\tau u T \frac{L(r_h)}{L(r)} g_h\right)\right] \tag{15}$$

where $\overline{\gamma} = \frac{P}{\sigma^2}$.

Since g_h is the Gamma random variable with a shape of Nm and a scale of $1/m$, its MGF is $M(s) = (1 + s/m)^{-Nm}$. Hence,

$$\mathcal{P}(T) = 1 - \sum_{u=0}^{mN} (-1)^u \mathbf{E}\left[\prod_{h\in\theta} \frac{1}{\left(1 + \frac{\tau u T}{m}\frac{L(r_h)}{L(r)}\right)^{Nm}}\right] \tag{16}$$

Utilizing the probability of the Moment Generating Function with recalling that the distance from user to its interfering users and to serving BS are independent random Variables, we obtain

$$\begin{aligned}\mathcal{P}(T) =& 1 - \sum_{u=0}^{mN} (-1)^u \mathbf{E}\left[\exp\left(-2\pi\lambda \int_0^\infty 1 - \frac{1}{\left(1 + \frac{\tau u T}{m}\frac{L(r_h)}{L(r)}\right)^{Nm}} r_h dr_h\right)\right] \\ =& 1 - \sum_{u=0}^{mN} (-1)^u \mathbf{E}\left[\prod_{k=0}^{mN-1} \exp\left(-2\pi\lambda \int_0^\infty \frac{\frac{\tau u T}{m}\frac{L(r_h)}{L(r)} r_h dr_h}{\left(1 + \frac{\tau u T}{m}\frac{L(r_h)}{L(r)}\right)^{k+1}}\right)\right]\end{aligned}$$

The Theorem is proved.

Lemma 1. *The closed-form expression of coverage probability of the user in the system with antenna selection under the Nakagami-m and regular path loss model* $L(r) = r^{-\alpha}$ *is given by*

$$\mathcal{P}(T) = \sum_{u=1}^{mN} (-1)^{u+1} \frac{1}{1 + \sum_{k=0}^{mN-1} \left[\frac{\tau u T}{m}\right]^{2-\alpha} \mathbf{B}(\alpha - 2, k - 2 + \alpha)} \tag{17}$$

Proof. Substituting $L(r) = r^{-\alpha}$ into Eq. 12, we obtain

$$\mathcal{P}(T) = \sum_{u=1}^{mN} (-1)^u \mathbf{E}\left[\prod_{k=0}^{mN-1} \exp\left(-2\pi\lambda \int_0^\infty \frac{\frac{\tau u T}{m}\frac{r_h^{-\alpha}}{r^{-\alpha}} r_h dr_h}{\left(1 + \frac{\tau u T}{m}\frac{L(r_h)}{L(r)}\right)^{k+1}}\right)\right]$$

Employing a change of variable $y = r/r_h$, then

$$\mathcal{P}(T) = \sum_{u=1}^{mN} (-1)^u \mathbf{E}\left[\prod_{k=0}^{mN-1} \exp\left(-\pi\lambda r^2 \int_0^\infty \frac{\frac{\tau u T}{m} y^{\alpha-3} dy}{\left(1 + \frac{\tau u T}{m} y^\alpha\right)^{k+1}}\right)\right]$$

Utilizing the result in Reference [1, p.315] with $\mu = a-2$, $\beta = \frac{\tau u T}{m}$ and $v = k+1$, the coverage probability is given by

$$\mathcal{P}(T) = \sum_{u=1}^{mN} (-1)^u \mathbf{E}\left[\prod_{u=0}^{mN-1} \exp\left(-\pi\lambda r^2 \left[\frac{\tau u T}{m}\right]^{2-\alpha} \mathbf{B}(\alpha-2, k-2+\alpha)\right)\right]$$

where $\mathbf{B}(.,.)$ is the Beta function.

Evaluating the expectation with respect to the random variable r whose PDF is $2\pi\lambda \exp\left(-\pi\lambda r^2\right)$, then

$$\begin{aligned}\mathcal{P}(T) =& \sum_{u=1}^{mN} (-1)^u \\ & \int_0^\infty \exp\left(-\pi\lambda r^2 \left[1 + \sum_{k=0}^{mN-1} \left[\frac{\tau u T}{m}\right]^{2-\alpha} \mathbf{B}(\alpha-2, k-2+\alpha)\right]\right) d(\pi\lambda r^2) \\ =& \sum_{u=1}^{mN} (-1)^u \frac{1}{1 + \sum_{k=0}^{mN-1} \left[\frac{\tau u T}{m}\right]^{2-\alpha} \mathbf{B}(\alpha-2, k-2+\alpha)} \end{aligned} \quad (18)$$

The theorem is proved.

