Near-Field Source Localization by Exploiting the Signal Sparsity

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Abstract. This work aims to study the source localization problem using a symmetric array in a near-field environment. To reduce the computational complexity, in this work, two spatial correlation signals are created in which each signal only depends on one parameter of direction of arrival (DOA) or range. In the development process, the each resulting signal still possesses the array spatial structure, and therefore, the atomic norm minimization is utilized to obtain the corresponding solutions. The utilization of atomic norm also allows one to avoid the off-grid problem when the sparse reconstruction concept is employed. The numerical studies demonstrate the proposed method provides a superior performance compared with other approaches.

Keywords: Near-field localization · Sparse reconstruction · Basis mismatch · Atomic norm

1 Introduction

Source localization plays a important role in a wide range of applications such as radar, sonar, oceanography, and seismology, to name a few [8–10,15]. In terms of the distance of the source signal, the localization technique is divided into two categories of far-field and near-field source localizations. In the far-field source localization, the wave front is assumed to be a plane wave, where only the direction of arrival (DOA) of the source is estimated. To estimate the DOA, in the past decades, many techniques have been developed, for example, multiple signal classification (MUSIC) algorithm [10], estimation of signal parameters via rotational invariance techniques (ESPRIT) [9], and source localization approaches based on compressed sensing (CS) [1,2,4,5].

For the near-field source localization, the signal wave is considered as a spherical wave front, and hence the DOA and range of the source need to be estimated to achieve localization [7]. By extending the original MUSIC algorithm into two-dimensional (2-D), 2-D MUSIC algorithm is developed to perform the near-field

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source localization, but this algorithm is known to be computationally intensive. Some suboptimal subspace-based methods with second-order statistics (SOS) are subsequently proposed for the mixed incident signals [6,13], but their estimation performances are inferior.

The objective of this work is to jointly estimate the DOA and range of the source signal to complete the near-field localization based on the sparse reconstruction concept. To do so, an over-complete dictionary can be constructed based on the DOA and range gridding, and then the sparse reconstruction method can be utilized to estimate the position parameters of the source signal. However, since this dictionary is 2-D, the computational complexity is high. Besides that, this gridding technique also creates the so-called off-grid issue. In this work, a special spatial relationship of the array outputs and the Fresnel approximation are explored to transform the 2-D problem into two one-dimensional (1-D) problems, where each 1-D problem is solved by atomic norm minimization. In doing so, the joint estimation of DOA and range is achieved, and the off-gird problem is also avoided.

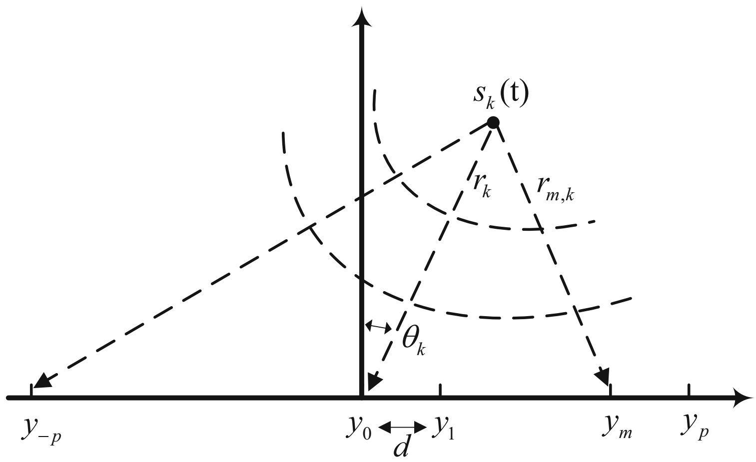


Fig. 1. Array geometry and signal illustration.

2 Signal Model

In Fig. 1, it is assumed that K near-field sources impinge on a symmetrical uniform linear array (ULA) with $M = 2p + 1$ sensors along the x-axis, where d is the sensor spacing, and θ_k, r_k respectively denote DOA and range of the k-th source, $k = 1, 2, \ldots, K$. The received signal of the m-th, $m = -p, \ldots, -1, 0, 1, \ldots, p$, sensor is

$$y_m(t) = \sum_{k=1}^{K} s_k(t) \exp(j\frac{2\pi}{\lambda}(r_{m,k} - r_k)) + w_m(t), \tag{1}$$

where $s_k(t)$ denotes the signal from the k-th source with power $\sigma_{s,k}^2$, $w_m(t)$ denotes the additive Gaussian white noise of the m-th sensor with variance σ_ω^2 and zero mean, λ is the wavelength. In (1), $r_{m,k}$ represents the distance from the k-th source signal to the m-th sensor, and according to the cosine theorem, it is

