A Factor Graph-Based Attitude Determination Approach for Microsatellite with Star Sensor and Gyros

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Abstract. Attitude determination is an important part of microsatellite attitude control system. Its determination accuracy directly affects microsatellite's attitude control performance. Hence, a novel attitude determination approach is presented in this work for microsatellites with star sensor and gyros fixed. The factor graph representing the probabilistic graphical model as a bipartite graph is applied to solve the attitude estimation problem. This allows multi-rate, asynchronous, and possible delayed measurements to be incorporated in a natural way. An incremental smoothing algorithm is then proposed to achieve optimal estimation. Simulation results are finally presented to verify that the proposed approach can significantly improve the attitude determination accuracy.

Keywords: Microsatellite · Attitude determination · Factor graph · Smooth optimization · Star sensor · Gyro

1 Introduction

When designing attitude control system for microsatellites, attitude determination should be carried out. It provides the attitude controller design with feedback measurements of the attitude control system's states. Moreover, it directly affects the attitude control performance. Hence, it is essential to obtain high-precision attitude information. However, microsatellites are limited by cost, power consumption, and mass. The attitude sensors fixed in them are with low power consumption and low precision. The development of novel approaches to achieve high-precision attitude determination is therefore critical for microsatellites with low-accuracy sensors.

The main task of attitude determination is to develop methods to estimate the attitude and the angular velocity states of microsatellite attitude control system by using the measurements provided by attitude sensors with noise.

Supported by the National Natural Science Foundation of China under Grants 11902359, 11972373, and 61873207.

S. Liu and X. Ma (Eds.): ADHIP 2021, LNICST 416, pp. 67–80, 2022.
https://doi.org/10.1007/978-3-030-94551-0_6

The attitude determination mainly consists of the measurement sensors and the corresponding information processing algorithm, namely attitude determination algorithm [1]. Therefore, the accuracy of attitude determination depends on the measurement accuracy of attitude sensor and the accuracy of attitude determination algorithm. The attitude determination algorithm is to process the attitude sensor's measurements and filter or estimate the attitude of the microsatellite.

The attitude determination methods can be divided into the deterministic method and the state estimation method. For the deterministic method, the attitude matrix of a microsatellite is obtained only from a set of vector measurements. An algorithm to determine the three-axis attitude with two or more vector observations was presented in [2]. That algorithm has high calculation efficiency and was described as the problem of finding the orthogonal matrix which minimizes the non-negative loss function. More typical deterministic algorithms include the TRIAD [3], the QUEST [4], the SVD [5], the FOAM [6], and Euler-q [7]. The characteristic of the deterministic method that it does not require the prior knowledge of the attitude. However, this method can only estimate the attitude information of the microsatellite. Due to the uncertainty of the measured reference vector, higher attitude accuracy may not be obtained. Moreover, when there is only one measurement vector, all the deterministic algorithms are unable to determine the attitude of satellite.

The state estimation algorithm uses the attitude measurement information of continuous-time states, and combines the motion model and the estimation algorithm of the satellite attitude to obtain the optimal attitude estimation. Its advantage is that it can not only estimate the attitude, but also can estimate the uncertain parameters and system errors. The attitude determination accuracy is thus improved. For example, the extended Kalman filter (EKF) [8], the unscented Kalman filter (UKF) [9], the federated filter [10], nonlinear predictive filter [11], and particle filter [12] are available to state estimation methods design. Since, the performance of microsatellites' sensors are inferior, those filtering methods are not capable of achieving stability and real-time performance of the optimizer. To solve the stability problem, more accurate model may be required. For the real-time problem, it is quite necessary to design the optimal estimation algorithm consuming simple calculation and less time. Hence, it is necessary to develop a new optimal attitude determination methods and theoretically break through the nonlinear and real-time problems of current filtering methods.

Motivated by solving the above problems, a factor graph-based approach is presented for the microsatellite attitude determination system in this work. The proposed factor graph method has a plug-and-play characteristics that can directly solve the problems of nonlinear and sensor asynchronous, even the system uncertainties. The rest of paper is organized as follows. In Sect. 2, the factor graph and the correlation between the factor graph and the attitude determination system are introduced. In Sect. 3, the modeling of sensors and microsatellite attitude determination are given. In Sect. 4, the approach of smoothing optimization is proposed. Simulation results presented in Sect. 5. Finally, concluding remarks are drawn in Sect. 6.

2 Factor Graph

2.1 Factor Graphs Theory

The factor graph is a graphical representation of a function that visualizes the relationships between variables in a model and between variables and factors [13,14]. Its essence is the function factorization. A complex global function consisting of multiple variables is represented by the product of multiple simple local sub-functions. The relationship between subfunctions and the corresponding variables is reflected in the factor graph model.

The factor graph algorithm describes the integrated navigation information fusion problem as the connection factor node in the factor graph model and solves the integrated navigation result through a factor graph model inference [15]. Good scalability and flexibility of the factor graph model can efficiently and quickly integrate asynchronous measurement information. It has received more and more attention in multi-sensor integrated navigation systems.

