Nodes Deployment Optimization for Indoor Localization Using FIR Filter

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Abstract. In order to improve the positioning accuracy of ultra-wideband (UWB) positioning system, it is usually to reduce the measurement error or choose a better positioning algorithm, and the geometric relationship between the blind node and the reference node is also an important factor affecting the positioning accuracy. In this paper, the minimum geometric dilution of precision (GDOP) of the mean value of the trajectory is proposed as the criterion for judging the optimal geometric layout of the reference node. After the optimal geometric layout of the reference node is obtained by the evaluation criterion, the finite impulse response (FIR) filter is selected to filter the observed values to remove noise and improve the system performance again. After simulation, it can be concluded that this method can effectively reduce the positioning error and improve the positioning accuracy.

Keywords: Mean geometric dilution of precision · Ultra-wideband · Finite impulse response

1 Introduction

Since the 20th century, due to the rapid development of information technology and artificial intelligence, the various walks of life have been constantly pushed forward, and location information has become increasingly important to people's daily life and work. Various positioning technologies on the market continue to emerge [4]. Based on the ultra-wideband (UWB) positioning technology in indoor positioning technology, it stands out among many indoor positioning technologies due to its strong penetration ability, high resolution, and centimeter-level positioning accuracy [5].

In UWB indoor positioning, the known coordinate points are called reference node (RN), and the points to be located are called blind node (BN) [2]. For its positioning accuracy, in addition to ranging accuracy and positioning algorithm, the geometric layout of the RNs is also an important factor affecting the positioning accuracy [1]. The geometric dilution of precision (GDOP) provides a theoretical basis for us to judge the geometric layout of the RNs. Originated from Loran-C, GDOP is widely used to test the influence of satellite geometry on single point positioning estimation and is a dimensionless number of signals,

B. Wang et al. (Eds.): ICMTEL 2023, LNICST 533, pp. 3–11, 2024.
https://doi.org/10.1007/978-3-031-50574-4_1

which is often used to estimate the expected positioning estimation accuracy and shows superior performance in wireless sensor network (WSN) applications [7]. For UWB positioning technology, GDOP is also applicable. For this article, the positioning area is a known trajectory, so the minimum mean GDOP of the trajectory is proposed as the criterion for judging the geometric layout of the RNs, and then the optimal geometric layout can be obtained [6]. Based on this premise, experimental simulation is carried out, and the finite impulse response (FIR) filter is used to process the observed values. For FIR filter, in the 1950s, FIR filter has attracted people's attention, under constant attention FIR filter has been further promoted [2]. The research shows that the robustness of FIR filter is higher than that of Kalman filter (KF) in the presence of external interference [8]. Therefore, this filter is selected in this paper to further improve the positioning accuracy.

The structure of this paper is as follows. In the second chapter, the definition and calculation method of GDOP are analyzed. The third chapter discusses the establishment of the optimal geometric model of the RNs and the reasons for selecting the FIR filter. The fourth chapter is software simulation, which verifies the method in this paper. The fifth chapter summarizes this article.

2 Analysis of Geometric Dilution of Precision

2.1 Positioning Error Analysis

Covariance matrix of positioning error [3]:

$$\mathrm{Var}(\boldsymbol{\Delta}\mathbf{x}) = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^{-1}\mathbf{R}\mathbf{H}\left(\mathbf{H}^T\mathbf{H}\right) = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\sigma_e^2 \tag{1}$$

where, $\mathbf{H}$ is the observation matrix of the system, $\mathbf{R}$ is the covariance matrix of the observation error. The following two assumptions are made for the observation error model:

1) It is assumed that the observation errors of the BNs to each RNs show the same normal distribution, the average value is zero and the σ_e^2 variance is Gaussian white noise.
2) Assume that the observations of the BNs to different base stations are not correlated with each other.

