Joint Power Allocation and Passive Beamforming Design for IRS-Assisted Cell-free Networks

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Abstract. This paper investigates multiple intelligent reflecting surface (IRSs) assisted cell-free networks, where multiple single antenna access points (APs) and IRSs are connected to a network controller, to serve multiple user equipment (UEs) simultaneously. Our objective is to maximize the sum-rate of the cell-free network by jointly designing the power allocation of APs and the passive reflecting beamforming of IRSs, while the constraints on the maximum transmit power of each AP and the phase of each phase shifter (PS) of IRS are satisfied. However, the problem is non-convex and challenging to solve. To this end, we propose an efficient framework to jointly design the power allocation vectors and the passive reflecting beamforming matrices. Particularly, we first reformulate the problem as a more tractable form by employing the fractional programming methods and then decompose the transformed problem into two subproblems. Finally, we propose an alternating iteratively (AI) algorithm to solve the two subproblems, which is guaranteed to converge to locally optimal solutions. Simulation results indicate that the advantages of leveraging IRSs in improving the performance of the conventional cell-free networks.

Keywords: Intelligent reflecting surface · Power allocation · Fractional programming · Sequential Optimization

1 Introduction

Cell-free networks comprise a large number of randomly located single-antenna access points (APs) connected to a controller, and the controller optimizes the

This work was supported in part by the National Natural Science Foundation of China under Grant 61701401 and Grant 61872295, in part by the Key Research and Development Project of Shaanxi Province under Grant 2021KWZ-07, and in part by the Nova Program of Science and Technology of Shaanxi Province under Grant 2019KJXX-061.

H. Gao et al. (Eds.): ChinaCom 2021, LNICST 433, pp. 264–274, 2022.
https://doi.org/10.1007/978-3-030-99200-2_21

power allocation to improve the network performance [1]. Compared with the cellular system, cell-free networks are no cells or cell boundaries and do not assign the user equipment (UE) to the particular AP, and APs in cell-free networks serve multiple UEs simultaneously and cooperatively. Besides, the number of APs is larger than the number of UEs [2].

Recently, intelligent reflecting surfaces (IRSs) have emerged as a promising technology to improve the performance of the communication system via mitigating the detrimental propagation conditions and strengthening the desired signal powers [3]. Generally, IRSs are composed a large number of phase shifters (PSs), each of which can reconfigure the incident signals to desired directions with significant beamforming gains. The various IRS-assisted communication systems have been extensively investigated, and one of the key problem is jointly designing the power allocation (the active transmitting beamforming) and the passive reflecting beamforming [4–8]. The authors in [4] studied a joint power allocation and passive beamforming design problem to maximize the physical-layer security. The work [5] maximized the sum-rate of the IRS-assited non-orthogonal multiple access (NOMA) system by jointly optimizing the power allocation, passive and hybrid beamforming. The work [6] proposed an efficient algorithm based on vector forms of the fractional programming methods [9,10], i.e., Lagrangian dual transform (LDT) and quadratic transform (QT), and through joint optimizing the active transmit beamforming and the passive reflecting beamforming to maximize the sum-rate. The authors in [7] studied the sum-rate maximization problem in a multi-cell scenario, and they employed the weighted minimum mean-square error (WMMSE) [11] technique to transform the original problem into an equivalent form and proposed a block coordinate descent (BCD) and the majorization-minimization (MM) method based algorithm to solve it. The authors in [8] deployed the IRS to assist the joint processing coordinated multipoint (JP-CoMP) transmission. They studied two cases, i.e., the single UE case and the multiple UEs case. Particularly, the authors proposed a computational efficient MM-based algorithm to solve the problem in a single UE case and by capitalizing on the second-order cone programming (SOCP) and semi-definite relaxation (SDR) [12] techniques to solve the problem under multiple UEs case.

Motivated by the discussions as mentioned above, in this paper, we consider maximizing the sum-rate of IRSs-assisted cell-free communication system by jointly optimizing the power allocation of APs and the passive reflecting beamforming of IRSs. Note that the power allocation and the beamforming are intricately coupled, and the constant modulus constraints of IRS, the formulated problem is non-convex and challenging to solve. As a compromise approach, we provide an efficient framework to jointly design the power allocation and the beamforming. We first transform the problem to an equivalent form by employing the fractional programming methods, e.g., LDT and QT, and then decompose the reformulated problem into two subproblems, i.e., the power allocation optimization and the passive reflecting beamforming optimization. Then, we propose an alternating iteratively (AI) algorithm to solve the two subproblems. Particularly, for the former subproblem, we reformulate it as a convex quadratic programming (QP) problem and propose a Lagrangian dual sub-gradient based algorithm to obtain nearly closed-form solutions. While for the latter subprblem, we recast it as a constant modulus constrained quadratic programming (CMC-QP) problem, and then we propose a computational efficient sequential optimization (SO) algorithm to solve it with locally optimal solutions in closed-forms.