$$r_{m,k} = \sqrt{r_k^2 + (md)^2 - 2mdr_k \sin(\theta_k)}. \tag{2}$$

In a matrix form, the received signal is rewritten as

$$\mathbf{y}(t) = \mathbf{A}(\theta, r)\mathbf{s}(t) + \mathbf{w}(t), \tag{3}$$

where $\mathbf{y}(t) = [y_{-p}(t), \ldots, y_0(t), \ldots, y_p(t)]^T$, $\mathbf{s}(t) = [s_1(t), \ldots, s_K(t)]$, and $\mathbf{w}(t) = [\omega_{-p}(t), \ldots, \omega_0(t), \ldots, \omega_p(t)]$. In (3) $\mathbf{A}(\boldsymbol{\theta}, \mathbf{r}) = [\mathbf{a}(\theta_1, r_1), \ldots, \mathbf{a}(\theta_K, r_K)]$ is the direction matrix, and steering vector is given by $\mathbf{a}(\theta_k, r_k) = [e^{(j\frac{2\pi}{\lambda}(r_{-p,k}-r_k))}, \ldots, e^{(j\frac{2\pi}{\lambda}(r_{0,k}-r_k))}, \ldots, e^{(j\frac{2\pi}{\lambda}(r_{p,k}-r_k))}]^T$.

3 The Proposed Algorithm

3.1 Signal Reformulation

The Fresnel approximation [11,14] is based on the second order Taylor expansion, and using that, (2) is approximated by

$$r_{m,k} \approx r_k - md\sin\theta_k + m^2 d^2 \left(\frac{\cos^2\theta_k}{2r_k}\right). \tag{4}$$

Substituting (4) into (1) yields

$$y_m(t) \approx \sum_{k=1}^{K} s_k(t) e^{j(m\omega_k + m^2\beta_k)} + w_m(t), \tag{5}$$

where $\omega_k = -\frac{2\pi d}{\lambda}\sin(\theta_k)$ and $\beta_k = \frac{\pi d^2}{\lambda r_k}\cos^2(\theta_k)$.

From (5), using the symmetric property of the array, a special spatial correlation is calculated by

$$\begin{aligned} Y_{1(-m,m)} &= \sum_{k=1}^{K} e^{j\left((-m-m)\omega_k + \left((-m)^2 - m^2\right)\beta_k\right)} \sigma_{s,k}^2 \\ &\quad + \sigma_w^2 \delta(-m-m) \\ &= \sum_{k=1}^{K} e^{j(-2m)\omega_k} \sigma_{s,k}^2 + \sigma_w^2 \delta(-2m), \end{aligned} \tag{6}$$

where $\sigma_{s,k}^2 = \mathbb{E}\{s_k(t)s_k^*(t)\}$ is the signal power of the k-th source and $\sigma_w^2 = \mathbb{E}\{\sigma_w(t)\sigma_w^*(t)\}$ represents the noise power at m-th sensor, and $\delta(\cdot)$ denotes the Dirac function.

Collecting all the spatial correlations at different sensor pairs, one obtains

$$\mathbf{Y}_1 = \mathbf{A}_\omega(\theta)\boldsymbol{\sigma}_s + \sigma_\omega^2\mathbf{e}, \tag{7}$$

where $\mathbf{Y}_1 = [Y_{1(0,0)}, \ldots, Y_{1(p,-p)}]$, and $\mathbf{A}_\omega(\theta) = [1, \ldots, e^{j(-2p)\omega_1}, 1, \ldots, e^{j(-2p)\omega_2}, \ldots, 1, \ldots, e^{j(-2p)\omega_K}]$, $\boldsymbol{\sigma}_s = [\sigma_{s,1}^2, \ldots, \sigma_{s,K}^2]$, and $\mathbf{e} = [1, 0, \ldots, 0]^T$. From both (6) and (7), it is seen that the correlation signals only involve one parameter ω_k, which depends on the DOA. This is to say that DOA estimation can be obtained by using (7).

3.2 Sparse Reconstruction

DOA Estimation of Near-Field Signals. From (6), it is now a 1-D problem. By gridding the angle space, a dictionary $\mathbf{A}_\omega(\overline{\theta})$, where $\overline{\theta} = [\overline{\theta}_1, \ldots, \overline{\theta}_{N_\theta}]$ and the interval size is Δ_θ, can be constructed. Based on sparse reconstruction, the following ℓ_1-regularized minimization problem can be utilized to obtain the DOA estimation

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{Y} - \mathbf{A}_\omega(\overline{\theta})\mathbf{x}\|_2^2 + \mu\|\mathbf{x}\|_1, \tag{8}$$

where μ is the penalty factor. The solution of $\mathbf{x}$ provides the DOA estimation.