According to assume 2, the covariance matrix of observation error is diagonal matrix, The specific form is shown in Eq. (2). In this paper, four RNs are used, so it is a 4-order diagonal matrix.

$$\mathbf{R} = \begin{bmatrix} \sigma_e^2 & 0 & 0 & 0 \\ 0 & \sigma_e^2 & 0 & 0 \\ 0 & 0 & \sigma_e^2 & 0 \\ 0 & 0 & 0 & \sigma_e^2 \end{bmatrix} = \sigma_e^2\mathbf{I} \tag{2}$$

where, $\mathbf{I}$ is a 4-order unit matrix. After the above two assumptions, Eq. (1) is further simplified to:

$$\mathrm{Var}(\boldsymbol{\Delta}\mathbf{x}) = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^{-1}\left(\sigma_e^2\mathbf{I}\right)\mathbf{H}\left(\mathbf{H}^T\mathbf{H}\right) = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\sigma_e^2 \tag{3}$$

It can be seen from the Eq. (3) that the covariance of the positioning error is the product of the matrix $\mathbf{G} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}$ and the variance of the observation error. Therefore, we can improve the positioning performance of the system from the following methods:

1) Observation error, That is, the larger the σ_e^2, the greater the positioning error.
2) Geometric layout of RNs, The matrix $\mathbf{G}$ depends on the number of RNs involved in the solution and its geometric layout relative to the BNs. The smaller the elements in the matrix $\mathbf{G}$, the smaller the amplification of the observation error, and the smaller the positioning error.

Therefore, there are two ways to reduce the positioning error: one is to reduce the observation error. The second is to improve the geometric distribution of RNs. For the first point, the error can be reduced by improving the performance of the equipment and improving the mathematical model. This paper will explain how to improve the geometric layout of the RNs from the second point.

2.2 Definition and Calculation of Geometric Dilution of Precision

In order to represent the magnification from observation error to positioning error, the concept of GDOP is proposed. In this paper, it is used as the standard to measure the layout of RNs. That is, under the condition of the same observation error, the larger the GDOP, the greater the positioning error, and the smaller the GDOP, the smaller the positioning error.

According to Eq. (3) in three-dimensional positioning:

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 & \sigma_{zt} \\ \sigma_{tx} & \sigma_{ty} & \sigma_{tz} & \sigma_t^2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \sigma_e^2 \tag{4}$$

where, σ_x^2 , σ_y^2 , σ_z^2 , σ_t^2 is each positioning error component in the covariance matrix of the positioning error, and $\mathbf{G}_{is}(i, s = 1, 2, 3, 4)$ is the element in the matrix $\mathbf{G}$.

The definition formula of GDOP can be obtained as follows:

$$GDOP = \frac{\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_t^2}}{\sigma_e} = \sqrt{G_{11} + G_{22} + G_{33} + G_{44}} = \sqrt{\mathrm{tr}\left(\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\right)} \tag{5}$$

Since this paper is a two-dimensional positioning, there is the following solution to the system observation matrix $\mathbf{H}$ [1]:

$$r_i' = \sqrt{\left(x - x_i\right)^2 + \left(y - y_i\right)^2} + ct_e, i = 1, 2, 3, 4. \tag{6}$$

where r_i' is the distance observation, (x, y) is the real coordinate of the BNs, (x_i, y_i) is the coordinates of RNs, and ct_e is the error caused by the time deviation between the BNs and the RNs.

The Taylor series linearization expansion of Eq. (6) is performed at the approximate position $(\hat{x}, \hat{y})$ of the BNs, and the components above the second order are ignored. Because the simulation area in this paper is small, the time delay error between the RNs and the BNs can be ignored, and we can get:

$$r_i' - \hat{r}_i = \frac{x_i - \hat{x}}{\hat{r}_i}(x - \hat{x}) + \frac{y_i - \hat{y}}{\hat{r}_i}(y - \hat{y}) \tag{7}$$

where, $\hat{r}_i = \sqrt{(\hat{x} - x_i)^2 + (\hat{y} - y_i)^2}$. Write Eq. (7) in matrix form as follows:

$$\mathbf{L} = \mathbf{H}\mathbf{X} \tag{8}$$

where, $\mathbf{H} = \begin{bmatrix} \frac{x_1 - \hat{x}}{\hat{r}_1} & \frac{y_1 - \hat{y}}{\hat{r}_1} \\ \frac{x_2 - \hat{x}}{\hat{r}_2} & \frac{y_2 - \hat{y}}{\hat{r}_2} \\ \frac{x_3 - \hat{x}}{\hat{r}_3} & \frac{y_3 - \hat{y}}{\hat{r}_3} \\ \frac{x_4 - \hat{x}}{\hat{r}_4} & \frac{y_4 - \hat{y}}{\hat{r}_4} \end{bmatrix}$, $\mathbf{L} = \begin{bmatrix} r_1' - \hat{r}_1 \\ r_2' - \hat{r}_2 \\ r_3' - \hat{r}_3 \\ r_4' - \hat{r}_4 \end{bmatrix}$, $\mathbf{X} = \begin{bmatrix} x - \hat{x} \\ y - \hat{y} \end{bmatrix}$.