Notations: Low case letters denote vectors and upper case bold letters stand for matrices. $\text{Re}\{a\}$ is the real part of a. $\mathcal{CN}(0,1)$ is the distribution of a circularly symmetric complex Gaussian variable with zero mean and unit variance.

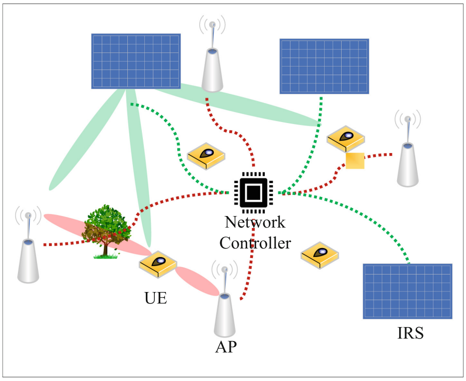


Fig. 1. System model of IRSs-assisted cell-free networks.

2 System Model and Problem Formulation

In this section, as shown in Fig. 1, we investigate an IRS-assisted cell-free networks, where $\mathcal{R}$ distributed IRSs are deployed to assisted $\mathcal{A}$ randomly distributed APs via connecting to a network controller for cooperatively serving $\mathcal{K}$ users. We assume that each AP and each UE are equipped with single antenna, and each IRS has $\mathcal{N}$ PSs.

The transmitted signal from the a-th AP is given as

$$x_a = \sum_{k=1}^{\mathcal{K}} \sqrt{p_{a,k}} t_{a,k}, \forall a \in \mathcal{A}, \tag{1}$$

where $t_{a,k} \sim \mathcal{CN}(0,1)$ represent the transmit signal from the a-th AP to the k-th UE and $p_{a,k}$ denotes the transmit power from the a-th AP to the k-th UE. Let $d_{a,k} \in \mathbb{C}$, $\mathbf{g}_{r,k} \in \mathbb{C}^{\mathcal{N}\times 1}$, and $\mathbf{s}_{a,r} \in \mathbb{C}^{\mathcal{N}\times 1}$ denote the channels of a-th AP to the k-th UE link, r-th IRS to the k-th UE link, and a-th AP to the r-th IRS link, respectively. Besides, the passive reflecting beamforming of IRSs are denoted as

$$\mathbf{\Theta}_r = \text{diag}\left(e^{j\phi_{r,1}}, e^{j\phi_{r,1}}, \cdots, e^{j\phi_{r,\mathcal{N}}}\right), \forall r \in \mathcal{R}. \tag{2}$$

The received signal at the k-th UE can be expressed as

$$y_k = \sum_{a=1}^{\mathcal{A}} \left(d_{a,k} + \sum_{r=1}^{\mathcal{R}} \mathbf{g}_{r,k}^{\mathrm{H}} \mathbf{\Theta}_r \mathbf{s}_{a,r} \right) x_a + n_k, \forall k \in \mathcal{K}, \tag{3}$$

where n_k is the noise at the k-th UE with distribution $\mathcal{CN}\left(0, \sigma_k^2\right)$. Hence, the signal-to-interference-plus-noise ratio (SINR) at the k-th UE can be written as

$$\gamma_k = \frac{\sum_{a=1}^{\mathcal{A}} p_{a,k} \left| d_{a,k} + \sum_{r=1}^{\mathcal{R}} \mathbf{g}_{r,k}^{\mathrm{H}} \mathbf{\Theta}_r \mathbf{s}_{a,r} \right|^2}{\sum_{a=1}^{\mathcal{A}} \sum_{i \neq k}^{\mathcal{K}} p_{a,i} \left| d_{a,k} + \sum_{r=1}^{\mathcal{R}} \mathbf{g}_{r,k}^{\mathrm{H}} \mathbf{\Theta}_r \mathbf{s}_{a,r} \right|^2 + \sigma_k^2}. \tag{4}$$

To facilitate expression, we define $h_{a,k} = d_{a,k} + \sum_{r=1}^{\mathcal{R}} \mathbf{g}_{r,k}^{\mathrm{H}} \mathbf{\Theta}_r \mathbf{s}_{a,r}$, which denotes the equivalent channel spanning from the a-th AP to the k-th UE.

