Extraction of the 1961—2020 Long Time Scale Climate Memory Signal in Qingdao

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Abstract. Based on the average temperature series during the past 60 years, the climate memory signals on different scales in Qingdao are extracted, to which autocorrelation function and fractional operator method analyses are then applied to fulfill the extraction. The result shows that: the climatic signals of long time scale have certain memory. The fractional integral operator $({}_0I_t^q)$ is used to extract the climate memory signals in the temperature series. This method is expected to play a role in the prediction of the climate in Qingdao.

Keywords: Fractional Operators · Long-term Memory Signals · Weather and Climate

1 Introduction

Although the degree of randomness of weather-scale and climate-scale stochastic time series are different, the relationship between them can be inscribed by introducing fractional order derivatives and integrals [1], and climate change on long time scales has significant long-term memory or persistence [2]. Although both weather and climate events follow the laws of hydrodynamics, they show chaos because both weather and climate are multi-scale phenomena that produce different degrees of rise and fall with different time scales [3]. As early as 1976, Hasselmann compared weather and climate to the relationship between micro-scale motion and macro-scale motion, and used the Langevin equation to establish the relationship between climate random variables and weather random variables, and proposed that the first-order derivatives of climate random variables with respect to time are a consequence of weather random variables [3]. However, recent studies have found that the simple first-order derivative is no longer a good model of the relationship between weather and climate, and that the relationship between the two is likely to be one of fractional order derivatives, and there have been studies that have confirmed the memorability of climate element sequences [4–7]. Since the Qingdao area is bordered by the Yellow Sea and influenced by the marine environment, this paper conducts an in-depth study on the climate memory characteristics of Qingdao area [8]. In this study, the daily, monthly and annual temperature data of Qingdao city from 1961 to 2020 are used to analyze the autocorrelation and climate memory signal characteristics of stochastic time series by using autocorrelation function

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and fractional order integration function, and the research results can provide technical guidance for local climate prediction and climate change research.

2 Data and Methods

2.1 Data Sources

The daily, monthly and annual temperature data of Qingdao City from 1961 to 2020 were used in the paper, and the information was obtained from the Shandong Meteorological Data Center. In order to ensure that the temperature data are true and valid, the temperature data are tested for temporal consistency and spatial consistency, abnormal data are removed, and linear interpolation is performed on the missing measurement data, etc. The daily, monthly, and annual mean temperature distance level data for 1961–2020 were obtained using quality-controlled temperature data at different time scales.The use of distance level series information can remove the trend disturbance of the actual observed data and has relative smoothness.

2.2 Correlation Analysis

Autocorrelation is the degree of dependence between the instantaneous values of two moments in a random time series, and the value of the autocorrelation function expresses the degree of correlation of the data. For stochastic time series, autocorrelation reflects the memorability of the stochastic time series. Let a time series be $T(\tau)$, then its autocorrelation function $R(\tau)$ can be expressed as:

$$R(\tau) = \frac{\langle T(t+\tau)T(\tau)\rangle}{T^2(\tau)} \tag{1}$$

where, $T(t+\tau)$ representing the sequence value of the interval $t+\tau$, $T(\tau)$ representing the present moment value, τ representing the time interval. If the autocorrelation function of a random time series is similar to white noise, it can be approximated as white noise.The value of the state at each moment of the white noise sequence is independent of the value of the state at other moments, which can also be referred to as a special kind of smooth time-random sequence without any temporal memorability.Therefore, the autocorrelation function should be used to determine whether the random time series is white noise [9].

2.3 Definition of Fractional Order Integral

In 1819, the French mathematician Lacroix used the Gamma function to express the fractional order derivative of a non-negative exponential power function [10]:

$$\frac{d^{1/2}x^{\alpha}}{dx^{1/2}} = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1)}x^{\alpha-1/2} \tag{2}$$