2.2 Formulations of Factor Graphs

The factor graph is actually a bipartite graph. Its model is represented by a set $G = (X, F, E)$. $X = \{X_1, X_2, \ldots, X_n\}$ is the variable node, $F = \{f_1, f_2, \ldots, f_n\}$ is the factor node, and the undirected edge E connecting the two nodes represents a functional relationship between the factor node and the variable node. The necessary and sufficient condition for the existence of an edge between the factor node f_j and the variable node X_k is that $X_k \in S_j$ exists. Suppose that there is a function $g(X_1, X_2, \ldots, X_n)$ factored into m factors as

$$g(X_1, X_2, \ldots, X_n) = \prod_{j=1}^{m} f_j(S_j) \tag{1}$$

where $S_j \subseteq \{X_1, X_2, \ldots, X_n\}$ is the j-th variable subset of X and f is a real value function.

Define the state of microsatellite attitude determination set $\boldsymbol{X}_k = \{x_i\}_{i=1}^{k}$. At the time t_k, $\boldsymbol{Z}_k = \{z_i\}_{i=1}^{k}$ represents all the current measurement sets. Then, the joint probability density function of the attitude system is expressed as $p(\boldsymbol{X}_k/\boldsymbol{Z}_k)$. The maximum posteriori estimate of the system states can be obtained as

$$\boldsymbol{X}_k^* = \arg\max_{\boldsymbol{X}_k} p(\boldsymbol{X}_k/\boldsymbol{Z}_k) \tag{2}$$

Let $p(\boldsymbol{X}_k)$ be used to denote $p(\boldsymbol{X}_k/\boldsymbol{Z}_k)$ and the local function f_i be utilized to represent the local probability. Then, (2) can be rewritten as

$$p(\boldsymbol{X}_k) \propto \prod_{i} f_i(\boldsymbol{X}_k^i) \tag{3}$$

where $\boldsymbol{X}_k^i$ represents a subset of the variable nodes, *i.e.*, $\boldsymbol{X}_k^i \subseteq \boldsymbol{X}_k$ exists. Each factor node f_i in (2) represents a local function constructed from the states of the attitude measurements.

Note that the function representation of the factor nodes depends on the particular measurement model. For Gaussian noise distribution, the general factor node is given by

$$f_i(\boldsymbol{X}_k^i) = \exp(-\frac{1}{2}||err_i(\boldsymbol{X}_k^i, \boldsymbol{z}_i)||^2_{\boldsymbol{\Sigma}_i}) \tag{4}$$

Define the cost function g_i as

$$g_i(\boldsymbol{X}_k^i) = d(err_i(\boldsymbol{X}_k^i, \boldsymbol{z}_i)) \tag{5}$$

where $d(\cdot)$ is the square of the Mahalanobis distance for the Gaussian noise distribution, *i.e.*, $d(\boldsymbol{a}) = ||\boldsymbol{a}||^2_{\Sigma} = \boldsymbol{a}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{a}$, $\boldsymbol{\Sigma}$ is the covariance matrix of the estimated measurements. To this end, the maximum posteriori estimate can be established by minimizing the following global cost function

$$\sum_i g_i(\boldsymbol{X}_k^i) = \sum_i ||err_i(\boldsymbol{X}_k^i, \boldsymbol{z}_i)||^2_{\boldsymbol{\Sigma}_i} \tag{6}$$

3 Modeling of Mircosatellite Attitude Determination System

3.1 Modeling of Star Sensor Measurement

The output of a star sensor needs to convert the vector measurements to the quaternion information $\boldsymbol{q}_s$. The measurement noise $\boldsymbol{v}_s$ also needs to be converted to quaternion which is denoted as $\boldsymbol{v}_{sc} = [\boldsymbol{v}_s^{\mathrm{T}} \quad 1]^{\mathrm{T}}$. Moreover, $\boldsymbol{v}_s$ is assumed as Gaussian white noise and can be expressed as

$$E[\boldsymbol{v}_s(t)\boldsymbol{v}_s^{\mathrm{T}}(\tau)] = \sigma_s^2(t)\delta(t-\tau) \tag{7}$$

where σ_s^2 is the variance of $\boldsymbol{v}_s$. Hence, the attitude measurement information represented by quaternion of star sensor modeled as

$$\boldsymbol{q}_{sc} = \boldsymbol{q}_s \otimes \boldsymbol{v}_{sc} \tag{8}$$

For the case that the installation matrix of the star sensor is in the body-fixed coordinate system of the microsatellite, then the output measurement of the star sensor can be modeled as

$$\boldsymbol{q}_{sc} = (\boldsymbol{q}_s \otimes \boldsymbol{v}_{sc}) \otimes \boldsymbol{q}_{sb} \tag{9}$$

where $\boldsymbol{q}_{sb}$ is the equivalent quaternion representing the installation error.

When applying the method of factor graph, the star sensor's measurement model should be transformed into an factor graph, which is given by

$$\boldsymbol{z}^s = h^s(\boldsymbol{x}_k) + \boldsymbol{n}^s \tag{10}$$

where h^s is the measurement function, $\boldsymbol{x}_k$ is the measurement vector, and $\boldsymbol{n}^s$ is the measurement noise of the star sensor.