It is the same as all the grid-based approaches, the solution in (8) suffers from off-grid problem. To circumvent this issue, in this work, the atomic norm minimization is utilized to produce the DOA estimation. With that spirit, (7) is rewritten as

$$\mathbf{Y} = \sum_{k=1}^{K} A_k\boldsymbol{\omega}_k + \sigma_w^2\mathbf{e}, \tag{9}$$

where $\boldsymbol{\omega}_k = [1, e^{-j\omega_k}, \cdots, e^{j(-2p)\omega_k}]^T$. To estimate $\boldsymbol{\omega}$, based on atomic norm, the following optimization is devised

$$\text{minimize}_{\mathbf{i}} \ \|\mathbf{Y} - \mathbf{i}\|_2^2 + \tau\|\mathbf{i}\|_{\mathcal{A}}, \tag{10}$$

where $\mathbf{i} = \sum_{k=1}^{K} A_k\boldsymbol{\nu}_k$ and $\|\cdot\|_{\mathcal{A}}$ is the atomic norm, which is defined by

$$\|\mathbf{i}\|_{\mathcal{A}} = \inf\left\{\sum_l c_l : \mathbf{i} = \sum_l c_l\boldsymbol{\nu}(f_l), c_l > 0, \boldsymbol{\nu}(f_l) \in \mathcal{A}\right\}, \tag{11}$$

where $\mathcal{A}$ is a collection of atoms and $\boldsymbol{\nu}(f_l) = [1, e^{-jf_l}, \cdots, e^{-j(2p)f_l}]^T$. To efficiently solve the atomic norm, it can be transformed into the following semidefinite programming (SDP) [12].

$$\begin{aligned} \|\mathbf{i}\|_{\mathcal{A}} &= \text{minimize}_{t,\mathbf{u}} \frac{1}{2}(t + u_1) \\ &\text{subject to } \begin{bmatrix} T(\mathbf{u}) & \mathbf{i} \\ \mathbf{i}^{\mathrm{H}} & t \end{bmatrix} \succeq \mathbf{0}, \end{aligned} \tag{12}$$

where $T(\mathbf{u})$ is an $N \times N$ Toeplitz matrix, given by

$$T(\mathbf{u}) = \begin{bmatrix} u_1 & u_2 & \cdots & u_N \\ u_2^* & u_1 & \cdots & u_{N-1} \\ \vdots & \vdots & \vdots & \vdots \\ u_N^* & u_{N-1}^* & \cdots & u_1 \end{bmatrix}, \tag{13}$$

where u_i is the ith component of $\mathbf{u}$.

By utilizing the SDP formulation in (12), the optimization problem (10) is

$$\begin{aligned} &\text{minimize}_{t,\mathbf{u},\mathbf{i}}\ \tau(t+u_1) + \|\mathbf{r}-\mathbf{i}\|_2^2 \\ &\text{subject to}\ \begin{bmatrix} T(\mathbf{u}) & \mathbf{i} \\ \mathbf{i}^{\mathrm{H}} & t \end{bmatrix} \succeq \mathbf{0}. \end{aligned} \tag{14}$$

Range Estimation of Near-Field Signals. From (5), another spatial correlation sequence can be constructed by utilizing different sensor outputs. That is,

$$Y_{2(m+1,m-1)} = \sum_{k=1}^{K} \sigma_{s,k}^2 \exp(j2\omega_k + j4m\beta_k). \tag{15}$$

Using the matrix representation, (15) is rewritten as

$$\mathbf{Y}_2 = \mathbf{A}_\vartheta(\vartheta)\mathbf{r}_s + \sigma_w^2\mathbf{e}, \tag{16}$$

where $\mathbf{Y}_2 = [Y_{2(1,-1)}, Y_{2(2,0)}, \cdots, Y_{2(p,p-2)}]^{\mathrm{T}}$, $\mathbf{r}_s = \mathrm{diag}(\sigma_1^2 e^{j2\omega_1}, \cdots, \sigma_K^2 e^{j2\omega_K})$, and the array manifold $\mathbf{A}_\vartheta(\vartheta) = [\mathbf{a}_\vartheta(\vartheta_1), \cdots, \mathbf{a}_\vartheta(\vartheta_K)]$ with the steering vector $\mathbf{a}_\vartheta(\vartheta_k) = [1, e^{j\vartheta_k}, \cdots, e^{jp\vartheta_k}]^{\mathrm{T}}$ with $\vartheta_k = 4\beta_k$.