This coincides with the result calculated from the now familiar definition of the Riemann-Liouville fractional order derivative, In addition, Abel used fractional order derivatives $\frac{d^{1/2}f(x)}{dx^{1/2}}$ to express the solution of integral equation in his study of the isochronous problem,

$$\mathrm{f(x)} = \int_0^x (x-t)^{1/2} g(t)dt \tag{3}$$

In the 1830s, Liouville proposed what is now called the Liouville-type fractional order derivative, and Riemann also gave an expression for the fractional order derivative, Later, Sonin, Letnikov, Laurent and other mathematicians improved and perfected it, and finally formed the Riemann-Liouville fractional order operator [12]. It is defined as

$${}_0I_t^q \mathrm{u} = \frac{1}{\Gamma(q)} \int_0^t (t-x)^{q-1} u(x)dx \tag{4}$$

where $0 \leq \mathrm{q} \leq 1$, $\Gamma(q)$ denotes the Gamma function, t denotes the current time point, x denotes the historical time, the fractional order integral starts from 0, represents the historical starting point, theoretically $-\infty$ should be used to denote the historical starting point, considering the time node of the beginning of the observed data and the technical implementation, choose $\mathrm{x} = 0$ as the historical starting point.As can be seen from the definition of the fractional order operator, the value of the integer order derivative of a function at a point depends only on the variability of the function near that point, whereas the value of the fractional order derivative of a function at that point is influenced by the variability of the function from almost the whole globe. In this sense, the integer order derivative operator can be considered as a local operator, while the fractional order operator is a non-local operator, which is the essential difference between the two.

2.4 Climate Memory Signal Intensity

Since weather and climate element series are multi-scale series, power spectra $\mathrm{S}(f)$ in frequency space are commonly used to portray stochastic time series of weather and climate.The power spectrum $\mathrm{S}(f)$ of a climate random sequence £(t) is the square of its Fourier transform coefficient $\hat{\text{£}}$(t), The formula is:

$$\mathrm{S}(f) = \left|\hat{\text{£}}(\mathrm{t})\right|^2 \propto f^{-\beta} \tag{5}$$

where $\hat{\text{£}}$(t) is the Fourier transform of £(t), Then $\mathrm{S}(f)$ is called the power spectrum of the climate-time random series, f is the frequency, β is the power spectrum index [12].

In analyzing the distance level information of weather and climate stochastic time series, the average of the squared temperature distance level differences $\Delta\mathrm{x}(\tau)$ at time intervals τ is often used, $\langle \Delta x(\tau)^2 \rangle$ called the second order structure function. The second-order structure function of the fractal system can be derived from the mean square displacement formula of Brownian motion, The equation is:

$$\Gamma(\tau) = \langle \Delta x(\tau)^2 \rangle \propto \tau^{2\alpha} \tag{6}$$

where α $(0 < \alpha < 1)$ is the Hurst index.

Using the second-order structure function and the fractional-order integral model, the relationship between the Hurst α and power spectrum indices β and the order of the fractional-order operator q is obtained as:

$$2q = \beta = 2\alpha + 1 \tag{7}$$

By selecting the appropriate intensity (q order) of the climate memory signal, it is possible to simulate the variation of the climate-scale stochastic sequence £(t). q $= 0$ represents a climate random sequence with no climate memory signal, behaving as white noise ($\beta = 0$). q > 0 represents the variation of the q-order integral operator to simulate a stochastic sequence of climate, cumulative climate memory signals and weather-scale signals are also introduced.

2.5 Climate Memory Signal Calculation

Decomposing the climate stochastic series into two components: the cumulative climate memory signal M(t) and the weather-scale signal ε(t). Before extract M(t) using the fractional order integral operator, Eq. (4) needs to be written in discrete form as follows:

$$\text{£(t)} = \mathrm{M(t)} + \varepsilon(\mathrm{t}) = \frac{1}{\Gamma(q)}\int_{x=0}^{t-\delta}(t-x)^{q-1}\varepsilon(x)dx + \varepsilon(\mathrm{t}) = \mathrm{K(q)}_t^{\delta} \otimes \Psi_0^{t-\delta} + \varepsilon(\mathrm{t}) \tag{8}$$

where $K(\mathrm{q})_t^{\delta} = \{k(q,t), k(q,t-\delta), \ldots, k(q,t-x), \ldots k(q,\delta)\}$ represents the climate memory signal kernel function, $\Psi_0^{t-\delta} = \{\varepsilon(0), \varepsilon(\delta), \ldots, \varepsilon(\mathrm{u}), \ldots \varepsilon(\mathrm{t}-\delta)\}$ representing at each time point in historical weather signals.Therefore, in order to extract the climate memory signal, it is necessary to determine $\mathrm{K(q)}_t^{\delta}$ and $\Psi_0^{t-\delta}$ and then calculate its convolution. In extracting the climate memory signal, the memory signal strength (q order) needs to be obtained by calculating the Hurst index α, and calculate the climate memory signal kernel function $\mathrm{K(q)}_t^{\delta} = \{k(q;t), k(q;t-\delta), \ldots, k(q;t-x), \ldots k(q;\delta)\}$ at each moment,