According to the measurement equation, the measurement factor of star sensor can be modeled as

$$\boldsymbol{f}^{\mathrm{STAR}}(\boldsymbol{x}_k) \triangleq d(\boldsymbol{z}^s - h^s(\boldsymbol{x}_k)) \tag{11}$$

Then, it Jacobian matrix is calculated as

$$\boldsymbol{J}^{\mathrm{STAR}}(\boldsymbol{x}_k) = \frac{\partial \boldsymbol{f}^{\mathrm{STAR}}}{\partial \boldsymbol{x}_k} = \boldsymbol{\Sigma}_s^{-\frac{1}{2}} \left\{ -\frac{\partial h^s}{\partial \boldsymbol{x}_k} \right\} \tag{12}$$

Moreover, the Jacobian matrix of the residual term can be expressed as

$$\boldsymbol{b}^{\mathrm{STAR}}(\boldsymbol{x}_k) = \boldsymbol{\Sigma}_s^{-\frac{1}{2}} \left\{ h^s(\boldsymbol{x}_k) - \boldsymbol{z}^s \right\} \tag{13}$$

3.2 Modeling of Gyro Measurement

The gyro directly measures the projection of the microsatellite's angular velocity in the inertial system. For simplicity, it is assumed that the gyro measurement coordinate system coincides with the body-fixed coordinate system of microsatellite. Then, the measurement model of the gyro is established as

$$\boldsymbol{\omega}_g = \boldsymbol{\omega} + \boldsymbol{b} + \boldsymbol{v}_g \tag{14}$$

$$\dot{\boldsymbol{b}} = \boldsymbol{v}_b \tag{15}$$

where $\boldsymbol{\omega} = [\omega_x \quad \omega_y \quad \omega_z]^{\mathrm{T}}$ is the angular velocity of the microsatellite and expressed in the body-fixed coordinate system. $\boldsymbol{b}$ is gyro drift and considered as a first-order random walk process driven by Gaussian white noise. $\boldsymbol{v}_g$ and $\boldsymbol{v}_b$ are irrelevant zero-mean Gaussian white noise. Moreover, the gyro error constantly satisfies

$$\begin{cases} E\{\boldsymbol{v}_g(t)\boldsymbol{v}_g{}^{\mathrm{T}}(\tau)\} = \sigma_g^2 \delta(t-\tau)\boldsymbol{I} \\ E\{\boldsymbol{v}_b(t)\boldsymbol{v}_b{}^{\mathrm{T}}(\tau)\} = \sigma_b^2 \delta(t-\tau)\boldsymbol{I} \end{cases} \tag{16}$$

where σ_g^2 and σ_b^2 are the variances of $\boldsymbol{v}_g$ and $\boldsymbol{v}_b$ respectively. $\boldsymbol{I}$ is the identity matrix with appropriate dimensions.

Given that the gyro's measurement value $\boldsymbol{z}_k$, the current state estimation value is $\boldsymbol{x}_k$, and the prediction value at the next time is $\boldsymbol{x}_{k+1}$, then the graph factor $\boldsymbol{f}^{\mathrm{GYRO}}$ of the gyro can be modeled as

$$\boldsymbol{f}^{\mathrm{GYRO}}(\boldsymbol{x}_{k+1}, \boldsymbol{x}_k) = d(\boldsymbol{x}_{k+1} - h(\boldsymbol{x}_k, \boldsymbol{z}_k)) \tag{17}$$

For the gyro factor $\boldsymbol{f}^{\mathrm{GYRO}}$, its Jacobian matrix is given by

$$\begin{cases} \boldsymbol{J}^{\mathrm{GYRO}}(\boldsymbol{x}_{k+1}) = \frac{\partial \boldsymbol{f}^{\mathrm{GYRO}}}{\partial \boldsymbol{x}_{k+1}} = \boldsymbol{\Sigma}^{-\frac{1}{2}} \left\{ -\frac{\partial h}{\partial \boldsymbol{x}_{k+1}} \right\} \\ \boldsymbol{J}^{\mathrm{GYRO}}(\boldsymbol{x}_k) = \frac{\partial \boldsymbol{f}^{\mathrm{GYRO}}}{\partial \boldsymbol{x}_k} = \boldsymbol{\Sigma}^{-\frac{1}{2}} \left\{ -\frac{\partial h}{\partial \boldsymbol{x}_k} \right\} \end{cases} \tag{18}$$

3.3 Modeling of Satellite Attitude Determination System

The star sensor, as the long-term attitude reference of the microsatellite, provides the three-axis attitude quaternion information of the satellite with a certain sampling frequency. Therefore, its measurement can be taken as the benchmark to correct the measurement information of the gyro. Using the measurements of the gyro and the star sensor, the attitude of satellite can be determined.

