An Extended TODIM Method for Multi-criteria Decision Making Under q-Rung Orthopair Fuzzy Environment

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Abstract. Multi-criteria fuzzy decision making theory is one of the important tools to solve modern decision making problems. Facing complex and changeable decision making problems in real life, decision makers evaluate and quantify various decisions based on expert index system. They evaluate and rank options through a series of methods to produce scientific and reasonable results. The purpose of this paper is to propose a new q-rung orthopair fuzzy number (q-ROFN) ranking method based on the analysis of the existing q-ROFN ranking methods, and apply this new proposal in the TOMID decision making method.

Keywords: q-rung orthopair fuzzy number · Multi-criteria decision making (MCDM) · Ranking function

1 Introduction

Decision making is essential to people's daily lives, Such as the choice of clothing, food, lifestyle and even major national decisions. How do people make decisions? Ancients often relied on their previous experience to make decisions, but in current society, Decisions are made based on various considerations. In this case, even for a simple decision, needs a comprehensive consideration, and all these considerations constitute the criteria for making certain decision. Due to the complexity of real-world problems and the limitations of the cognitive level of experts, evaluating quasi is not completely objective, there is widespread uncertainty and hesitation when considering problems. In 1965, Zadeh [1] proposed the theory of Fuzzy set (Fuzzy set, FS), the membership degree

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$\mu(x)$ indicates the degree of support for an attribute of x. This is used to quantify the options and help make decisions. In 1970, Bellman and Zadeh [2] introduced fuzzy set theory into multi-criteria decision making, they proposed the concepts and models of fuzzy decision making analysis and used this theory to solve the problem of uncertainty in practical decision making. However, the single membership μ of the fuzzy set can only represent the content of "if it is non-black $(1 - \mu(x))$ then it be white $(\mu(x))$", so it cannot describe a neutrality state. Due to this limitation, in 1983, the Bulgarian scholar Atanassov [3] proposed the concept of Intuitionistic Fuzzy Set (Intuitionistic Fuzzy Set, IFS). In contrast to the fuzzy set with a single scale measurement, Atanassov introduces the notion of non-membership degree $\nu(x)$ and the hesitation degree, and defined that an IFS must meet the sum of membership, non-membership, and hesitation degree $\pi(x)$ is less than or equal to 1. Thus, the IFS can simultaneously express support (membership degree) $\mu(x)$, , against (non-membership degree) $\nu(x)$, , and no support or against (hesitation degree) $\pi(x)$. To enrich the expression form of the criterion, Atanassov [4] proposed the concept of Interval-valued intuition fuzzy sets (Interval-valued intuition fuzzy sets, IVIFS). It means we can represent membership and non-membership in the form of interval values. The introduction of this concept increases the validity of decision making and attracts attention from many experts and scholars [5, 6] who apply it to teaching practice [7], smart medical [8], financial investment [9], evaluation system [10] and other fields [11]. But this expression form also has some limitations in the range of representation of the data. To expand the range of expression of intuitionistic fuzzy numbers, Yager [12] introduced the concept of Pythagorean fuzzy set (Pythagorean fuzzy set, PFS), and defined that the sum of the square of membership degree and non-membership is less than or equal to 1. Yager extended the triangular region of the intuitive fuzzy set to the sector region of the Pythagoras fuzzy set. The proposal of this concept provided an extended direction for the study of fuzzy sets. Yager [13] proposed a series of operators including weighted average operators, weighted geometry operators, weighted power average operators, weighted power geometric operators, and ordered weighted average operators of PF, to solve MCDM problems. Gou et al. [14] defined the PF functions, and further investigated the continuity, derivability, and differentiability of the PF functions, and enriched the PFS theory. Peng and Yang [15] defined some new operating methods of Pythagoras fuzzy clustering operators, and discussed their boundedness, idempotent, and monotonicity, etc. With the abundance of PF related operators, the MCDM related with PF was widely awakened by experts. Zhang and Xu [16] proposed new ranking methods that expand the TOPSIS decision making method to solve the MCDM problem by computing the distance to the positive and negative ideal solutions. Zhang [17] innovated the idea of relative progress and proposed a ranking method of the Pythagorean fuzzy numbers and the Interval-valued Pythagorean fuzzy numbers (Interval-valued Pythagorean fuzzy numbers, IVPFNs). Ho et al. [18] proposed a Pearson-based association measure method to obtain the ranking results by scoring the interval-valued Pythagoras fuzzy numbers. And Qiu [19] proposed a new ranking method about Interval-valued Pythagoras fuzzy ranking methods. These methods enrich the fuzzy decision making theory, broaden the application scope of fuzzy decision making, and greatly enhance the vitality of fuzzy decision making.

