Wavelet Threshold Denoising for High Speed Satellite Communication

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Abstract. The principle of wavelet threshold de-noising is described in detail. Daubechies wavelet function is selected to denoise the sinusoidal signal with noise. The relationship between the order of wavelet function, the number of decomposition layers, the number of signal samples and the signal-to-noise ratio of denoised signal is demonstrated. At the same time, the calculation methods of fixed threshold, unbiased risk estimation threshold, mixed threshold and mini threshold max threshold are summarized, and then the denoising effects of hard threshold, soft threshold and semi soft threshold functions are compared. By analyzing the distribution of wavelet decomposition coefficients and combining with the characteristics of other threshold functions, a new wavelet threshold function is designed and compared with other threshold functions.

Keywords: Wavelet threshold denoising · Threshold function

1 Introduction

Generally speaking, after the data acquisition system collects the data, we need to process the data. In the process of signal detection and transmission, we will inevitably suffer from noise interference, so the measurement accuracy and accuracy in complex environment become an important factor affecting its practical application, so it is necessary to filter the noise.

Traditional signal theory, mainly based on Fourier analysis theory, directly or indirectly transforms the signal from time domain to frequency domain for analysis and processing. Because the set of analysis functions selected by Fourier analysis is a sinusoidal function set in an infinite time domain, Fourier transform lacks the ability to distinguish signals in time domain, that is, to know the frequency or frequency distribution of signals at different times. In view of the shortcomings of Fourier transform, the short-time Fourier transform (STFT) has been developed, which solved the problem of the lack of time resolution of Fourier transform to a certain extent, but because the

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width of the sliding window function of STFT is fixed, the short-time Fourier Transform (STFT) has insufficient time domain resolution when the signal frequency changes rapidly and irregularly with time [1].

Wavelet transform is a new method of signal analysis. It decomposes the signal by using the wavelet function which decays rapidly in the whole-time domain, the average amplitude is 0, and the time-frequency window is variable, so as to realize the signal analysis in time domain and frequency domain. Wavelet transform can be divided into two categories: continuous wavelet transform (CWT) and discrete wavelet transform (DWT). In the field of digital signal processing, discrete wavelet transform (DWT) decomposes digital signal by discrete scale and discrete translation wavelet function. By processing the decomposed wavelet coefficients and reconstructing them, signal processing tasks such as signal compression and denoising can be completed. Among them, the wavelet threshold denoising algorithm is a filtering algorithm that thresholds the discrete wavelet transform coefficients of the digital signal, and then reconstructs the signal to achieve the denoising effect. Compared with other filtering methods, the wavelet threshold denoising algorithm has a better effect for the denoising problem of time-varying and non-stationary random signals such as sound [2]. Next, the process of wavelet threshold de-noising is described step by step, and some skills in the algorithm are demonstrated. Finally, a new threshold function and its denoising effect are shown.

2 Wavelet Threshold Denoising Process

Assume that the signal with Gaussian white noise is:

$$f(t) = s(t) + n(t) \tag{1}$$

Where $s(t)$ is the original signal without noise, $n(t)$ is the Gaussian white noise of variance σ^2, that is to say, it obeys $N\left(0, \sigma^2\right)$ distribution. The discrete-time signal $f[n] = f(nT_s)$ is obtained by Nyquist sampling with sampling interval $T_s \leq \frac{2}{f_{max}} (f_{max}\ is\ the\ maximum\ frequency\ of\ f)$.

Generally speaking, compared with noise, the energy of signal is larger and its distribution is more concentrated. Because wavelet transform has the characteristics of sparsity and decorrelation [3], most of the energy of the signal is concentrated in the wavelet approximation coefficients, while the wavelet detail coefficients are the coefficients after the decomposition of the signal high frequency part and high frequency noise, and the energy of signal wavelet transform is concentrated in the low frequency part. The wavelet transform of Gaussian noise is still Gaussian distribution, the energy of noise is more evenly distributed on all wavelet coefficients, and the corresponding wavelet coefficient amplitude is also small. Therefore, the appropriate threshold is selected in different wavelet transform scales, and the corresponding wavelet coefficients are processed by threshold quantization. Finally, the denoising signal is obtained by inverse wavelet transform, so that the noise can be effectively suppressed. On the other hand, compared with FIR filter or IIR filter, wavelet threshold de-noising can also preserve the peak and mutation of the signal. Wavelet threshold denoising process can be divided into the following three steps [4]:

1. Discrete wavelet transform for noisy signal $f[n]$. According to the characteristics of the signal, the appropriate wavelet function is selected, and the decomposition level $Ndec$ is determined, then the decomposition calculation is carried out to obtain a group of wavelet detail coefficients $cD_j(j = 1 \cdots Ndec)$ and wavelet approximation coefficients cA_{Ndec}.
2. The detail coefficients after wavelet decomposition are quantized by threshold. Selecting a suitable threshold value for the detail coefficients cD_j of each decomposition scale to quantize the threshold value and get the processed wavelet coefficients sD_j.
3. The signal is reconstructed by inverse wavelet transform to get the denoised signal. According to the approximation coefficient cA_{Ndec} of wavelet decomposition and the detail coefficient sD_j after threshold quantization of each layer, the estimated signal s is obtained, that is, the denoising signal.

The process of wavelet threshold denoising is shown in Fig. 1.

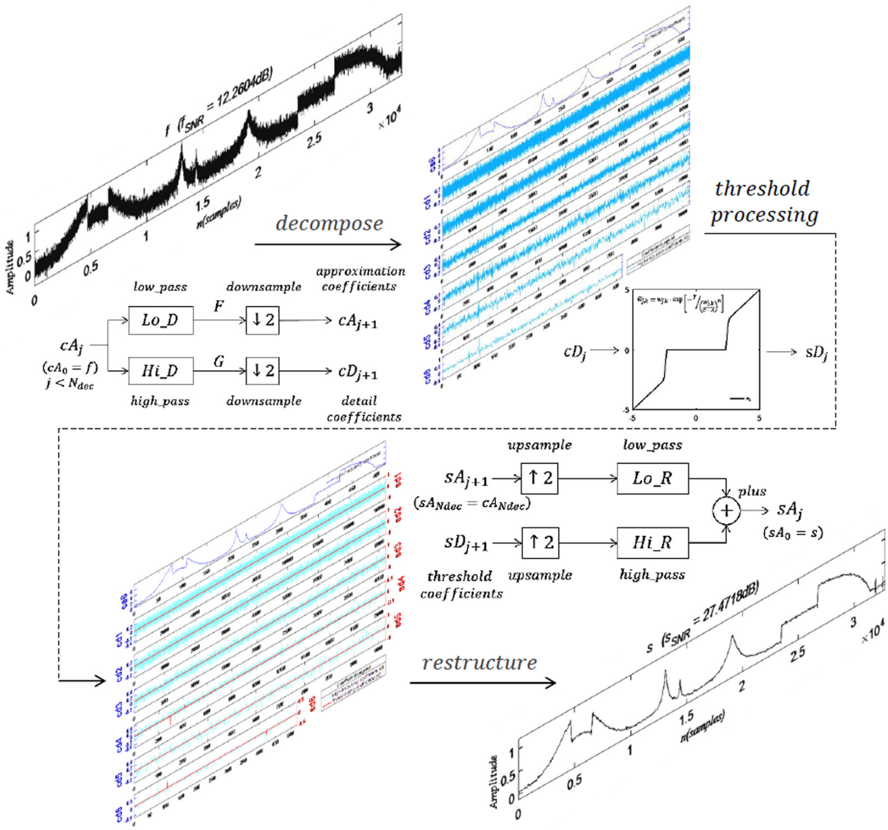

Fig. 1. The process of wavelet threshold denoising

Mallat algorithm is used for wavelet decomposition and reconstruction [5].

3 Selection of Wavelet and Determination of Decomposition Level

According to the properties of wavelet function, it can be seen that the vanishing moment reflects the smoothness of the wavelet function. The higher the vanishing moment, the smoother the wavelet, but it also means that the tightly supported interval is larger, which is not conducive to localized analysis [6]. For fast wavelet transform (FWT), the longer the filter length required, the greater the amount of calculation, and the denoising effect cannot be greatly improved. Therefore, for smoother signals, wavelet functions with higher vanishing moments should be selected appropriately. For example, for *Daubechies* wavelets, *Symlet* wavelets and *Coiflet* wavelets, wavelet functions with higher orders should be selected. On the contrary, for abrupt signal, the wavelet function with small vanishing moment and short support length should be selected.