Define the quaternion as $\boldsymbol{Q} = [p_0\ \boldsymbol{p}^{\mathrm{T}}]^{\mathrm{T}} \in \Re^4, \boldsymbol{p} \in \Re^3, p_0^2 + \boldsymbol{p}^{\mathrm{T}}\boldsymbol{p} = 1$, where

$$\boldsymbol{p} = [p_1\ p_2\ p_3]^{\mathrm{T}}, [\boldsymbol{p}\times] = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}$$

According to the multiplicative relationship among error quaternion, real attitude quaternion and estimated attitude quaternion, the attitude deviation quaternion is defined as follows

$$\boldsymbol{q} = \hat{\boldsymbol{q}} \otimes \boldsymbol{q}_e \tag{19}$$

The equation of satellite attitude motion is defined as

$$\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{q} \otimes [0\ \boldsymbol{\omega}] = \frac{1}{2}\begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix}\begin{bmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{20}$$

According to the properties of quaternion multiplication, the derivative of formula (19) can be obtained as follows

$$\dot{\boldsymbol{q}} = \dot{\hat{\boldsymbol{q}}} \otimes \boldsymbol{q}_e + \hat{\boldsymbol{q}} \otimes \dot{\boldsymbol{q}}_e \tag{21}$$

Combining with formulas (20) and (21), we can obtain

$$\frac{1}{2}\boldsymbol{q} \otimes [0\ \boldsymbol{\omega}] = \frac{1}{2}\hat{\boldsymbol{q}} \otimes [0\ \hat{\boldsymbol{\omega}}] \otimes \boldsymbol{q}_e + \hat{\boldsymbol{q}} \otimes \dot{\boldsymbol{q}}_e \tag{22}$$

Substituting (19) into (22) yields

$$\dot{\boldsymbol{q}}_e = \frac{1}{2}\boldsymbol{q}_e \otimes [0\ \boldsymbol{\omega}] - \frac{1}{2}[0\ \hat{\boldsymbol{\omega}}] \otimes \boldsymbol{q}_e \tag{23}$$

Suppose the measured value of the gyros is $\boldsymbol{\omega}_{meas} = \boldsymbol{\omega} - \boldsymbol{b} - \boldsymbol{v}$, and the estimated value of the gyro is $\hat{\boldsymbol{\omega}} = \boldsymbol{\omega} - \hat{\boldsymbol{b}}$, then

$$\Delta\boldsymbol{\omega} = \boldsymbol{\omega}_{meas} - \hat{\boldsymbol{\omega}} = -(\boldsymbol{b} - \hat{\boldsymbol{b}}) - \boldsymbol{v} = -\Delta\boldsymbol{b} - \boldsymbol{v} \tag{24}$$

Combining with (23), simplifying and omitting higher-order terms yield

$$\dot{\boldsymbol{Q}}_e = -[\hat{\boldsymbol{\omega}}\times]\boldsymbol{Q}_e - \frac{1}{2}(\Delta\boldsymbol{b} + \boldsymbol{v}) \tag{25}$$

where $\boldsymbol{q}_e = [q_{e0}\boldsymbol{Q}_e^{\mathrm{T}}]^{\mathrm{T}}$, $\dot{q}_{e0} = 0$, and

$$\Delta\dot{\boldsymbol{b}} = \boldsymbol{v}_b \tag{26}$$

The star sensor is used to compensate the gyro drift. Let the estimate of the gyro drift be $\hat{\boldsymbol{b}}$, then the equation of state for the attitude determination is obtained by combining formulas (25) and (26) as follows

$$\begin{bmatrix} \dot{\boldsymbol{Q}}_e \\ \Delta\dot{\boldsymbol{b}} \end{bmatrix} = \begin{bmatrix} -[\boldsymbol{\omega}\times] & -\frac{1}{2}\boldsymbol{I}_3 \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q}_e \\ \Delta\boldsymbol{b} \end{bmatrix} + \begin{bmatrix} -\frac{1}{2}\boldsymbol{I}_3 & 0_{3\times3} \\ 0_{3\times3} & \boldsymbol{I}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_g \\ \boldsymbol{v}_b \end{bmatrix} \tag{27}$$

Define the state vector as $\boldsymbol{X}_k = [\,\delta\boldsymbol{q}\ \Delta\boldsymbol{b}\,]^{\mathrm{T}} = [\,q_{e1}\ q_{e2}\ q_{e3}\ \Delta b_1\ \Delta b_2\ \Delta b_3\,]^{\mathrm{T}}$. The measurement equation for attitude determination can be obtained from the output of the star sensor

$$\boldsymbol{Z}_k = \boldsymbol{H}_k\boldsymbol{X}_k + \boldsymbol{D}_k\boldsymbol{V}_k \tag{28}$$

where $\boldsymbol{H}_k = [\,\boldsymbol{I}_{3\times3}\ 0_{3\times3}\,]$, $\boldsymbol{V}_k = [\,v_{sc1}\ v_{sc2}\ v_{sc3}\,]$.

To determine the attitude of satellite via factor graph, the Jacobian matrix needs to calculated to update the state. Since the gyro's measurement frequency is higher than that of the star sensor, a state connection is established with the star sensor, and the Jacobian matrix between time $t + \Delta t$ and time t can be expressed as

$$\boldsymbol{J}_{t+\Delta t} = (\boldsymbol{I} + \boldsymbol{F}\cdot\Delta t)\boldsymbol{J}_t \tag{29}$$

where $\boldsymbol{F} = \begin{bmatrix} -[\omega\times] & -\frac{1}{2}\boldsymbol{I}_3 \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}$, the initial value of the Jacobian matrix $\boldsymbol{J}_t$ is the identity matrix, and Δt is the time interval of the gyro measurement output.