In this paper, our objective is to maximize the sum-rate of the cell-free network, i.e., $R(\mathbf{p}, \mathbf{\Theta}) = \sum_{k=1}^{\mathcal{K}} \log(1 + \gamma_k)$, by jointly designing the power allocation vector $\mathbf{p} = [p_{1,1}, \cdots, p_{1,\mathcal{K}}, p_{2,1}, \cdots, p_{\mathcal{A},\mathcal{K}}]^{\mathrm{T}}$, and the passive reflecting beamforming matrix $\mathbf{\Theta} = \mathrm{diag}\{\mathbf{\Theta}_1, \mathbf{\Theta}_2, \cdots, \mathbf{\Theta}_{\mathcal{R}}\}$. Therefore, the sum-rate maximization problem can be formulated as

$$\max_{p, \mathbf{\Theta}} \quad R(p, \mathbf{\Theta}) = \sum_{k=1}^{\mathcal{K}} \log(1 + \gamma_k) \tag{5}$$

$$\text{s.t.} \quad \sum_{k=1}^{\mathcal{K}} p_{a,k} = P_{a,\max}, \forall a \in \mathcal{A}, \tag{5a}$$

$$|\mathbf{\Theta}_{r,n}| = 1, \forall r \in \mathcal{R}, \forall n \in \mathcal{N}, \tag{5b}$$

where the constraint (5a) limits the maximum transmitting power of a-th AP and the constraint (5b) represents the constant modulus constraint of each PS at IRSs. Since the variables $\mathbf{p}$ and $\mathbf{\Theta}$ are intricately coupled, the problem is non-convex and intractable. To this end, in the following section, we propose an efficient framework to solve the problem.

3 Jointly Design Framework

First, we employ the fractional programming methods, i.e., LDT [10, Theorem 4] and QT [10, Corollary 1], to equivalently transform the original problem to a more tractable form. By introducing auxiliary variable vectors $\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_{\mathcal{K}}]^{\mathrm{T}}$ and $\beta = [\beta_1, \beta_2, \cdots, \beta_{\mathcal{K}}]^{\mathrm{T}}$, and according to the LDT and QT methods, the original problem can be reformulated as

$$\max_{\mathbf{p}, \mathbf{\Theta}, \alpha, \beta} \quad f(\mathbf{p}, \mathbf{\Theta}, \alpha, \beta)$$

$$\text{s.t.} \quad (5a), (5b), \tag{6}$$

where $f(\mathbf{p}, \mathbf{\Theta}, \alpha, \beta)$ is given as $f(\mathbf{p}, \mathbf{\Theta}, \alpha, \beta) = \sum_{k=1}^{\mathcal{K}} \log(1+\alpha_k) - \sum_{k=1}^{\mathcal{K}} \alpha_k + \sum_{k=1}^{\mathcal{K}} 2\beta_k \sqrt{(1+\alpha_k)\sum_{a=1}^{\mathcal{A}} |h_{a,k}|^2 p_{a,k}} - \sum_{k=1}^{\mathcal{K}} |\beta_k|^2 \left(\sum_{a=1}^{\mathcal{A}} \sum_{i=1}^{\mathcal{K}} |h_{a,k}|^2 p_{a,i} + \sigma_k^2\right)$. For detailed transformation of Problem 6, the readers are referred to [9]. Based on the fact that the objective function of the problem 6 is convex with respect to any one of the four variables $\mathbf{p}$, $\mathbf{\Theta}$, α, and β, while fixing the other three, the problem can be effectively solved in an alternating optimization manner. The solutions after the u-th iteration are denoted by $(\cdot)^{(\mathrm{u}+1)}$.

Note that α and β only appear in the objective function of Problem 6 and do not exist in any constraints. Therefore, the optimal values of α and β can be obtained by setting the partial derivatives of $f(\mathbf{p}, \mathbf{\Theta}, \alpha, \beta)$ with respect to $\alpha_k, \forall k$ and $\beta_k, \forall k$ to be zeros, respectively. First, with fixed $\mathbf{p}$, $\mathbf{\Theta}$, and β, we have

$$\alpha_k^{(\mathrm{u}+1)} = \frac{\sum_{a=1}^{\mathcal{A}} |h_{a,k}|^2 p_{a,k}}{\sum_{a=1}^{\mathcal{A}} \sum_{i \neq k}^{\mathcal{K}} |h_{a,k}|^2 p_{a,i} + \sigma_k^2}, \forall k \in \mathcal{K}, \tag{7}$$

and then with fixed $\mathbf{p}$, $\mathbf{\Theta}$, and $\alpha_k^{(\mathrm{u}+1)}$, we have

$$\beta_k^{(\mathrm{u}+1)} = \frac{\sqrt{(1+\alpha_k)\sum_{a=1}^{\mathcal{A}} |h_{a,k}|^2 p_{a,k}}}{\sum_{a=1}^{\mathcal{A}} \sum_{i \neq 1}^{\mathcal{K}} |h_{a,k}|^2 p_{a,i} + \sigma_k^2}, \forall k \in \mathcal{K}. \tag{8}$$