Therefore, with $\mathbf{G} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$, the algorithm for obtaining the geometric accuracy factor in this paper is as follows:

$$GDOP = \sqrt{\mathrm{tr}\left((\mathbf{H}^T\mathbf{H})^{-1}\right)} = \sqrt{G_{11} + G_{22}} \tag{9}$$

3 Analysis of Optimal Geometric Layout Model

3.1 Establishment of Objective Function

From the above analysis, it is known that the value of GDOP is a function of RNs coordinates and BNs coordinates, and is independent of other factors. The GDOP formula obtained from the systematic observation matrix **H** shows that the GDOP is for each BN. However, the indoor location service space belongs to a region, and in this paper, the trajectory in the location area is solved. Therefore, the minimum GDOP of the mean value of the track is proposed as a criterion to evaluate the optimal geometric layout of the RNs.

When the RNs location is known, a GDOP(x, y) value can be calculated for each BN in the location space. Assume that there are N track coordinate sampling points, calculate each track coordinate point through Eq. (9) to obtain GDOP $(x_j, y_j)(j = 1, 2, \cdots N)$. In order to optimize the RNs geometry layout of UWB positioning system, the minimum mean value of GDOP is used as the criterion, Eq. (10) and Eq. (11) are respectively used to calculate the mean value and the minimum mean value of GDOP.

$$E\left(GDOP_1, GDOP_2, \cdots, GDOP_j\right) = \frac{1}{N}\sum_{j=1}^{N} GDOP_j \tag{10}$$

$$\mathrm{F} = \min \frac{1}{N}\sum_{j=1}^{N} GDOP_j \tag{11}$$

3.2 Control Variable

Taking the RN D coordinate (x_4, y_4) as the control variable of the optimal geometric layout model, the optimal solution of RN D coordinate, namely (x_m, y_m) , can be obtained by solving Eq. (11).

3.3 Restraint Condition

In this paper, the positioning area is a square with an area of 6*6, so RN D is divided into squares with an area of 3*3, which accounts for 1/4 of the area of the positioning area.

The constraints of RN D are as follows:

$$\begin{cases} -3 \leq x_4 \leq 0 \\ \ \ 3 \leq y_4 \leq 6 \end{cases} \tag{12}$$

3.4 Finite Impulse Response Filtering Algorithm

KF is usually used to process data in satellite positioning and navigation. However, this kind of filter has a disadvantage. In order to obtain accurate estimation of position information and state variables, it is necessary to have an accurate noise statistic for system noise in the state equation and observation noise in the observation equation to improve the accuracy of the system model. This accurate noise statistics is difficult to achieve in the real environment. The FIR filter selected in this paper is less affected by noise, has stable linear phase, and does not need feedback. Research shows that the robustness of FIR filter is higher than that of KF in the presence of external interference.

The state equation of the system is as follows:

$$\underbrace{\begin{bmatrix} P_{x,t} \\ V_{x,t} \\ P_{y,t} \\ V_{y,t} \end{bmatrix}}_{\mathbf{x}_t} = \underbrace{\begin{bmatrix} 1 & \nabla t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \nabla t \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{F}_{t-1}} \underbrace{\begin{bmatrix} P_{x,t-1} \\ V_{x,t-1} \\ P_{y,t-1} \\ V_{y,t-1} \end{bmatrix}}_{\mathbf{x}_{t-1}} + \mathbf{W}_t \tag{13}$$

where, $P_{x,t}$ and $P_{y,t}$ are the positions of BNs at time t, and $V_{x,t}$ and $V_{y,t}$ are the speeds of BNs at time t, and $\mathbf{W}_t \sim N\left(0, \mathbf{Q}_t\right)$ is the system noise.