The Jacobian matrix is used to establish the relationship between the state at time $k + 1$ and time k as follows

$$\boldsymbol{X}_{k+1} = \boldsymbol{J}_{\mathrm{state}}\boldsymbol{X}_k \tag{30}$$

where $\boldsymbol{J}_{\mathrm{state}} = \begin{bmatrix} \boldsymbol{J}_{\Delta q}^{\Delta q} & \boldsymbol{J}_{\Delta b}^{\Delta q} \\ \boldsymbol{J}_{\Delta q}^{\Delta b} & \boldsymbol{J}_{\Delta b}^{\Delta b} \end{bmatrix}$.

Since $\boldsymbol{J}_{\Delta q}^{\Delta b} = 0$, $\boldsymbol{J}_{\Delta b}^{\Delta b} = \boldsymbol{I}$, we can obtain

$$\begin{cases} \delta\boldsymbol{q}_{k+1} = \boldsymbol{J}_{\Delta q}^{\Delta q}\delta\boldsymbol{q}_k + \boldsymbol{J}_{\Delta b}^{\Delta q}\Delta\boldsymbol{b}_k \\ \Delta\boldsymbol{b}_{k+1} = \Delta\boldsymbol{b}_k \end{cases} \tag{31}$$

Then we can get the gyro factor f^{GYRO} as

$$\boldsymbol{f}^{\mathrm{GYRO}} = \begin{bmatrix} \delta\boldsymbol{q}_{k+1} - \boldsymbol{J}_{\Delta q}^{\Delta q}\delta\boldsymbol{q}_k + \boldsymbol{J}_{\Delta b}^{\Delta q}\Delta\boldsymbol{b}_k \\ \Delta\boldsymbol{b}_{k+1} - \Delta\boldsymbol{b}_k \end{bmatrix} \tag{32}$$

Assuming that the current value of the star sensor is $\boldsymbol{Q}_{k+1}$, the star sensor factor $\boldsymbol{f}^{\mathrm{STAR}}$ can be established as

$$\boldsymbol{f}^{\text{STAR}} = \boldsymbol{Q}_{k+1} \otimes \left[\begin{bmatrix} \delta \boldsymbol{q}_{k+1} \\ 1 \end{bmatrix} \otimes \hat{\boldsymbol{Q}}_{k+1|k} \right]^{-1} \tag{33}$$

where $\hat{\boldsymbol{Q}}_{k+1|k}$ is the predicted value at the current time, and $\delta \boldsymbol{q}_{k+1}$ is the state at the current time.

Combining the proposed attitude measurement model of the star sensor and gyro, a star sensor/gyro attitude measurement system based on factor graph can be constructed. The factor graph representation is shown in Fig. 1.

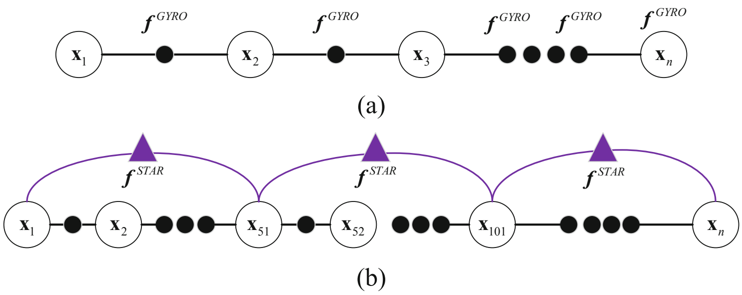

Fig. 1. Factor graph-based attitude determination system with star sensor and gyros.

4 Smoothing Optimization

Let $\boldsymbol{\Theta} = \boldsymbol{X}_k = [q_{e1}\ q_{e2}\ q_{e3}\ \Delta b_1\ \Delta b_2\ \Delta b_3]^{\text{T}}$, then for factor graph G define the factorization of function $f(\boldsymbol{\Theta})$ as

$$f(\boldsymbol{\Theta}) = \prod_i f_i(\boldsymbol{\Theta}_i) \tag{34}$$

The goal of attitude determination is to find the variable assignment $\boldsymbol{\Theta}^*$ of maximized formula (34) as follows

$$\boldsymbol{\Theta}^* = \arg\max_{\boldsymbol{\Theta}} f(\boldsymbol{\Theta}) \tag{35}$$

When assuming that the Gaussian measurement model is

$$f_i(\boldsymbol{\Theta}_i) \propto \exp(-\frac{1}{2}||h_i(\boldsymbol{\Theta}_i) - \boldsymbol{z}_i||^2_{\boldsymbol{\Sigma}_i}) \tag{36}$$

The decomposition objective function of the maximized (35) corresponds to the nonlinear least square criterion and can be expressed as

$$\arg\min_{\boldsymbol{\Theta}}(-\log f(\boldsymbol{\Theta})) = \arg\min_{\boldsymbol{\Theta}} \frac{1}{2} \sum_i ||h_i(\boldsymbol{\Theta}_i) - \boldsymbol{z}_i||^2_{\boldsymbol{\Sigma}_i} \tag{37}$$

where $h_i(\boldsymbol{\Theta}_i)$ is a measurement function, $\boldsymbol{z}_i$ is a measurement value, and $||\boldsymbol{e}||^2_{\boldsymbol{\Sigma}} \triangleq \boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{e}$ is defined as the square of the Mahalanobis distance of the covariance matrix $\boldsymbol{\Sigma}$.