Meanwhile, with fixed $\mathbf{\Theta}$, and the so-obtained optimal $\alpha_k^{(\mathrm{u}+1)}$ and $\beta_k^{(\mathrm{u}+1)}$, the objective function of Problem 6 can be reformulated as $f(\mathbf{p}, \mathbf{\Theta}, \alpha, \beta) = f_{\mathbf{p}}(\mathbf{p}) + \mathrm{const}\left(\mathbf{\Theta}, \alpha^{(\mathrm{u}+1)}, \beta^{(\mathrm{u}+1)}\right)$, where $\mathrm{const}\left(\mathbf{\Theta}, \alpha^{(\mathrm{u}+1)}, \beta^{(\mathrm{u}+1)}\right) = \sum_{k=1}^{\mathcal{K}} \log(1+\alpha_k) - \sum_{k=1}^{\mathcal{K}} \alpha_k - \sum_{k=1}^{\mathcal{K}} |\beta_k|^2 \sigma_k^2$, which is the irrelevant constant term with respect to $\mathbf{p}$ and has no impact on the optimization of $\mathbf{p}$. Then, we have the following optimization subproblem of $\mathbf{p}$, which is given as

$$\begin{aligned} \mathbf{p} = \arg\max_{\mathbf{p}} \quad & f_{\mathbf{p}}(\mathbf{p}) \\ \text{s.t.} \quad & (5\text{a}), \end{aligned} \tag{9}$$

where $f_{\mathbf{p}}(\mathbf{p}) = \sum_{k=1}^{\mathcal{K}} 2\beta_k \sqrt{(1+\alpha_k)\sum_{a=1}^{\mathcal{A}} |h_{a,k}|^2 p_{a,k}} - \sum_{k=1}^{\mathcal{K}} |\beta_k|^2 \sum_{a=1}^{\mathcal{A}} \sum_{i=1}^{\mathcal{K}} |h_{a,k}|^2 p_{a,i}$. It can be verified that the objective function and the constraints of Problem 9 are both convex, which yields that Problem 9 is convex and can be solved by employing convex solver tools, e.g. CVX. Note that the dual gap of the above problem is guaranteed to be zero, therefore, instead of relying on the generic solver with high computational complexity, we propose a Lagrangian dual sub-gradient based algorithm to optimize $\mathbf{p}$.

The Lagrangian dual function of Problem 9 is defining as

$$\mathcal{L}(\mathbf{p}, \lambda) = f_{\mathbf{p}}(\mathbf{p}) - \sum_{a=1}^{\mathcal{A}} \lambda_a \left(\sum_{k=1}^{\mathcal{K}} p_{a,k} - P_{a,\max}\right), \tag{10}$$

where $\lambda = [\lambda_1, \lambda_2, \cdots, \lambda_{\mathcal{A}}]^{\mathrm{T}}$ is the dual vector with $\lambda_a \geq 0, \forall a \in \mathcal{A}$ is introduced for enforcing the maximal power in the a-th AP. The variables $\mathbf{p}$ and λ can be obtained in an alternating manner. First, with fixed λ^{q}, the optimal power allocation vector $\mathbf{p}^{\mathrm{q}+1}$ can be optimized by solving the following sub-problem

$$\mathbf{p}^{\mathrm{q}+1} = \arg\max_{\mathbf{p}} \mathcal{L}\left(\mathbf{p}, \lambda^{\mathrm{q}}\right). \tag{11}$$

By setting the first-order partial derivative of $\mathcal{L}\left(\mathbf{p}, \lambda^{\mathrm{q}}\right)$ with respect to $p_{a,k}, \forall \{\mathcal{A}, \mathcal{K}\}$ to be zeros, we have

$$p_{a,k}^{\mathrm{q}+1} = \frac{(1+\alpha_k)\,\beta_k^2\,|h_{a,k}|^2}{\left(\sum_{i=1}^{\mathcal{K}} \beta_i^2\,|h_{a,i}|^2 + \lambda_a\right)^2}, \forall a \in \mathcal{A}, \forall k \in \mathcal{K}. \tag{12}$$

The dual variable λ can be determined by solving the following dual optimizing problem, which is given as

$$\lambda^{\mathrm{q}+1} = \arg\min_{\lambda}\max_{\mathbf{p}}\ \mathcal{L}\left(\mathbf{p}^{\mathrm{q}+1}, \lambda\right). \tag{13}$$

By defining $f_a\left(\lambda_a^{\mathrm{q}}\right) = \sum_{k=1}^{\mathcal{K}} p_{a,k} - P_{a,\max}$, which is a monotonically decreasing function for $\lambda_a \geq 0$, we propose a sub-gradient based method to update λ_a^{q+1}. Particularly, with the fixed $p_{a,k}^{\mathrm{q}+1}$, the dual variable λ_a can be performed as follows

$$\lambda_a^{q+1} = \left[\lambda_a^q + \tau f_a\left(\lambda_a^q\right)\right]^+, \forall a \in \mathcal{A}, \tag{14}$$

where τ_a denotes the positive step for updating λ_a^{q+1} and $[x]^+ = \max\{x, 0\}$. With the number of iterations increasing, the solutions finally converge to the locally optimal solution, and we have $\mathbf{p}^{(\mathrm{u}+1)} = \mathbf{p}^{\mathrm{q}^*}$.