The selection of decomposition layers is related to the maximum frequency and sampling frequency of the signal. There are too many decomposition layers. The cut-off frequency of the wavelet decomposition low-pass filter is less than the maximum frequency of signal. The signal will be divided into the high-frequency detail wavelet coefficient part. Because the wavelet threshold denoising will process the wavelet coefficient, the high frequency detail wavelet coefficients will be attenuated or eliminated, and the signal will be distorted after wavelet reconstruction. Similarly, if the number of decomposition layers is too small, too much high-frequency noise will be classified into the low-frequency part of wavelet decomposition.

Therefore, the selected number of wavelet decomposition layers should make the maximum frequency of the useful signal fall on the low-frequency coefficient part of wavelet decomposition [7]. Next, the influence of wavelet order *Nr*, decomposition level *Ndec* and sampling number *N* on wavelet threshold denoising is studied quantitatively.

As shown in Fig. 2. The variation of *SNR* with wavelet order *Nr* and decomposition level *Ndec* is shown in the figure. The original signal is a sinusoidal signal with period 1, and the sampling number is $N = 2^{15}$, sampling 10 periods. We use *Daubechies* wavelet function with different orders and different decomposition layers to perform wavelet threshold denoising on the noisy signal, and we calculate the signal-to-noise ratio of the denoising signal. For better observation, three views of the three-dimensional coordinate diagram are shown.

For the signal with specific frequency and sampling number, it can be seen from the figure, there is a corresponding decomposition level, which makes the signal-to-noise ratio of wavelet threshold denoising best after selecting the wavelet function. At the same time, the influence trend of the order of wavelet on the signal-to-noise ratio is generally the same as that mentioned before, but the signal-to-noise ratio (SNR) oscillates with the wavelet order due to the mismatch between the scale of wavelet function and the scale of signal. Therefore, the matching problem of wavelet function scale and signal should be considered in the selection of wavelet order.

The effect of the number of samples *N* on the signal-to-noise ratio *SNR* is shown in Fig. 3. The signal is the same as that in Fig. 2, but the number of samples *N* of the signal is changing. With the increase of *N*, the optimal decomposition level increases, and the corresponding signal-to-noise ratio also increases. In fact, for a discrete-time sinusoidal signal with a fixed number of periods, its frequency and sampling rate are equivalent, so the number of samples increases and the signal frequency decreases. From the previous

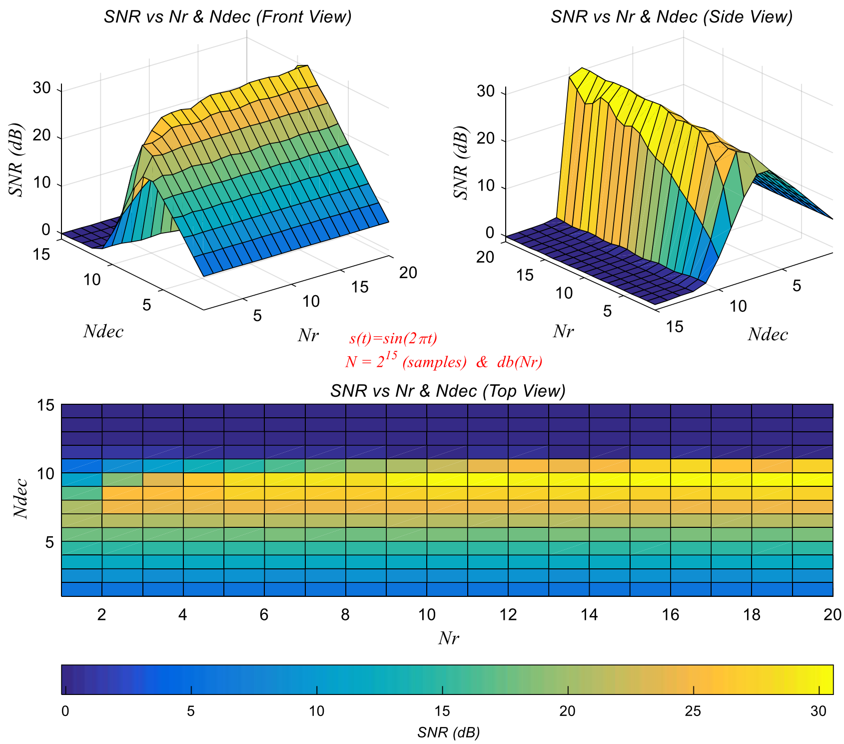

Fig. 2. The influence of *Ndec* and *Nr* on *SNR*

analysis, it can be seen that the number of decomposition levels should be increased. The increase of signal-to-noise ratio is due to the decrease of noise power spectral density with the increase of sampling rate while the signal power remains unchanged.