It is seen from (16) that the signal structure is the same as the DOA estimation in (7) and it only depends on the unknown parameter β. Therefore, the atomic norm minimization can be applied to eliminate the off-grid issue.

4 Simulation Results

To access the performance of the proposed method, the simulations are conducted in this section and comparisons with popular methods are also provided. The root mean square error (RMSE) as a performance indicator is utilized. In the experiment, $d = \frac{\lambda}{4}$ is set and the number of sensor is $M = 15$, unless stated otherwise.

In the first experiment, three near-field narrowband sources are located at $(-36°, 6\lambda)$, $(6°, 10\lambda)$, $(40°, 20\lambda)$, and the estimation results are provided in Fig. 2. To utilize the sparse construction idea, the DOA of $[-90°, 90°]$ and the range of $[5\lambda, 24\lambda]$ are uniformly gridded with the interval sizes of $\varDelta_\theta = 1°$ and $\varDelta_r = 0.5\lambda$, respectively. From Fig. 2a, when there is no off-grid, the estimations match the

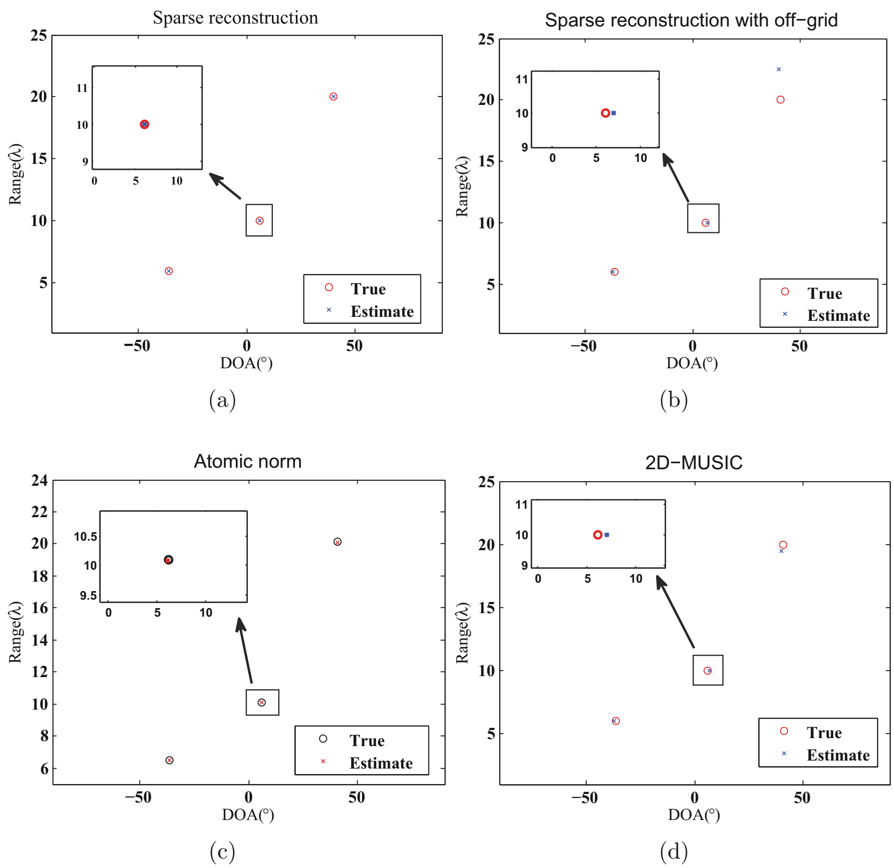


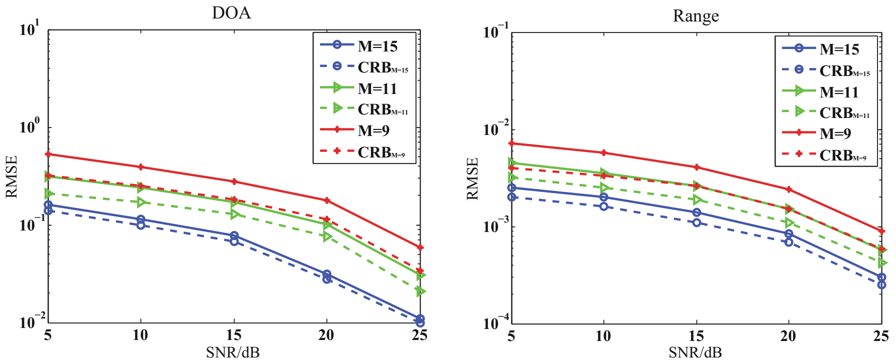
Fig. 2. Near-field source localization with snapshot 100 and SNR = 20 dB. (a) sparse reconstruction, (b) basis mismatch, (c) proposed method, (d) 2D-MUSIC.

true values perfectly, whereas when there is a off-grid, the performance deteriorates, demonstrated in Fig. 2b. The proposed atomic norm minimization approach achieves the full sparse reconstruction performance, regardless the off-gird, shown in Fig. 2c. It is seen that 2D-MUSIC algorithm also cannot perfectly estimate the source locations, depicted in Fig. 2d.