Let $\boldsymbol{J}_i(\hat{\boldsymbol{X}}_k)$ and $\boldsymbol{b}_i$ represent the Jacobian matrix and the residual of a particular factor node respectively, the updated variable $\boldsymbol{\Delta}_i$ of the factor node is obtained as a form that can be normalized to a standard least squares

$$\begin{aligned} {\boldsymbol{\Delta}_i}^* &= \arg\min_{\boldsymbol{\Delta}_i} ||\boldsymbol{J}_i\boldsymbol{\Delta}_i - \boldsymbol{b}_i||^2_{\boldsymbol{\Sigma}_i} \\ &= \arg\min_{\boldsymbol{\Delta}_i} ||\boldsymbol{\Sigma}_i^{-\frac{\mathrm{T}}{2}}(\boldsymbol{J}_i\boldsymbol{\Delta}_i - \boldsymbol{b}_i)||^2 \end{aligned} \tag{38}$$

According to the above formula, the linearization solution for updating the increment $\boldsymbol{\Delta}$ is a standard least squares problem. Generally, the Cholesky decomposition or the QR decomposition is needed. Hence, the QR decomposition is used to update the linear solution of the increment $\boldsymbol{\Delta}$. For the convenience of calculation, $\boldsymbol{A} \in \Re^{m\times n}(m \geq n)$ is employed to measure the Jacobian matrix.

The QR decomposition is firstly applied to the matrix $\boldsymbol{A}$, then one has

$$\boldsymbol{A} = \boldsymbol{Q}\begin{bmatrix}\boldsymbol{R}\\0\end{bmatrix}, \boldsymbol{A} \in \Re^{m\times n}, \boldsymbol{Q} \in \Re^{m\times m}, \boldsymbol{R} \in \Re^{n\times n} \tag{39}$$

where $\boldsymbol{Q}$ is an m-order unitary matrix and $\boldsymbol{R}$ is an n-order upper triangular matrix. Then, solving the least squares solution is equivalent to minimizing $||\boldsymbol{A\Delta} - \boldsymbol{b}||^2_{\boldsymbol{\Sigma}}$

$$\begin{aligned} ||\boldsymbol{A\Delta} - \boldsymbol{b}||^2 &= \left\|\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{Q}\begin{bmatrix}\boldsymbol{R}\\0\end{bmatrix}\boldsymbol{\Delta} - \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{b}\right\|^2 \\ &= \left\|\begin{bmatrix}\boldsymbol{R}\\0\end{bmatrix}\boldsymbol{\Delta} - \begin{bmatrix}\boldsymbol{d}\\\boldsymbol{e}\end{bmatrix}\right\|^2 \\ &= ||\boldsymbol{R\Delta} - \boldsymbol{d}||^2 + ||\boldsymbol{e}||^2 \end{aligned} \tag{40}$$

Define $[\boldsymbol{d} \quad \boldsymbol{e}]^{\mathrm{T}} = \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{b}$, $\boldsymbol{d} \in \Re^n$, $\boldsymbol{e} \in \Re^{m-n}$, then if and only if $\boldsymbol{R\Delta} = \boldsymbol{d}$, the above formula is the smallest, i.e., $||\boldsymbol{e}||^2$ is the residual of the least squares solution. Therefore, the QR decomposition simplifies the least squares solution problem to solve linear equations with unique solutions, as shown below

$$\boldsymbol{R\Delta}^* = \boldsymbol{d} \tag{41}$$

where $\boldsymbol{\Delta}^*$ is the updated increment of all state variables.

5 Simulation Results

To verify the effectiveness of the proposed factor graph method, this paper uses the EKF method for comparison. In the simulation, the initial quaternion of the satellite attitude is $[0 \quad 0.7071 \quad 0.7071 \quad 0]^{\mathrm{T}}$, The initial error covariance of the attitude is 3.0462×10^{-6}. The simulation parameters of star sensor are: The measurement of the star sensor takes the z-axis as the visual axis, the sampling period of the star sensor is 1 s, the field of view of the star sensor is 6 deg, the amplitude threshold of the star sensor is 6 deg, and the standard deviation of the star sensor is $\sigma_s = 2.9089 \times 10^{-5}$ rad. The simulation parameters of gyro are: The sampling frequency of the gyro 50 Hz, the standard deviation of the gyro bias noise $\sigma_u = 3.1623 \times 10^{-10}$, the standard deviation of the gyro noise is $\sigma_v = 3.1623 \times 10^{-7}$, and the initial error covariance of the gyro deviation is 9.4018×10^{-13}.