4 Threshold and Threshold Function

4.1 Threshold Calculation

Fixed Threshold Criterion (Sqtwolog)

$$Tr = \sigma\sqrt{2\log N} \tag{2}$$

N is the signal length and Tr is the global threshold. In soft thresholding, the fixed thresholding can get the visual direct denoising effect, so it is called VisuShrink [8].

Unbiased Risk Assessment Criteria (Rigrsure)
That is, an adaptive threshold selection method based on Stein's unbiased likelihood estimation principle. For each threshold value, the corresponding risk value is calculated. The one with the lowest risk is selected. The specific algorithm is as follows.

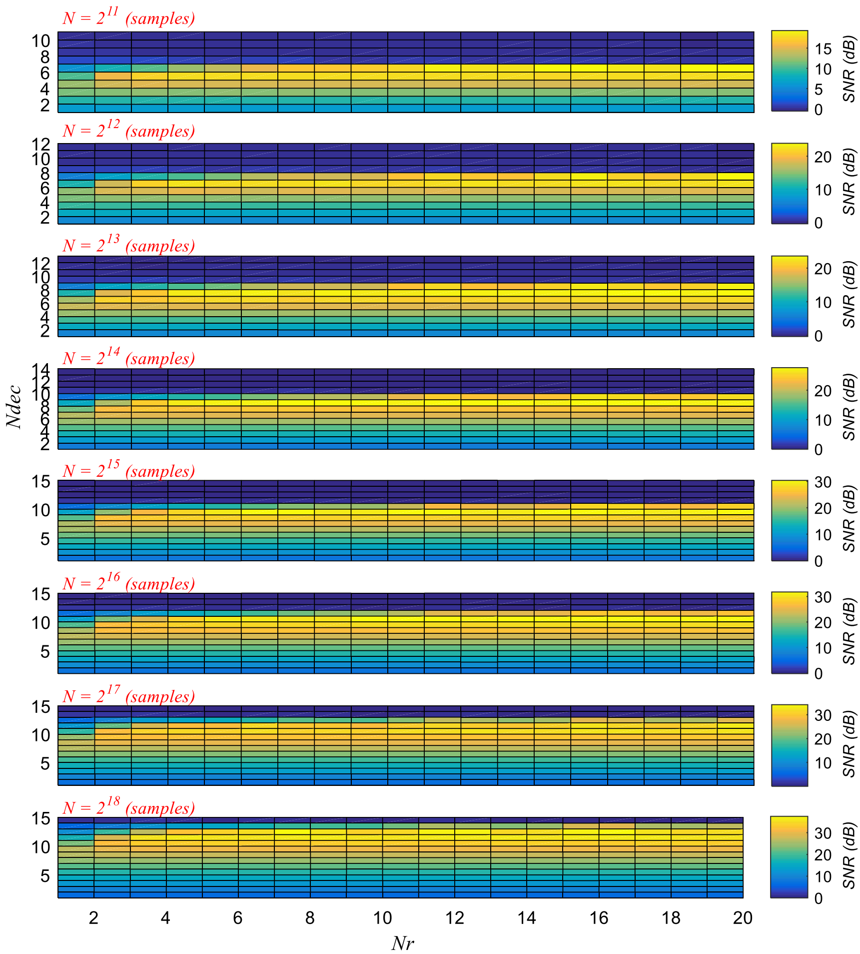

Fig. 3. The influence of sampling number N on SNR

- The wavelet coefficient vector (whose length is n) used to estimate the threshold value is squared, and then sorted from small to large to get a new vector NV to be estimated.
- For each element subscript k of NV, calculate the risk vector as follows:

$$Risk(k) = \frac{n - 2k + \sum_{i=1}^{k} NV(i) + (n-k) \cdot NV(k)}{n} \quad (3)$$

- The value of subscript k corresponding to the minimum point of risk vector *Risk* is obtained, and the threshold *Tr* is obtained as follows:

$$Tr = \sqrt{NV(k)} \tag{4}$$

Mixed Criterion (Heursure)
It is a mixture of fixed threshold criteria and unbiased risk estimation criteria. When the signal-to-noise ratio is very low, unbiased risk estimation criteria has a lot of noise, so fixed threshold is used. The calculation method of the threshold value is: first, judge the size of two variables *Eta* and *Crit*, their expressions are respectively.