In Fig. 3, the RMSEs of DOA and range estimations of the proposed method with different number of sensors M versus SNR are provided. As expected, in the case where the number of sensors M is constant, the performance improves as SNR increases. However, the proposed method outperforms other methods. Specifically, Fig. 4 displays that the RMSEs vary from the numbers of sources. Although the performance of algorithms has degraded with the increase of K, the proposed algorithm is better than other algorithms. Finally, in Fig. 5, the performances of different methods are plotted versus the number of snapshots.

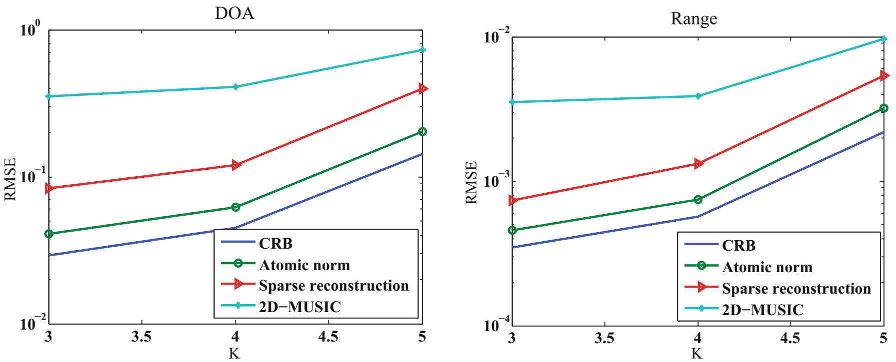
It is noticed that the estimation error of the proposed method is the lowest and approaches the CRB [3].

Finally, the proposed algorithm is analyzed for complexity. The running time of different methods is provided in Table 1, using a Intel Dual Core i3 with CPU 2.4 GHz and MATLAB. The results show that running time of all the Methods increases as the number of sensors increases. In particular, compared with Sparse reconstruction based method, the proposed Atomic norm based approach is faster than the Sparse one. Since Atomic norm method does not require grid search, the method greatly reduces time consumption.



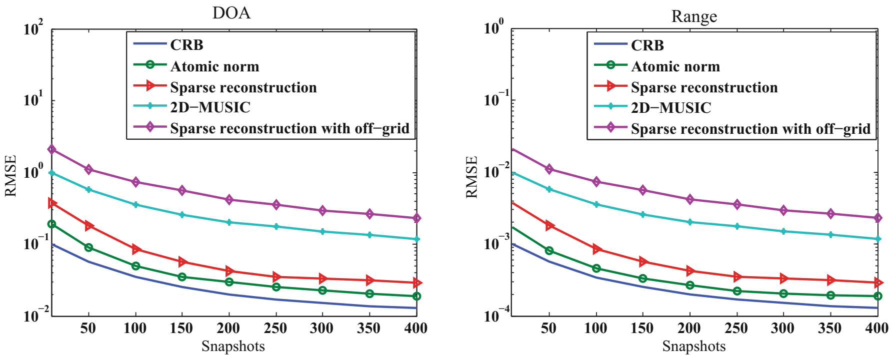
(a) (b)

Fig. 3. Performance evaluations of the proposed method versus SNR.



(a) (b)

Fig. 4. Performance comparisons versus K.



(a) (b)

Fig. 5. Performance comparisons versus snapshots.

Table 1. Running times of different methods versus number of sensors (seconds).

M	3	5	7	9
2D-MUSIC	1.012	1.536	2.563	4.807
Sparse reconstruction	3.917	5.773	7.088	7.296
Atomic norm	1.503	1.761	2.153	2.721

5 Conclusion

In this work, the near-field source localization problem is investigated, designed to estimate DOA and range. By exploiting the symmetric property of the array, two special spatial correlation sequences are constructed. With that, near-field source localization problem is separated into two subproblems, where each subproblem is solved by atomic norm minimization. Because of this decouple operation, the computational complexity is obviously reduced since the search only confines to one parameter. In doing so, the off-grid problem is eliminated, and the numerical results indicate the effectiveness of the proposed method.

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