$$k(q,t-x) = \frac{1}{\Gamma(q)(t-x)^{1-q}} \tag{9}$$

2.6 Historical Weather Signal Calculations

According to the study of the relationship between weather and climate series, the following fractional order relationship between weather and climate time series is found [13]:

$$\frac{d^q x}{dx^q} = \varepsilon(t) \tag{10}$$

The variation of the climate stochastic series can be regarded as the q order integral of the weather stochastic series, and the method can express the long-tail phenomenon of the climate stochastic series more precisely [14]. Fractional-order integration models split the climate stochastic time series into cumulative climate memory signals M(t) and weather-scale signals ε(t).

$$£(t) = M(t) + ε(t) \tag{11}$$

The cumulative climate memory signal M(t) represents the cumulative long-tail impact of the historical weather signal, and the weather-scale signal ε(t) represents the impact of the current weather signal. At the same time the cumulative climate memory signal M(t) and the weather scale signal ε(t) make up the current climate state, and the current weather scale signal continues to have an impact on the climate state. If the climate memory signal M(t) can be calculated and the impact of the historical memory signal can be further quantified, it will improve the climate prediction technology, therefore, how to extract its signal is the focus of the research.

If the current climate state £(t) is unknown, the cumulative climate memory signal M(t) can be expressed by the historical weather signal $\Psi_0^{t-\delta}$ using a fractional order integration model as follows:

$$M(t) = \frac{1}{\Gamma(q)} \int_{x=0}^{t-\delta} (t-x)^{q-1} ε(x) dx \tag{12}$$

ε(0), ε(δ), ..., ε(u), ... ε(t − δ) representing at each time point in historical weather signals, δ is the time interval of the random time series, calculating the cumulative climate memory signal M(t) and the historical random series £(0), £(δ), ⋯ , £(t − δ), the corresponding weather signal ε(0), ε(δ), ..., ε(t − δ) can be further extracted, Assuming a historical starting point is t = 0 and ignoring the effect of the current moment, we have ε(0) = £(0), For the next moment t = δ in time, due to the corresponding historical time node $x = 0$, ε(δ) can be calculated from Eq. (8) as:

$$ε(\delta) = £(\delta) - K(q)_t^{\delta} \otimes \Psi_0^0 = £(\delta) - k(q;\delta) \times ε(0) \tag{13}$$

By analogy, we get the corresponding weather signal for the historical time point ε(0), ε(δ), ..., ε(t − δ), where $k(q, t-x) = \frac{1}{\Gamma(q)(t-x)^{1-q}}$, $\Psi_0^{t-\delta}$ = {ε(0), ε(δ), ..., ε(u), ... ε(t − δ)}, $k(q, t-x)$ is determined by the strength (order) of the climate memory signal, Calculate $k(q, t-x)$ to get an accurate estimate of the historical weather stochastic time series, Inverting the values of the climate random sequence (£(0), £(δ), ..., £(u), ... £(t − δ)) according to Eq. (8), Also using Eq. (12) to obtain the climate memory signal, although the value of the weather signal at the current moment cannot be estimated, the time stochastic series of the historical moment is still important and significant for climate prediction.

3 Results

3.1 Stability Test of Temperature Data

The amplitude of the daily, monthly and annual mean temperature range series of Qingdao from 1961 to 2020 (see Fig. 1), the amplitude of the daily temperature range series is around 10°C, the amplitude of the monthly temperature range series is about 4°C, and the amplitude of the annual temperature range series is only around 1°C, the degree of fluctuation of the daily temperature range series is relatively large, while the monthly and annual temperature series are relatively small. The smoothness of the year-by-year, monthly, and daily temperature distance series was tested by the unit root ADF method, and the results showed that the p-values of the year-by-year, monthly, and daily temperature distance series were close to 0, which was much smaller than the p-value level of the original hypothesis significance test of 1%, and the original hypothesis was significantly rejected, so the distance series of daily, monthly, and annual average temperature in Qingdao from 1961 to 2020 was judged to be a smooth time series.