$$\begin{cases} Eta = \frac{\sum_{j=1}^{n} |w_{j,k}|^2 - n}{n} \\ Crit = \sqrt{\frac{1}{n}(log_2 n)^3} \end{cases} \tag{5}$$

Where n is the length of the wavelet coefficient vector to be estimated, if $Eta < Crit$, the fixed threshold is selected, otherwise, the smaller of unbiased risk estimation criterion and fixed threshold criterion is selected as the threshold of this criterion.

Minimax Criterion (Minimaxi)
Minimax criterion is also a fixed form of threshold selection, which is used in statistics for the design of estimators. Since the denoising signal can be assumed to be the estimator of unknown regression function, the minimax estimator is the one to achieve the minimum mean square error under the worst condition [9]. The calculation formula of the threshold value is [9]:

$$Tr = \begin{cases} 0, \ N \le 32 \\ 0.3936 + 0.1829 \cdot log_2 N, \ N > 32 \end{cases} \tag{6}$$

The above threshold is for Gaussian white noise with standard deviation (wavelet domain) of 1, so the actual threshold should be $Tr{\cdot}\sigma$, where σ is the standard deviation of noise. Generally speaking, the wavelet coefficients on the minimum scale are mostly caused by noise, so the estimated value is used. The estimation method is as follows: if M_x is the median of absolute value vector of wavelet coefficient on the minimum scale of noisy signal, then:

$$\sigma = \frac{M_x}{0.6745} \tag{7}$$

The mixed criterion and minimax criterion in the above criteria are relatively conservative (only part of the coefficients are set to zero), so these two thresholds are more suitable when a small part of the high-frequency information of the signal is in the noise range. The other two threshold selection rules, especially the fixed threshold method, can eliminate more noise, but may also remove the high-frequency part of the useful signal [10].

4.2 Selection of Threshold Function

Hard-threshold function:

$$\hat{w}_{j,k} = \begin{cases} w_{j,k}, \left|w_{j,k}\right| \geq \lambda \\ 0, \left|w_{j,k}\right| < \lambda \end{cases} \tag{8}$$

Soft threshold function:

$$\hat{w}_{j,k} = \begin{cases} sign\left(\left|w_{j,k}\right|\right)\left(\left|w_{j,k}\right| - \lambda\right), \left|w_{j,k}\right| \geq \lambda \\ 0, \left|w_{j,k}\right| < \lambda \end{cases} \tag{9}$$

Semi-soft threshold function:

$$\hat{w}_{j,k} = \begin{cases} 0, \left|w_{j,k}\right| \leq \lambda_1 \\ sign\left(\left|w_{j,k}\right|\right)\frac{\lambda_2\left(\left|w_{j,k}\right| - \lambda_1\right)}{\lambda_2 - \lambda_1}, \lambda_1 < \left|w_{j,k}\right| \leq \lambda_2 \\ w_{j,k}, \left|w_{j,k}\right| > \lambda_2 \end{cases} \tag{10}$$

And λ_2 is the upper threshold and λ_1 is the lower threshold [11].

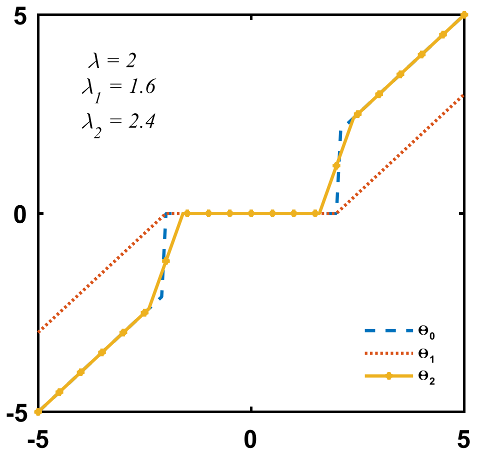

Fig. 4. Threshold function

In Fig. 4, Θ_0, Θ_1, Θ_2 are hard threshold function, soft threshold function and semi-soft threshold function curves respectively.

The following figure shows the processing effect of each threshold function under the fixed threshold value.