4 Simulation and Analysis

4.1 Verification of Theoretical Analysis

This section utilizes Monte Carlo to verify the analytical result in Theorem 1 in the case of SPLM. Particularly, the SPLM parameters are selected at $\alpha = 0.5$ and $\beta = 0.5$; Nakagami-m with $m = 2$; the density of active BSs is $\lambda = 500/1e6$ (BS/km^2). The uplink user coverage probability is plotted with different values of SINR threshold T. As seen from Fig. 1, the Monte Carlo simulation curves visually match the theoretical ones, which illustrates the accuracy of the analytical approach and results.

Since the SINR threshold is the minimum value of uplink SINR that the BS requires to successfully detect the signal from the user, a larger value of SINR means a stronger requirement of the BS on uplink SINR quality. In other words, the probability of successful signal detection of the BS reduces as SINR threshold increases. Therefore, the uplink user coverage probability has a fast decline trend as SINR threshold increases as illustrated in Fig. 1. For example, when SINR threshold T increases from $T = -18$ dB to $T = -12$ dB, the coverage probability reduces approximately 25% in the case of $m = 1, N = 1$. This fast decline trend can be seen from Eq. 12 where the coverage probability likely exponentially reduce with SINR threshold T.

5 Coverage Probability Vs Number of Antennas

Figure 2 examines the effects of number of antennas that the user uses to convey the signal to its serving BS. While most of works in the literature proved that

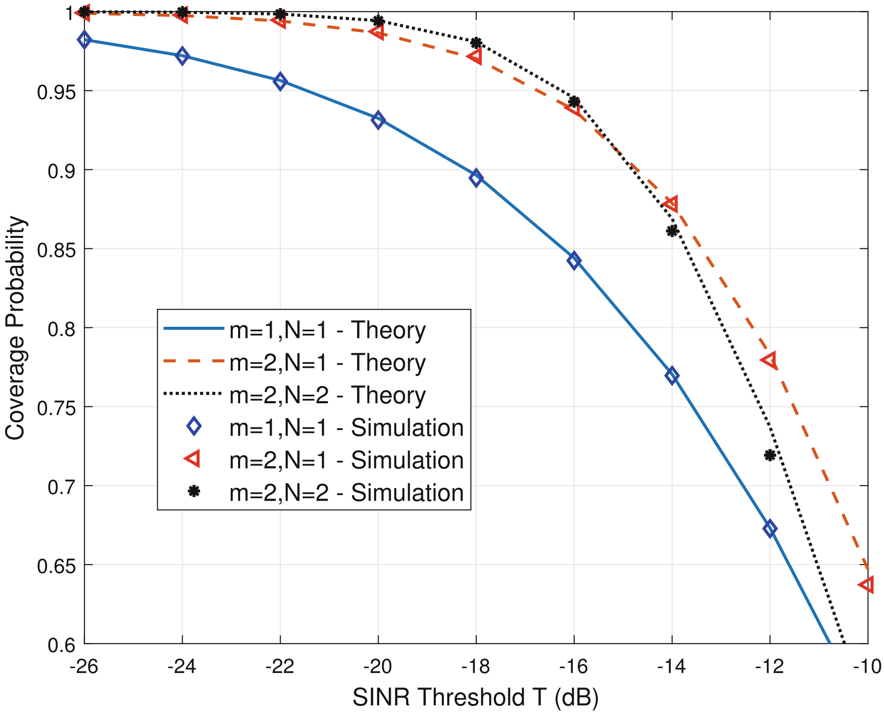

Fig. 1. Coverage Probability vs SINR threshold T (dB)

the utilization of antenna selection technique can improve the user performance, Fig. 2 illustrates the user coverage probability reduces as the number of antennas increases.

Particularly, when the number of antennas N increases from 4 to 10, the uplink coverage probability reduces by 53.6% from 0.41 to 0.19. This decline trend can be explained as follows: When the number of antennas at the user increases, the BSs has more change to receive the signal with the highest quality. However, due to the policy of the antenna selection, the user only receives the best signal from the transmitted antenna. Furthermore, a higher number of antennas also causes a worse wireless condition, particularly a higher intercell interference. When the benefit of a higher antenna utilization can not overcome the intercell interference, the coverage probability of the user decline. Thus, to improving the performance of the multi-antenna ultra dense network, the following critical problems need to be studied

- *Interference Mitigation* This is one of the most popular techniques to improve the uplink SINR of the user. In the previous cellular system, the interference coordination technique is the recommended solution to reduce the intercell interference. Thus, the combination of this technique and multi-antenna one should be studied.