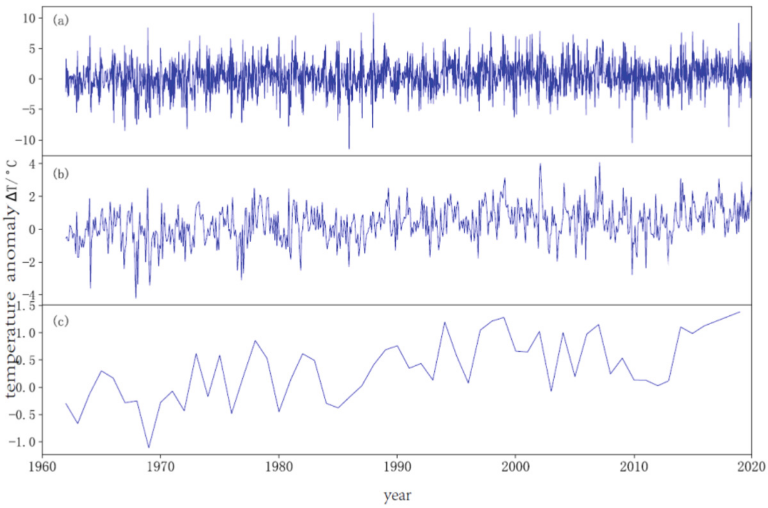

Fig. 1. Average air temperature anomaly in Qingdao (1951–2020) on different time scales: a) daily scale, b) monthly scale, c) yearly scale).

3.2 Correlation Analysis

The autocorrelation function R(τ) was calculated for the daily, monthly and annual mean temperature distance level series for Qingdao 1961–2020 (Fig. 2), and it can be seen from Fig. 2a that the curve fluctuates around the value of 0. Therefore, the daily mean temperature can be considered as a white noise series [14], which is not memorable. In contrast, compared to the daily distance level series, the amplitude of the autocorrelation values of the distance level series of monthly (Fig. 2b) and annual (Fig. 2c) mean temperatures range from 0.2 to 0.6, both of which have a certain degree of autocorrelation, and there is a law power phenomenon [15], which means that although the climate signal tends to decay with time, some of the characteristics do not disappear with the smoothing of the weather signal, but remain in the climate signal, which makes the monthly, annual or longer time scales are combined, so that the climate signals on long time scales show a certain degree of memory.

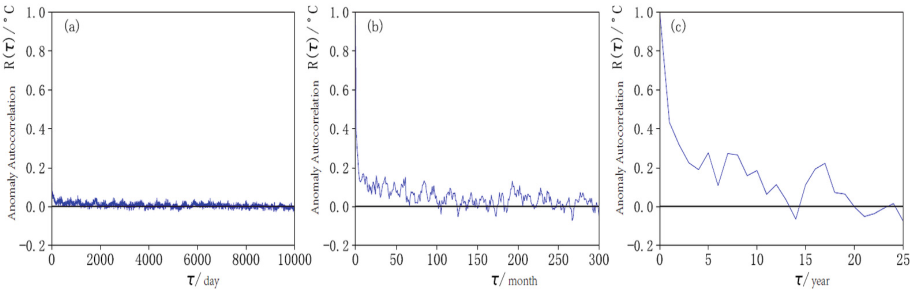

Fig. 2. Autocorrelation functions of the average air temperature anomaly series in Qingdao (1951–2020) on daily (a), monthly (b), and yearly scales(c).

3.3 Historical Weather Signals and Climate Memory Signals

Assuming t = 0 as the starting point for the beginning of history, and ε(0) = £(0), for the next moment t = δ can be calculated from Eq. (13), so the weather signal at the node of the historical moment can be inferred point by point. Determining the starting point of the beginning of history is the key to extracting historical weather signals. Although theoretically chosen $-\infty$ to represent a distant historical starting point, it is often chosen t = 0 as the starting point of history in practical calculations, and the influence of previous historical temperature data on the accuracy of the calculation cannot be theoretically ignored. Since meteorological observations began only more than 100 years ago, there are not enough observations to record previous climate states, yet observations recording previous climate states will inevitably have an impact on current climate states.

The 5560th point to 5700th point of the daily average temperature distance level series in Fig. 1 is selected as the experimental object, and it can be seen from Fig. 3 that the error of the random time series is relatively large at the starting point 5560, and the error gradually decreases with time. Assuming that 5560 is the starting point of the observation series, the estimation of the historical weather signal starts at 5560, and the values before 5560 need to be ignored, and since the estimation starts at 5560, the appearance of errors is understandable. The historical observed values before 5560 are ignored in the calculation, which is the reason for the large error at the beginning. As time goes on, the influence of the ignored part will become smaller and its error will become smaller, and after 5580 point, the error is basically close to 0. Therefore, the calculation method of estimating the historical weather signal is considered reliable, and in later simulations, due to the influence of the unobserved time series, the Some data at the beginning need to be discarded to ensure the reliability.