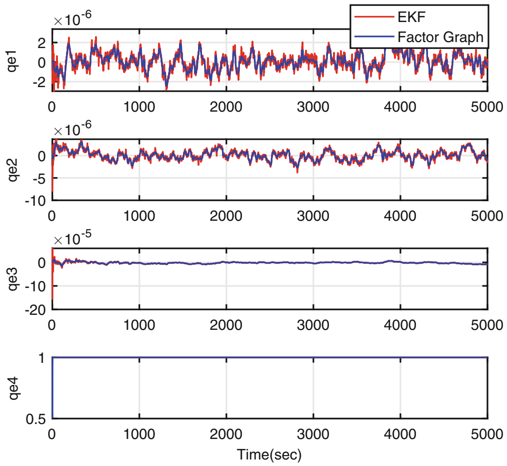

Fig. 2. The error of attitude quaternion.

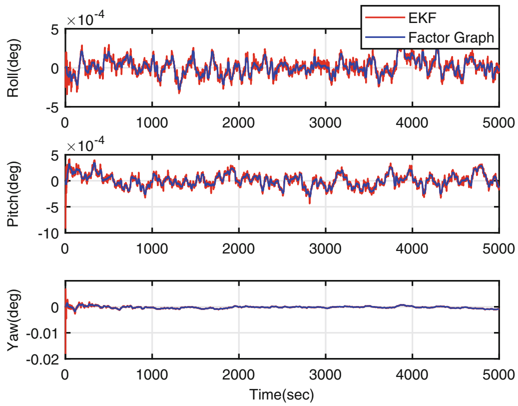

Fig. 3. The error of attitude angle.

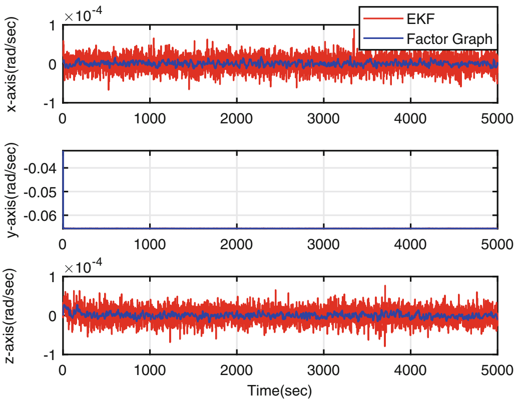

Fig. 4. The error of attitude angular velocity.

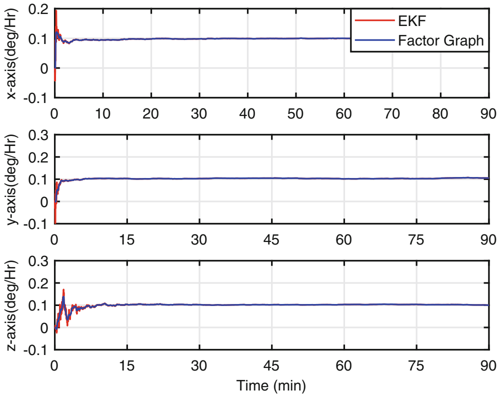

Fig. 5. The bias of gyro.

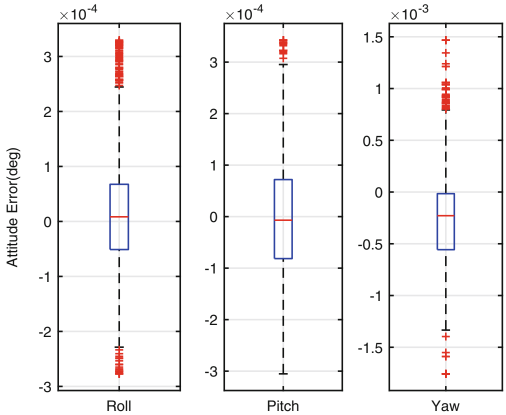

Fig. 6. The statistics of attitude angle erro.r

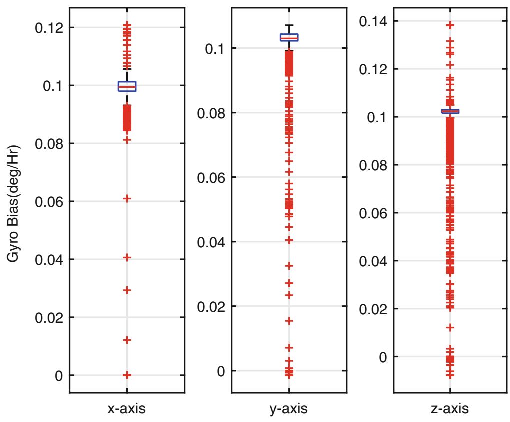

Fig. 7. The statistics of gyro bias.