Finally, we consider to optimize the passive reflecting beamforming matrix $\mathbf{\Theta}$. With the fixed $\mathbf{p}^{(\mathrm{u}+1)}$, $\alpha^{(\mathrm{u}+1)}$, and $\beta^{(\mathrm{u}+1)}$, the corresponding subproblem for optimizing $\mathbf{\Theta}_r, \forall r \in \mathcal{R}$ of IRSs are given as

$$\begin{aligned} \mathbf{\Theta} = \arg\max_{\mathbf{\Theta}} \quad & f\left(\mathbf{p}^{(\mathrm{u}+1)}, \mathbf{\Theta}, \alpha^{(\mathrm{u}+1)}, \beta^{(\mathrm{u}+1)}\right) \\ \text{s.t.} \quad & (5b). \end{aligned} \tag{15}$$

By defining $\mathbf{d}_k = [d_{1,k}, d_{2,k}, \cdots, d_{\mathcal{A},k}]$, $\mathbf{g}_k = \left[\mathbf{g}_{1,k}^{\mathrm{T}}, \mathbf{g}_{2,k}^{\mathrm{T}}, \cdots, \mathbf{g}_{\mathcal{R},k}^{\mathrm{T}}\right]^{\mathrm{T}}$, $\mathbf{p}_k = [p_{1,k}, p_{2,k}, \cdots, p_{\mathcal{A},k}]^{\mathrm{T}}$,

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_{1,1}^{\mathrm{T}} & \mathbf{s}_{1,2}^{\mathrm{T}} & \cdots & \mathbf{s}_{1,\mathcal{R}}^{\mathrm{T}} \\ \mathbf{s}_{2,1}^{\mathrm{T}} & \mathbf{s}_{2,2}^{\mathrm{T}} & \cdots & \mathbf{s}_{2,\mathcal{R}}^{\mathrm{T}} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{s}_{\mathcal{A},1}^{\mathrm{T}} & \mathbf{s}_{\mathcal{A},2}^{\mathrm{T}} & \cdots & \mathbf{s}_{\mathcal{A},\mathcal{R}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \tag{16}$$

we have $\sum_{a=1}^{\mathcal{A}}\left(d_{a,k} + \sum_{r=1}^{\mathcal{R}} \mathbf{g}_{r,k}^{\mathrm{H}} \mathbf{\Theta}_r \mathbf{s}_{a,r}\right)\sqrt{\mathbf{p}_{a,k}} = \mathbf{d}_k\sqrt{\mathbf{p}_k} + \mathbf{g}_k^{\mathrm{H}} \mathbf{\Theta} \mathbf{S} \sqrt{\mathbf{p}_k}$. To make above expression more tractable, we further define $\theta =$

$[\theta_{1,1}, \theta_{1,2}, \cdots, \theta_{1,\mathcal{N}}, \cdots, \theta_{\mathcal{R},\mathcal{N}}]^{\mathrm{T}} \in \mathbb{C}^{\mathcal{RN}\times 1}$, and $\mathbf{H}_k = \mathrm{diag}(\mathbf{g}_k)\,\mathbf{S} \in \mathbb{C}^{\mathcal{RN}\times\mathcal{A}}$, and then $\mathbf{g}_k^{\mathrm{H}}\mathbf{\Theta}\mathbf{S} = \theta^{\mathrm{H}}\mathbf{H}_k$. Hence, with fixed $\mathbf{p}^{(\mathrm{u}+1)}$, $\alpha^{(\mathrm{u}+1)}$, and $\beta^{(\mathrm{u}+1)}$, we have $f\left(\mathbf{p}^{(\mathrm{u}+1)}, \theta, \alpha^{(\mathrm{u}+1)}, \beta^{(\mathrm{u}+1)}\right) = f_\theta(\theta) + \mathrm{const}\left(\mathbf{p}^{(\mathrm{u}+1)}, \alpha^{(\mathrm{u}+1)}, \beta^{(\mathrm{u}+1)}\right)$, where

$$f_\theta(\theta) = \sum_{k=1}^{\mathcal{K}} 2\beta_k\sqrt{1+\alpha_k}\mathrm{Re}\left\{\theta^{\mathrm{H}}\mathbf{H}_k\sqrt{\mathbf{p}_k}\right\} - \sum_{k=1}^{\mathcal{K}} 2\left|\beta_k\right|^2\left\{\theta^{\mathrm{H}}\mathbf{H}_k\sum_{i=1}^{\mathcal{K}}\sqrt{\mathbf{p}_i}\sqrt{\mathbf{p}_i}^{\mathrm{H}}\mathbf{d}_k^{\mathrm{H}}\right\}$$
$$-\sum_{k=1}^{\mathcal{K}}\left|\beta_k\right|^2\theta^{\mathrm{H}}\mathbf{H}_k\sum_{i=1}^{\mathcal{K}}\sqrt{\mathbf{p}_i}\sqrt{\mathbf{p}_i}^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\theta, \tag{17}$$