In the above discussion, it is found that the climate memory signal is objectively present in the long time scale stochastic series and has an impact on the later climate state, so quantifying the strength of the climate memory signal has a positive impact on climate prediction [16]. In Fig. 4, the time-stochastic series is divided into three stages: stage I (1961–1970), stage II (1970–2010), and stage III (2010–2020), and stage III is used as the test stage, in order to exclude the error caused by the historical observation data errors arising from inadequate data, using the data of Phase II as the experimental

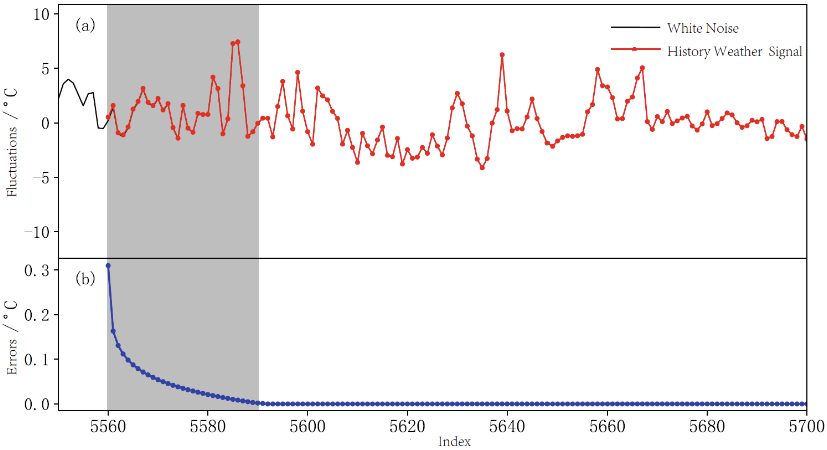

Fig. 3. Verification of the extracted historical weather signal in Qingdao.

sequence to extract the historical memory signal can ensure the reliability of the extracted historical weather signal.

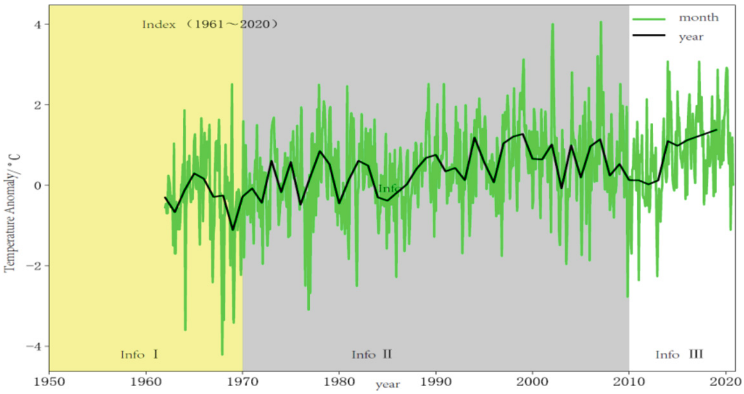

Fig. 4. The annual and monthly weather signals in Qingdao (1961–2020).

The historical weather signal is extracted by backpropagating (8), and considering the influence of unobserved historical data, the data of stage I (1961–1970) is discarded to ensure the reliability of historical weather signal calculation, so stage II (1970–2010) is selected to extract the historical weather signal, and the extraction of historical weather signal is completed by using Eq. (13), and the the intensity of the climate memory signal in Qingdao area q = 0.53 can be obtained by the calculation of hurst index. Stage III (2010–2020) is used as the test interval (See Fig. 5), the red curve represents the memory signal fluctuation, and the error fluctuation between the extracted memory signal and the historical weather signal can be visualized by Fig. 5b. A large number of experiments show that the memory signal extraction method works more obviously in unstable signals such as extreme weather and other abnormal temperatures.