In Fig. 5, s_0 is the original signal, f is the noisy signal, $SNR = 12.2049$ dB, $s_{\theta 0}$ is the reconstructed signal processed by hard threshold function, and the *SNR* is 24.4302 dB. $s_{\theta 1}$ is the reconstructed signal processed by the soft threshold function with an *SNR* of 25.3501 dB. $s_{\theta 2}$ is the reconstructed signal processed by the semi-soft threshold function, and the *SNR* is 24.4488 dB. From the aspect of *SNR*, the processing effect of soft threshold and semi-soft threshold function is better than that of hard threshold function.

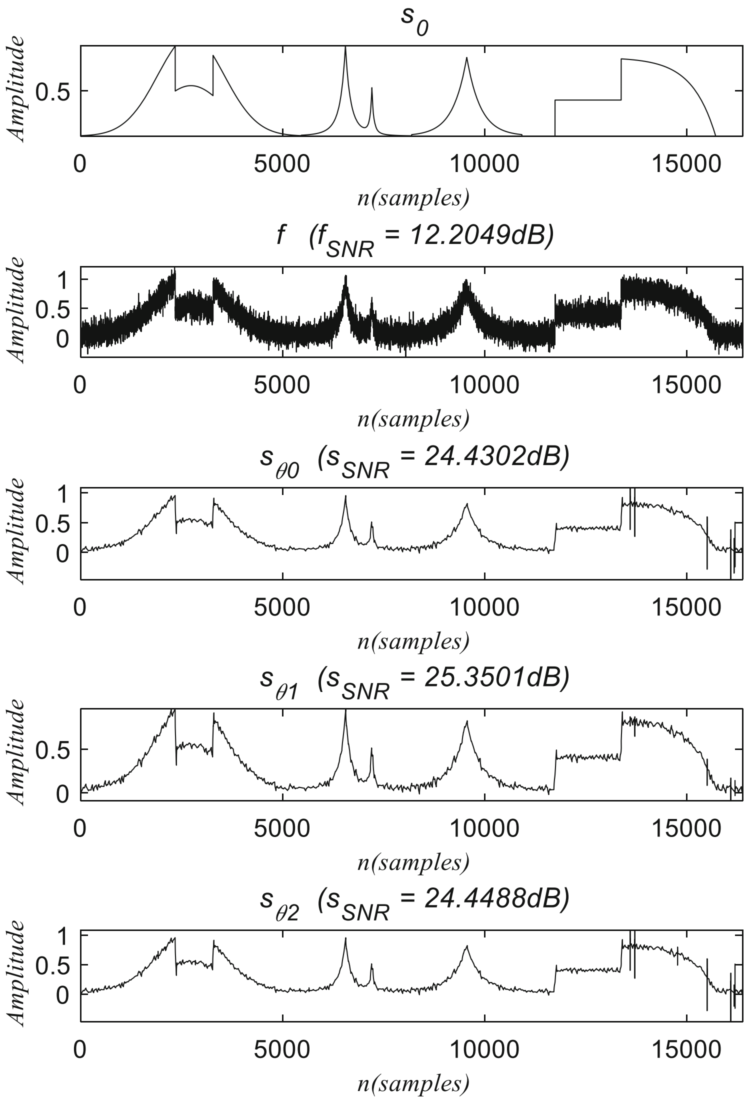

Fig. 5. Processing effect of different threshold functions

5 Improved Design of Threshold Function

The hard threshold function can easily cause Pseudo-Gibbs oscillation because of its discontinuity at the threshold. The soft threshold function has good continuity at the threshold, but the constant deviation of the quantized wavelet coefficients leads to the loss of some useful signals. Aiming at the dual problem of large variance of the hard threshold denoising result and fixed deviation of the soft threshold, the semi-soft threshold method is proposed by improving the threshold function, which can reduce the distortion of the denoising result better [4]. However, ordinary semi-soft threshold function has a few parameters and is not flexible to be used. Moreover, its linear Transition section cannot precisely control the wavelet coefficient near the threshold. Therefore, this paper designs a new semi-soft threshold function aiming at the shortcomings of ordinary semi-soft threshold function above in the processing of a high-frequency wavelet coefficient [12].

$$\hat{w}_{j,k} = w_{j,k} \cdot \exp\left[-\gamma \Big/ \left(\frac{w_{j,k}}{c \cdot \lambda}\right)^n\right] \tag{11}$$

Where λ is the threshold, n is the adjustment order, γ, c is the adjustment parameter, and the larger n is, the closer the function is to the hard threshold function; conversely, the smaller n is, the closer the function is to the soft threshold function. By adjusting γ, c to adjust the shape of the transition section of the function. If the wavelet coefficient $w_{j,k}$ is greater than the threshold, the function will converge to $w_{j,k}$ quickly. In a certain range where the wavelet coefficient $w_{j,k}$ is less than the threshold value λ, the function will quickly converge to 0, which is in line with the characteristics of the threshold function. Its function curve is shown in Fig. 6.