and $\mathrm{const}\left(\mathbf{p}^{(\mathrm{u}+1)}, \alpha^{(\mathrm{u}+1)}, \beta^{(\mathrm{u}+1)}\right)$ is the irreverent constant term about θ. After omitting the terms, the subproblem of optimizing θ can be expressed as

$$\theta = \arg\max_{\theta}\ -\theta^{\mathrm{H}}\mathcal{Z}\theta + 2\mathrm{Re}\left\{\theta^{H}\omega\right\} \tag{18}$$
$$\text{s.t.}\ \ |\theta_i| = 1, \forall i \in \hat{\mathcal{N}}, \tag{18a}$$

where $\hat{N} = \mathcal{RN}$, and

$$\mathcal{Z} = \sum_{k=1}^{\mathcal{K}}\left|\beta_k\right|^2\mathbf{H}_k\sum_{i=1}^{\mathcal{K}}\sqrt{\mathbf{p}_i}\sqrt{\mathbf{p}_i}^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}},$$
$$\omega = \sum_{k=1}^{\mathcal{K}}\beta_k\sqrt{1+\alpha_k}\mathbf{H}_k\sqrt{\mathbf{p}_k} - \left|\beta_k\right|^2\mathbf{H}_k\sum_{i=1}^{\mathcal{K}}\sqrt{\mathbf{p}_i}\sqrt{\mathbf{p}_i}^{\mathrm{H}}\mathbf{d}_k^{\mathrm{H}}. \tag{19}$$

However, Problem 18 is a non-convex CMC-QP problem, which is still hard to obtain the optimal solution. Note that the objective function and the constant modulus constraints are separable with respect to $\theta_i, \forall i \in \hat{\mathcal{N}}$ [13], therefore, we can decompose Problem 18 into $\hat{\mathcal{N}}$ separate subproblems and solve them one-by-one. Particularly, we have

$$\theta^{\mathrm{H}}\omega = \sum_{n=1}^{\hat{\mathcal{N}}}\theta_n^*\omega_n = \theta_i^*\omega_i + \sum_{n=1,n\neq i}^{\hat{\mathcal{N}}}\theta_n^*\omega_n. \tag{20}$$

Meanwhile, $\theta^{\mathrm{H}}\mathcal{Z}\theta$ can be expanded as

$$\theta^{\mathrm{H}}\mathcal{Z}\theta = \sum_{\substack{n=1\\n\neq i}}^{\hat{\mathcal{N}}}\theta^{\mathrm{H}}\mathbf{z}_n\theta_n + \theta^{\mathrm{H}}\mathbf{z}_i\theta_i = \sum_{\substack{n=1\\n\neq i}}^{\hat{\mathcal{N}}}\theta_i^*z_{i,n}\theta_n + \theta^{\mathrm{H}}\mathbf{z}_i\theta_i + \sum_{\substack{m=1\\m\neq n}}^{\hat{\mathcal{N}}}\sum_{\substack{p=1\\p\neq i}}^{\hat{\mathcal{N}}}\theta_m^*z_{m,p}\theta_p$$
$$= \theta_i^*z_{i,i}\theta_i + \sum_{\substack{n=1\\n\neq i}}^{\hat{\mathcal{N}}}\left(\theta_i^*z_{i,n}\theta_n + \theta_i z_{n,i}\theta_n^*\right) + \sum_{\substack{m=1\\m\neq n}}^{\hat{\mathcal{N}}}\sum_{\substack{p=1\\p\neq i}}^{\hat{\mathcal{N}}}\theta_m^*z_{m,p}\theta_p, \tag{21}$$

where $\mathcal{Z} = \left[\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_{\hat{\mathcal{N}}}\right]$ and $\mathbf{z}_n = \left[z_{1,n}, z_{2,n}, \cdots, z_{\hat{\mathcal{N}},n}\right]^{\mathrm{T}} \in \mathbb{C}^{\mathcal{N}\times 1}$. By using the property $z_{i,n} = z_{n,i}^*$ and basing on the fact that $\mathcal{Z}$ is a positive semi-definite matrix, we have $-\theta^{\mathrm{H}}\mathcal{Z}\theta+\theta^{\mathrm{H}}\omega+\omega^{\mathrm{H}}\theta = 2\mathrm{Re}\left\{\theta_i^*\omega_i + \sum_{n=1,n\neq i}^{\hat{\mathcal{N}}}\theta_n^*\omega_n\right\} - \theta_i^* z_{i,i}\theta_i - 2\mathrm{Re}\left\{\sum_{\substack{n=1\\n\neq i}}^{\hat{\mathcal{N}}}\theta_i^* z_{i,n}\theta_n\right\} - \sum_{\substack{m=1\\m\neq n}}^{\hat{\mathcal{N}}}\sum_{\substack{p=1\\p\neq i}}^{\hat{\mathcal{N}}}\theta_m^* z_{m,p}\theta_p$. Then, $f_7(\theta)$ can be recast as