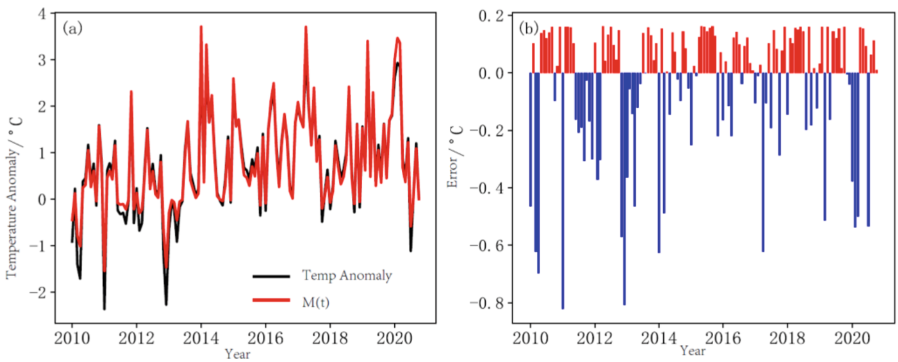

Fig. 5. A comparison of the extracted climate memory signal to the historical weather signal in Qingdao (2010–2020).

4 Discussions

In this paper, we use fractional order integral model to quantitatively extract climate memory signal in the historical time stochastic series of qingdao area, and use this method to estimate the long-term impact of historical climate state on the generation of future climate change, if the historical time stochastic series is regarded as a memory signal characterized by long time scale, the climate memory signal can explain climate change to some extent [17–20], as shown in Fig. 5, the red curve representing the climate memory signal accounts for a large proportion of the variation in the Stochastic Time Series, in the process of simulating climate change. The climate memory signal M(t) determines on what basis the stochastic time series continues to change, while the stochastic perturbation, which $\varepsilon(t)$ represents the change in the short-term weather scale, generates the trend of the studied stochastic time series. The proportion of M(t) in the climate-time stochastic series is determined by the strength of the climate memory signal [21–23], but not all observed climate variables are characterized by strong climate memorability, which may be due to slow-responding subsystems such as the ocean. As shown in Fig. 5, besides indicating the climate memory signal of qingdao city, the weather-scale perturbation signal is also given, as seen in Figs. 5a and 5b in the past 20 years, the calculated weather perturbation signal is mostly negative, which indicates that the weather-scale perturbation is highly likely to be negative. In the experimental results, it is confirmed that the weather-scale perturbations of the temperature series for the past two decades are negative, and although the climate change is influenced by the historical climate memory signal, the climate conditions of qingdao city considered are also influenced by factors such as the topography of Qingdao.

The simulation of long-range correlated processes using this method was developed from fractional-order Brownian motion, which describes continuous and long-range correlated physical processes, unlike the processes of ordinary brownian motion, and considering that many physical phenomena in nature are long-range correlated, self-similar (Fractal) features in nature can be better explained using fractional-order brownian motion. Mandelbrot and Van Ness used fractional-order integral operators in Brownian motion to extract and model fractional-order Brownian motion. using the fractional-order integral operator to inscribe the historical time stochastic series can easily divide the current climate state in Qingdao area into two parts: the climate memory signal generated by the cumulative historical weather signal and the weather-scale perturbation signal, which can also be regarded as an independent homogeneously distributed noise sequence; therefore, the fractional-order integral model can be used not only as a fractional-order prediction model to improve the accuracy of climate prediction to simulate the long-term memory signals, but also to understand the process of physical changes from the causes of long-term memory signals.

5 Conclusion

(1) The monthly scale and annual scale mean temperature spacing series of Qingdao city from 1961 to 2020 have a certain degree of autocorrelation, and the climate signal tends to decay with time, but some of the characteristics do not disappear with the weather signal smoothing, but still exist in the climate signal, so that the long time scale climate signal presents a certain degree of memory, and the annual scale of mean temperature spacing in Qingdao area has more memory characteristics compared with the moThe intensity of climate memory signal in Qingdao region is 0.53, which can be regarded as a characteristic of the whole climate system in the time range from month to year, and can explain climate change to some extent.nthly scale and daily scale.

(2) By estimating the historical weather signal by this method, the weather signal of the historical moment node obtained by the inverse of Eq. (8) can be obtained based on the climate state of the current moment. Later, when using this method for climate prediction, some data from the beginning need to be discarded to ensure reliability due to the influence of unobserved time series.

(3) The intensity of climate memory signal in Qingdao region is 0.53, which can be regarded as a characteristic of the whole climate system in the time range from month to year, and can explain climate change to some extent.

Although some results were obtained by this method to analyze the mesoscale climate characteristics of Qingdao area, further analysis is needed to investigate the intensity differences of climate memory signals for different regions and the methods to calculate climate memory signals with higher accuracy, which is the focus of the next research.

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