It is seen in Fig. 2 that the magnitude of quaternion for the attitude error is 10^{-4}~10^{-6}. The mean value of the attitude quaternion is $[0 \quad 0 \quad 0 \quad 1.0000]^{\mathrm{T}}$. The mean square deviation of the attitude quaternion is $[8.3783\times10^{-7} \quad 1.0137\times10^{-6} \quad 3.4553\times10^{-6} \quad 1.2120\times10^{-11}]^{\mathrm{T}}$. Figures 3 and 4 show the attitude angle error and attitude angular velocity error, respectively. Figure 6 is the statistics of attitude angle error. According to the statistical characteristics of the three-axis attitude angle error shown in Table 1, it can be known that the mean square error (MSE) of the three-axis attitude angle error less than 4.2304×10^{-4} deg. Therefore, compared with EKF, the attitude determination accuracy of the proposed algorithm is higher than that of EKF. Figures 5, 6 and 7 are the gyro bias and the satistics of gyro bias, respectively. Combined with the MSE of gyro bias in Table 2, it can be seen that the cumulative error of the three-axis gyroscope is about 0.1 deg/Hr, and the mean square error is less than 0.0116 deg/Hr, the gyro deviation of the proposed method is less than EKF. It is shown that the proposed method ensures that the gyro bias also fluctuates within a certain range, and will not increase the error, thereby improving the accuracy of satellite attitude determination.

Table 1. The MSE of attitude angle

Value	Roll (deg)	Pitch (deg)	Yaw (deg)
EKF	1.0368×10^{-4}	1.2550×10^{-4}	5.3336×10^{-4}
FGs	9.4190×10^{-5}	1.1642×10^{-4}	4.2304×10^{-4}

Table 2. The MSE of gyro bias

Value	X-axis (deg/Hr)	Y-axis (deg/Hr)	Z-axis (deg/Hr)
EKF	0.0061	0.0099	0.0121
FGs	0.0052	0.0077	0.0116

6 Conclusion

In this paper, a factor graph-based attitude determination approach for satellites with star sensor and gyros was presented. The star sensor, gyro and satellite attitude dynamic model based on factor graph were given. The problem of satellite attitude determination was modeled as a factor graph optimization. Moreover, the attitude was determined through a smoothing optimization algorithm. The numerical simulations was developed to evaluate the optimization performance, and compared with the existing EKF method. The good scalability and flexibility of the proposed approach make it possible to efficiently and quickly integrate asynchronous measurement information. Mostly, it is possible to achieve plug and play of the sensor.

References

1. Lin, Y., Deng, Z.: Extended Kalman filtering for satellite orbital attitude estimation based on Gibbs vector. J. Guid. Control Dyn. **27**(3), 509–511 (2004)
2. Wahba, G.: A least squares estimate of satellite attitude. SIAM Rev. **7**(3), 409 (1965)
3. Shuster, M.D., Oh, S.D.: Three-axis attitude determination from vector observations. J. Guid. Control **4**(1), 70–77 (1981)
4. Shuster, M.: A simple Kalman filter and smoother for spacecraft attitude. J. Astronaut. Sci. **37**(1), 89–106 (1989)
5. Markley, F.L.: Attitude determination using vector observations and the singular value decomposition. J. Astronaut. Sci. **36**(3), 245–258 (1988)
6. Markley, F.L.: Attitude determination using vector observations: a fast optimal matrix algorithm. J. Astronaut. Sci. **41**(2), 261–280 (1993)
7. Mortari, D.: Euler-q algorithm for attitude determination from vector observations. J. Guid. Control Dyn. **21**(2), 328–334 (1998)
8. Kang, G., Xiaojing, D., Xinyuan, M.: Research on multi update rate method of precise satellite attitude determination based on gyro and star-sensor. In: AIAA Guidance, Navigation, and Control Conference, pp. 6645. AIAA, Portland, Oregon (2011)
9. Li, H., Sun, Z.-W., Zhang, S.-J.: Satellite attitude estimation UKF algorithm based on star-sensor. In: The Proceedings of the Multiconference on "Computational Engineering in Systems Applications", pp. 799–802. IEEE, Beijing (2006)
10. Cao, L., Yang, W., Chen, X., Huang, Y.: Application of multi-sensors data fusion based on improved federal filtering in micro-satellite attitude determination. In: 2011 International Workshop on Multi-Platform/Multi-Sensor Remote Sensing and Mapping, pp. 1–5. IEEE (2011)

11. Mook, D.J., Junkins, J.L.: Minimum model error estimation for poorly modeled dynamic systems. J. Guid. Control Dyn. **11**(3), 256–261 (1988)
12. Zhiru, F. and Jing, Y.: A research of gyro/star-sensor integrated attitude determination based on particle filter. In: 3th International Conference on Instrumentation, Measurement, Computer, Communication and Control, pp. 256–261, Shenyang (2013)
13. Frey, B.J., Kschischang, F.R., Loeliger, H.-A., Wiberg, N.: Factor graphs and algorithms. In: Proceedings of the Annual Allerton Conference on Communication Control and Computing, vol. 35, pp. 666–680. University of Illinois (1997)
14. Kschischang, F.R., Frey, B.J., Loeliger, H.-A.: Factor graphs and the sum-product algorithm. IEEE Trans. Inf. Theory **47**(2), 498–519 (2001)
15. Indelman, V., Williams, S., Kaess, M., and Dellaert, F.: Factor graph based incremental smoothing in inertial navigation systems. In: 15th International Conference on Information Fusion, pp. 2154–2161. IEEE, Singapore (2012)