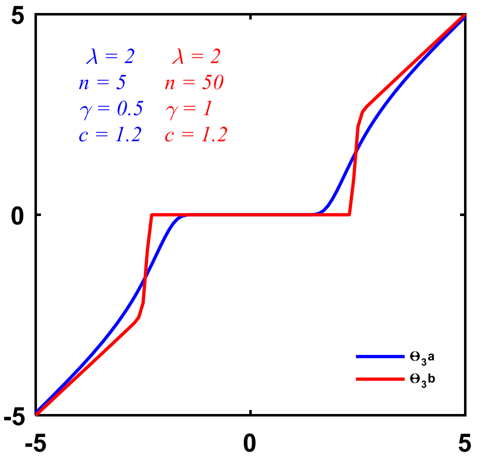

Fig. 6. New threshold function

The main advantage of this threshold function is that the function can be adjusted flexibly, which fully combines the advantages of soft and hard thresholds, and the function quickly converges to zero in the range of $[-\lambda, \lambda]$, and the cut-off effect is obvious.

Of course, the disadvantage is that in theory, using this threshold function for threshold processing cannot completely filter out high-frequency noise, because the function is only zero at the zero points, but because the value within the threshold range is very small (10^{-9}), so In practical applications, in addition to the accuracy of the algorithm or the accuracy of the machine, it can be set to zero, to achieve the purpose of making the high-frequency noise completely zero.

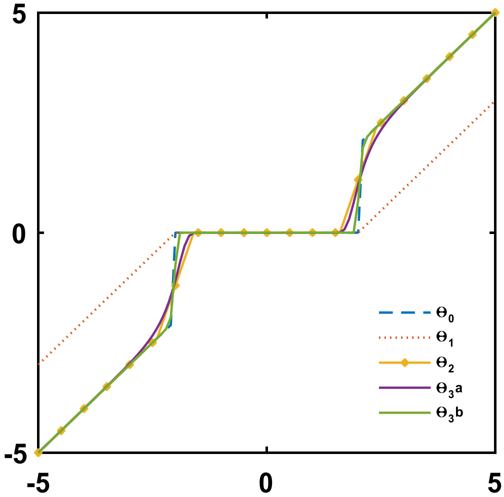

Fig. 7. Comparison of the new threshold function with the general threshold function

As shown Fig. 7, Θ_0, Θ_1, Θ_2 respectively hard threshold function, soft thresholding function, and semi-soft threshold function curve, $\Theta_3 a$, $\Theta_3 b$ is the new threshold function curve under different order numbers, you can see, choose different parameters, a new threshold function is a good way to approach the hard threshold function, and semi-soft threshold function, according to the different parameters, can approximate the optimal threshold function.

The specific denoising effect is shown in Fig. 8.

Compared with the hard threshold functions, the newly designed threshold function has a better processing effect, and it is equivalent to that of the soft threshold function.