$$f_8(\theta) = \sum_{i=1}^{\hat{\mathcal{N}}} 2\mathrm{Re}\left\{\theta_i^*\mu_i\right\} + \xi, \tag{22}$$

where $\mu_i = \omega_i - \sum_{\substack{n=1\\n\neq i}}^{\hat{\mathcal{N}}} z_{i,n}\theta_n$ and $\xi = 2\mathrm{Re}\left\{\sum_{n=1,n\neq i}^{\hat{\mathcal{N}}}\theta_n^*\omega_n\right\} - \sum_{\substack{m=1\\m\neq n}}^{\hat{\mathcal{N}}}\sum_{\substack{p=1\\p\neq i}}^{\hat{\mathcal{N}}}\theta_m^* z_{m,p}\theta_p - \theta_i^* z_{i,i}\theta_i$, where ξ is the irreverent constant term with regard to θ_i (e.g., $\theta_i^* z_{i,i}\theta_i = z_{i,i}\left|\theta_i\right|^2 = z_{i,i}$), which do not affect the optimal value of θ_i. Therefore, we can only investigate $\mathrm{Re}\left\{\theta_i^*\mu_i\right\}$ for optimizing θ_i and sequentially optimize each element while fixing the remaining $\hat{\mathcal{N}}-1$ elements. Problem 18 can be equivalently transformed as

$$\max_{\theta_i} \quad \mathrm{Re}\left\{\theta_i^*\mu_i\right\} \tag{23}$$

$$\text{s.t.} \quad |\theta_i| = 1, \tag{23a}$$

An equivalent expression for Problem 23 is given by

$$\max_{\phi_i} \quad \cos\left(-\phi_i + \eta_i\right) \tag{24}$$

$$\text{s.t.} \quad \phi_i \in [0, 2\pi], \tag{24a}$$

where η_i and $-\phi_i$ are the phases of μ_i and θ_i^*, respectively. Consequently, Problem 24 has a closed-form optimal solution whose phase is given by $\phi_i = \eta_i, \forall i \in \hat{\mathcal{N}}$. Consequently, we have

$$\theta_i = e^{j\eta_i}, \forall i \in \hat{\mathcal{N}}. \tag{25}$$

Based on the above discussions, the procedure of sequentially optimizing θ_1, θ_2, $\cdots$, $\theta_{\hat{\mathcal{N}}}$ and then repeatedly until convergence is attained. The complexity of the sequential optimization (SO) algorithm is $\mathcal{O}\left(\mathcal{I}_{\mathrm{SO}}\mathcal{R}^2\mathcal{N}^2\right)$, where $\mathcal{I}_{\mathrm{SO}}$ denotes the number of iterations when the SO algorithm converges.

The proposed AI algorithm is summarized in Algorithm 1, and we have the following lemma

Lemma 1. *The proposed Algorithm 1 is guaranteed to converge to locally optimal solutions.*

Proof. The proof is similar to that of [14], hence it is omitted for simplicity.

4 Numerical Simulation

As follows, simulation results are provided to evaluate the performance of the proposed alternating iteratively (AI) algorithm to evaluate its effectiveness and

Algorithm 1. Proposed AI algorithm

Input: $p_{a,k}^{(1)}, \forall \{a,k\}$, $\mathbf{\Theta}^{(1)}$; threshold ε.

1: **Update** $\alpha_k^{(\mathrm{u}+1)}, \forall k \in \mathcal{K}$ by using (7);
2: **Update** $\beta_k^{(\mathrm{u}+1)}, \forall k \in \mathcal{K}$ by using (8);
3: **Update** $p_{a,k}^{(\mathrm{u}+1)}, \forall a \in \mathcal{A}, \forall k \in \mathcal{K}$ by using (12);
4: **Update** $\mathbf{\Theta}_i^{(\mathrm{u}+1)}, \forall i \in \hat{\mathcal{N}}$ by using (25);
5: **IF** $\left| R^{(\mathrm{u}+1)} - R^{(\mathrm{u})} \right| / R^{(\mathrm{u})} \leq \upsilon$, **terminate**;
Otherwise, set $u = u + 1$ and go to Step 1.