The original observation value is distance. The location information of the target is obtained from the original distance data by least squares solution. This location information is used as the observation value in the observation equation, so the following observation equation can be obtained:

$$\underbrace{\begin{bmatrix} P'_{x,t} \\ P'_{y,t} \end{bmatrix}}_{\mathbf{Y}_t} = \underbrace{\begin{bmatrix} 1\,0\,0\,0 \\ 0\,0\,1\,0 \end{bmatrix}}_{\mathbf{M}_t} \underbrace{\begin{bmatrix} P_{x,t} \\ V_{x,t} \\ P_{y,t} \\ V_{y,t} \end{bmatrix}}_{\mathbf{x}_t} + \mathbf{V}_t \tag{14}$$

where $P'_{x,t}$ and $P'_{y,t}$ are the observation positions of BNs on X and Y axes at time t, and $\mathbf{V}_t \sim N\left(0, \mathbf{R}_t\right)$ are the observation noise. Pseudo code with algorithm 1 as FIR filter.

Algorithm 1: FIR filter

Input: $\mathbf{Y}_t$,W,b
Output: $\hat{\mathbf{X}}_t$
for $t = W - 1 : number$ **do**
  $m = t - W + 1, s = m + b - 1$
  $\hat{\mathbf{X}}_s = \begin{cases} \mathbf{Y}_s, & \text{if } s < W - 1 \\ \tilde{\mathbf{X}}_s, & \text{if } s \geq W - 1 \end{cases}$
  $\mathbf{G}_s = \mathbf{I}$
  for $s = m + b : t$ **do**
    $\tilde{\mathbf{X}}_{s|s-1} = \mathbf{F}_s \tilde{\mathbf{X}}_{s-1}$
    $\mathbf{G}_s = \left[\mathbf{M}_s^T \mathbf{M}_s + \left(\mathbf{F}_s \mathbf{G}_{s-1} \mathbf{F}_s^T\right)^{-1}\right]^{-1}$
    $\mathbf{K}_s = \mathbf{G}_s \mathbf{M}_s^T$
    $\tilde{\mathbf{X}}_s = \tilde{\mathbf{X}}_{s|s-1} + \mathbf{K}_s \left[\mathbf{Y}_s - \mathbf{M}_s \left(\tilde{\mathbf{X}}_{s|s-1}\right)\right]$
  end
  $\hat{\mathbf{X}}_t = \tilde{\mathbf{X}}_s$
end

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4 Experimental Simulation Analysis

In the experimental simulation, four RNs are used to locate the BNs. Firstly, RN A, RN B, RN C are used as fixed points, and RN D is used as moving points. In the given constraints, the best coordinate point of RN D is selected according to the criterion of minimum mean GDOP. Then, the coordinate points of the four RNs are substituted into the least squares algorithm, and the coordinate values of the BNs are calculated. Then, the FIR filter is used to remove the noise of the BNs coordinates to further improve the positioning accuracy.

The positioning area is set to a square with an area of 6*6, so the constraining area of base station D is set to a square with an area of 3*3, and the constraining area is gridded evenly into 900 subareas, that is, there are 900 sample points in the constraining area of RN D. By substituting the 900 sample points into Eq. (9), an array of 900*1 is obtained, and then the minimum mean GDOP is filtered out by Eq. (11). The corresponding solution is that we want to get RN D.

The software simulation shows that the RN coordinates selected according to the minimum mean GDOP and the addition of FIR filter can improve the performance of the positioning system. You can see the error CDF diagrams in Fig. 3. Table 1 shows the RMSEs for UWB and FIR in different mean GDOPs.

Figure 1 shows the simulation when the mean GDOP reaches the maximum. The result is 1.5718, and the coordinate value of the corresponding RN D is (−2.4828, 4.2414).

Figure 2 shows the simulation when the mean GDOP reaches the maximum. The result is 1.7497, and the coordinate value of the corresponding RN D is (−0.10345, 6).