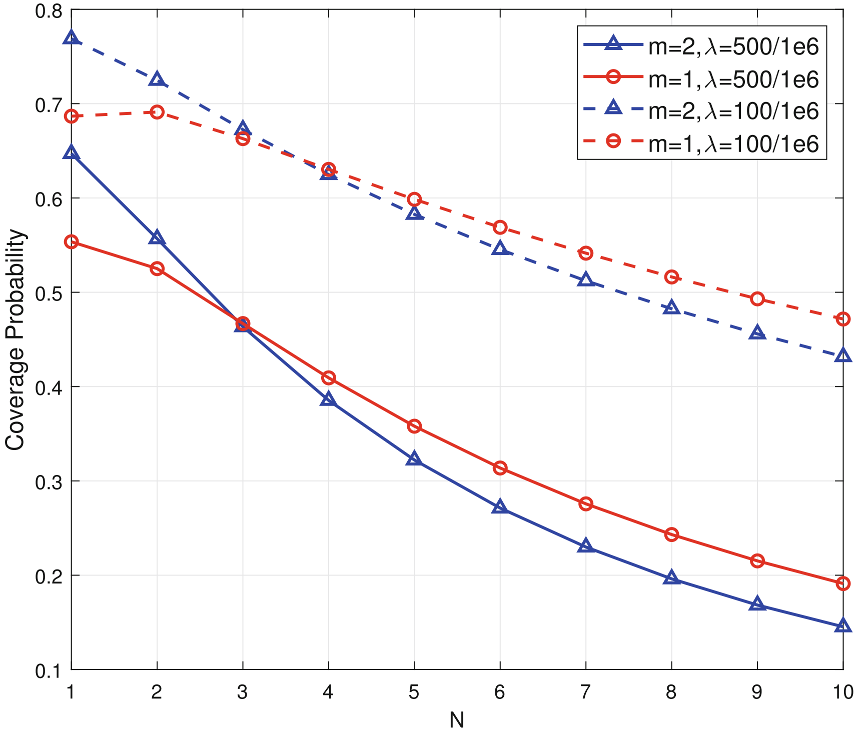

Fig. 2. Coverage probability vs number of antennas

– *Desired signal power improvement* Due to the deployment of the antenna selection technique, the user only receives and processes the signal from the best antenna. To improve the quality of the desired signal, the more effective technique such as Maximum Ratio Combining may be used to combine the signal from different antennas, so that the desired signal power can significantly improve. Hence, the feasibility of Maximum Ratio Combining in the ultra dense network with multi-antenna technique should be examined.

6 Conclusion

In this paper, the performance of ultra dense networks that utilize the antenna selection is evaluated under conditions of Nakagami-m fading. In this system, the user utilizes more than one antennas to convey the messages to its serving BSs, while the BSs compares the signal power from different antennas and select the strongest signal to perform further process. The uplink coverage probability of the user is derived in the case of SPLM and in closed-form when the path loss follows the regular model. While most of the related works showed that the user perform with increases with the number of antennas, the analytical results in this paper show that due to the rapid increase of intercell interference, the coverage probability reduces with the number of antennas.

References

1. Gradshteyn, I.S., Ryzhik, I.M.: Table of Integrals, Series, and Products Academic Press (2007)
2. Chafii, M., Bariah, L., Muhaidat, S., Debbah, M.: Twelve scientific challenges for 6G: rethinking the foundations of communications theory. In: IEEE Communications Surveys & Tutorials, vol. 25, no. 2, pp. 868–904. Secondquarter (2023). https://doi.org/10.1109/COMST.2023.3243918
3. Lee, S., Huang, K.: Coverage and economy of cellular networks with many base stations. IEEE Commun. Lett. **16**(7), 1038–1040 (2012). https://doi.org/10.1109/LCOMM.2012.042512.120426
4. AlAmmouri, A., Andrews, J.G., Baccelli, F.: SINR and throughput of dense cellular networks with stretched exponential path loss. IEEE Trans. Wirel. Commun. **17**(2), 1147–1160 (2018). https://doi.org/10.1109/TWC.2017.2776905
5. Alzer, H.: On some inequalities for the incomplete gamma function. Math. Comput. **66**(218), 771–778 (1997)
6. Chen, D., Li, J., Hu, J., Zhang, X., Zhang, S.: Multi-antenna jammer-assisted secure short packet communications in IoT networks. Future Internet **15**(10), 320 (2023). https://doi.org/10.3390/fi15100320
7. Krauss, R., Peron, G., Brante, G., Souza, R.D.: Area energy efficiency of antenna selection in limited feedback device-to-device networks. IEEE Wirel. Commun. Lett. **8**(3), 949–952 (2019). https://doi.org/10.1109/LWC.2019.2901475
8. AbdelNabi, A.A., Al-Qahtani, F.S., Shaqfeh, M., Ikki, S.S., Alnuweiri, H.M.: Performance analysis of MIMO multi-hop system with TAS/MRC in Poisson field of interferers. IEEE Trans. Commun. **64**(2), 525–540 (2016). https://doi.org/10.1109/TCOMM.2015.2496291
9. Do, D.-T., Nguyen, T.-L., Lee, B.M.: Transmit antenna selection schemes for NOMA with randomly moving interferers in interference-limited environment. Electronics **9**(1), 36 (2020). https://doi.org/10.3390/electronics9010036