show the performance gains achieved by the distributed IRSs. We consider six-APs and four-UEs are equipped with single antenna, and three-IRSs are composed of 60 PSs, and APs, UEs, and IRSs are randomly distributed in a circle with a radius of 100 m. Besides, we assumed that a uniform planar array (UPA) at the IRS. The large-scale path loss is given as $L(d) = L_0 + 10 \log \left(\frac{d}{D_0} \right)^{\rho}$, where L_0 is the path-loss at the reference distance $D_0 = 1\,\mathrm{m}$, d denotes the distance, and ρ is the path loss exponent. We set the path loss exponent between APs and UEs, between APs and IRSs, and between IRSs and UEs are $\rho_{au} = 3.75$, $\rho_{ai} = 2.2$, $\rho_{iu} = 2.2$, respectively, which is based on the fact that the location of IRSs are appropriately chosen for ensuring a free-space. Meanwhile, the maximum transmit power of APs are $P_{a,\max} = 20\,\mathrm{dBm}, \forall a \in \mathcal{A}$ and the noise power $\sigma^2 = \sigma_k^2 = -80\,\mathrm{dBm}, \forall k \in \mathcal{K}$.

As shown in Fig. 2, we study the sum-rate achieved by the proposed AI algorithm under the different number of PSs, e.g. $\mathcal{N} = 30$ and $\mathcal{N} = 60$. It is observed that the sum-rate achieved by the AI algorithm converges to corresponding stationary point after a few iteration, which demonstrates that the proposed AI algorithm has good convergence behaviour. Besides, the convergence speed of "AI alg. $\mathcal{N} = 60$" is slower than "AI alg. $\mathcal{N} = 30$", which indicated that the convergence speed is sensitive to the size of IRSs.

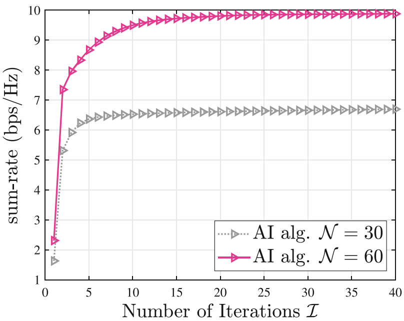
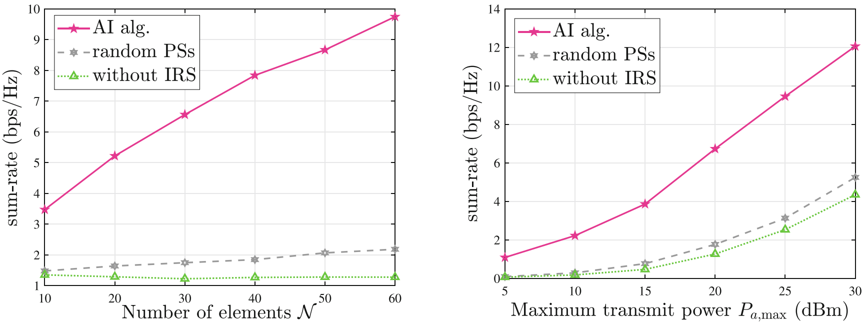


Fig. 2. Sum-rate versus the number of iterations.

Moreover, we introduce two baseline schemes to validate the performance of "AI alg.", i.e., "without IRS" and "random PSs" [4–8]. As illustrated in Fig. 3(a), we investigate the impact of the number of PSs of IRS on the performance. It is observed that the "AI alg." scheme achieves a significant performance gain compares with benchmark schemes. Besides, the "random PSs" scheme achieves a higher sum-rate than the "without IRS" scheme, which demonstrates that the IRS can improve the performance of the cell-free system even though the passive reflecting beamforming of IRSs are without optimizing. As shown in Fig. 3(b) illustrates the sum-rate achieved by the "AI alg." scheme and benchmark schemes over the maximum transmit power of APs. It is observed that the "AI alg." scheme outperforms both the "random PSs" and the "without IRS" scheme considerably, which also indicates the advantages of deploying IRSs into cell-free networks.



(a) Sum-rate vs. $\mathcal{N}$

(b) Sum-rate vs. $P_{a,\max}, \forall a \in \mathcal{A}$

Fig. 3. Comparison of performance.

5 Conclusion

In this paper, we proposed an efficient jointly power allocation and passive reflecting beamforming design framework to maximize the sum-rate of the IRSs-assisted cell-free networks. Due to the problem was non-convex, we first transformed the original problem to an equivalent and tractable form, and then decomposed the reformulated problem into two subproblems. Finally, the subproblems were solved in an alternating optimization manner. The proposed AI algorithm was guaranteed to converge to locally optimal solutions. Simulation results demonstrated that IRSs can improve the performance of the conventional cell-free networks significantly and the proposed AI algorithm achieves considerably performance gains than benchmark schemes.

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