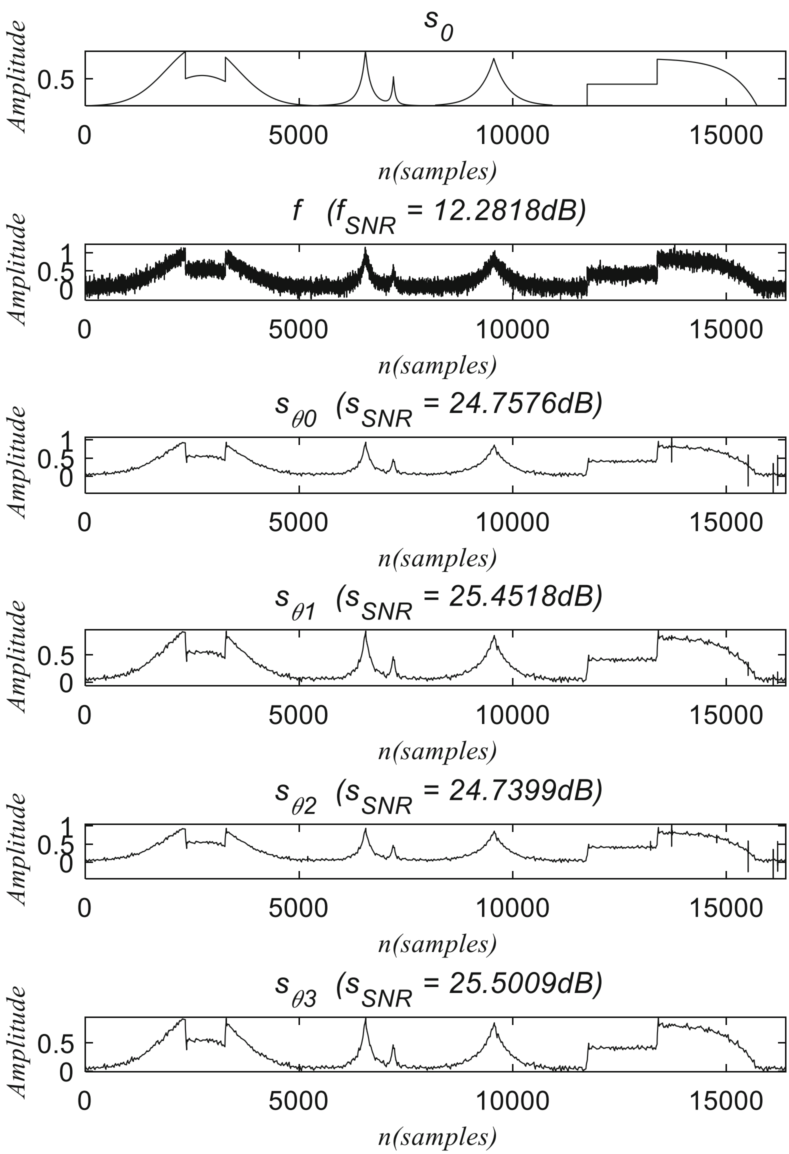

Fig. 8. Processing effect of the new threshold function

6 Conclusion

This paper mainly discusses the basic principle of wavelet threshold denoising, and analyzes the choice of wavelet function: that is, for the signal with higher smoothness, the wavelet function with higher disappearance moment should be selected, and for the sudden signal, the choice of small disappearance moment and Support a short wavelet function. Discuss the choice of decomposition layers: The choice of decomposition layers is related to the frequency of the signal and the sampling frequency. Too many decomposition layers. The cutoff frequency of the wavelet decomposition low-pass filter is greater than the signal frequency. The signal will be divided into high-frequency wavelet coefficients. Because wavelet threshold denoising will process wavelet coefficients and high-frequency wavelet coefficients will be eliminated, the signal will be eliminated, and the signal will not be restored during wavelet reconstruction; too few decomposition layers will cause excessive high-frequency noise Into the low-frequency part of the wavelet decomposition so that the reconstructed signal will contain these noises. Therefore, the number of wavelet decomposition layers should be selected so that the coefficient of the maximum frequency component of the useful signal falls exactly on the low-frequency coefficient part of the wavelet decomposition.

Then the characteristics of the threshold function are studied, and a new type of threshold function is designed. The function is flexible and convenient and can approximate other ordinary threshold functions. That is, by changing the parameters, the new threshold function has the characteristics of its ordinary threshold function. Experiments show that the new threshold function has a better denoising effect than other ordinary threshold functions.

Wavelet threshold denoising is a flexible and effective denoising method. Threshold calculation is the key problem of wavelet threshold denoising. If the threshold calculation is too large, the high-frequency details of the signal will be lost. If the threshold calculation is too small, there will be too many high-Frequency noises is reconstructed into the signal. At present, wavelet thresholds have fixed thresholds, unbiased risk estimation thresholds, heuristic thresholds, and maximum and minimum thresholds. Different threshold calculation methods should be selected for different types of noise and noise levels. Threshold function is a major focus of wavelet threshold denoising, and its function curve around the threshold determines the difference between this threshold function and other threshold functions. From my point of view, the key to improving the effect of wavelet threshold denoising is the threshold function. On the other hand, for the fine selection of wavelet coefficients, can we use neural networks instead of wavelet threshold functions to obtain the optimal threshold function for signals? This is a question to be demonstrated.

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