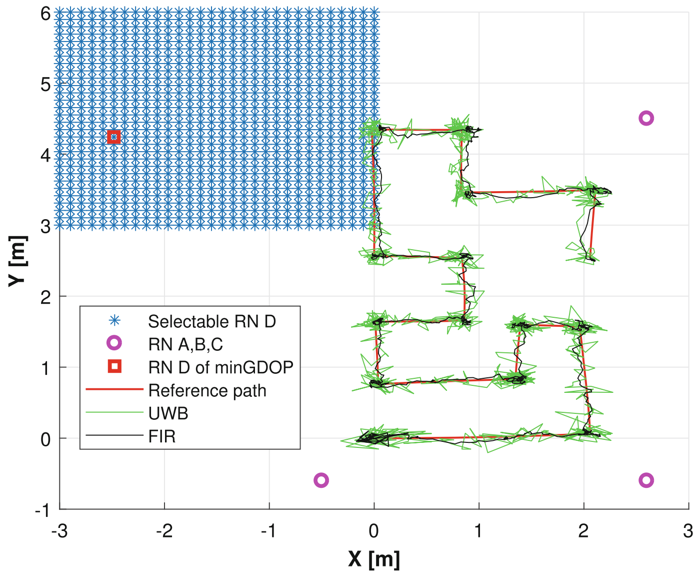

Fig. 1. Mean GDOP = 1.5718.

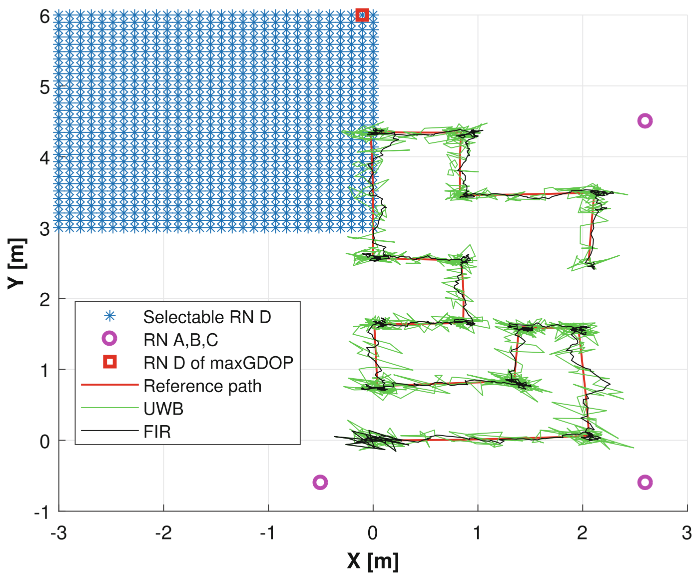

Fig. 2. Mean GDOP = 1.7497

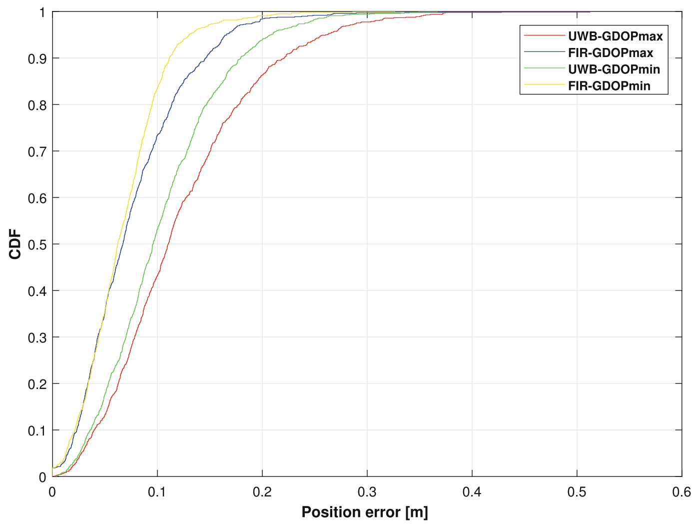

Fig. 3. CDFs of UWB and FIR under different mean GDOP.

Table 1. The RMSE(m) generated by FIR and UWB under different mean GDOP

Mean GDOP	RMSE	
	UWB	FIR
1.5718	0.11814	0.079218
1.7497	0.14112	0.089583

5 Conclusion

First of all, this paper analyzes the positioning error of the system. Secondly, on this basis, the concept of minimum mean GDOP is proposed, which is used to measure the geometric layout of RNs, and then the optimal layout is selected. Finally, FIR filter is introduced to filter the observed values. It can be seen that the addition of FIR filter has played a positive role in improving the positioning accuracy.

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