Although the PF fuzzy set extends the triangular region of the intuitive fuzzy set to the sector region. The Pythagoras fuzzy set still has some limitations. For example, the number (0.8, 0.7) goes beyond the scope of the Pythagorean fuzzy set, $0.8^2 + 0.7^2 > 1$. Therefore, in 2017, Yager [20] proposed the concept of Orthopair Fuzzy Set (Orthopair Fuzzy Set, OFS) and Interval-Valued Orthopair Fuzzy Set (Interval-Valued Orthopair Fuzzy Set, IVOFS). The q-rung orthopair fuzzy set extended the range of membership and non-membership to square regions of edge length 1, thus it greatly expanded the scope of the research. Yager [20] proposed the concept of q-rung Orthopair Fuzzy Set (q-rung Orthopair Fuzzy Set, or q-ROFS), which was a generalization of the fuzzy set. When $q = 1$, it represents the intuitive fuzzy set; when $q = 2$, it indicates the Pythagorean fuzzy set; When $q \geqslant 3$, it can indicate the (0.8, 0.7) and more fuzzy numbers. So far, some scholars have conducted some theoretical studies on q-ROFS [21–23].

To provide practical decision making schemes, experts and scholars have conducted specific studies on the ranking problem of q-ROFNS. Yager [20] proposed a score function based on membership and non-membership, but since it was difficult for this method to distinguish the degree of merits and demerits of two q-ROFNs in some cases, Yager proposed a further precise function in [24], Post-supplementary sequencing method. This method overcame the problem of failing to distinguish certain fuzzy numbers in [20], but there are still deficiencies in the judgment of the advantages and disadvantages of fuzzy numbers. Thus, Liu and Wang [25] proposed a score function for the sum of informative quantities. Peng [26] improved Yager's ranking method, To avoid the loss of information, Peng et al. aggregated the hesitation value information and proposed new scoring functions and ranking methods, but this method Could not compare two fuzzy numbers with the same membership and non-membership values. And this problem also exists in [27]. In order to solve the above problem, Mi [28] defined a score function for q-ROF environments, Peng [29] added the hesitation degree to the operation and Wang [30] proposed a new ranking method to rank q-ROFNs with high accuracy, in which they solved this problem to different degrees. However, the change in q-values in the q-ROFNS environment, when calculated by these three methods, will cause the completely opposite results. These methods still need to be perfected. Therefore, more research is necessary in the q-ROFNS ranking problem.

In terms of expanding decision making methods, Garg [31] combined the AHP and TOPSIS fuzzy decision making theory to present a new solution to the interval value q-ROFS fuzzy decision problem. But in terms of q-ROF fuzzy decision making issues, the current study only extends it to the TOPSIS method, and the problem with the TOPSIS method is its failure to reflect the proximity of each solution to the ideal solution. TODIM reflects experts' different preferences for earnings and loss. It constructs the advantage matrix of pairwise comparison in which the attribute advantages of each candidate are combined to be the basis of further ranking of these candidates. As we can see, TODIM decision method provides a ranking method based on prospect theory. This ranking method makes the decision more rational. This prospect theory considers that the judgment of decision making experts may have certain limitations at the rational level, and this consideration reduces the risk caused by experts misjudgment and then, more and more experts and scholars started studying the TODIM approach [32–35].

Through a review of the above studies, we found that the ranking function for q-ROF environments have partial drawbacks and decision making methods which suitable for q-ROF environments are yet to be developed. This paper will focus on the relevant ranking method and the distance formula of the q-ROFNs. The main contributions of this article are as follows:

1) proposed a new q-ROFNs ranking method and based on the analysis of the existing q-ROFNs ranking method;
2) Defined a new distance measure for q-ROFNs;
3) The TODIM method is extended to q-ROFNs environment by combing traditional TODIM with the proposed ranking method and the distance formula.

The rest of the paper is organized as follows: Sect. 2 mainly reviews the basic concepts of q-ROFS and introduces some existing research foundations; Sect. 3 presents a new q-ROFNS-based scoring function and ranking method for information reliability; Sect. 4 presents new distance formulas based on q-ROFNs; Sect. 5 generalizes the TODIM approach to q-ROF decision making environments.

2 Preliminary

This section introduces some relevant definitions and computational rules of q-ROFS, and introduces a traditional distance formula. These basic theories lay the theoretical basis for the study of q-ROFS.

Definition 1. [20] Let X be an ordinary set, then a q-ROFS A defined on X is given by

$$A = \langle x, \mu_A(x), \nu_A(x) | x \in X \rangle \tag{1}$$

where $\mu_A(x)$ and $\nu_A(x)$ are the membership and non-membership degrees, respectively, and $0 \leqslant \mu_A(x) \leqslant 1, 0 \leqslant \nu_A(x) \leqslant 1$ and $0 \leqslant \mu_A(x)^q + \nu_A(x)^q \leqslant 1$. And $\pi_A(x)$ is the hesitancy degree, $0 \leqslant \pi_A(x) \leqslant 1$ and $\pi_A(x) = (1 - (\mu_A(x)^q + \nu_A(x)^q))^{1/q}$.

Definition 2. [25] Given three q-ROFNs $A = (\mu, \nu), A_1 = (\mu_1, \nu_1)$ and $A_2 = (\mu_2, \nu_2)$, and they satisfy the following operational rules.

1. $A_1 \oplus A_2 = \left(\left(\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q \right)^{1/q}, \nu_1 \nu_2 \right)$
2. $A_1 \otimes A_2 = \left(\mu_1 \mu_2, \left(\nu_1^q + \nu_2^q - \nu_1^q \nu_2^q \right)^{1/q} \right)$
3. $\lambda A = \left(\left(1 - (1 - \mu^q)^\lambda \right)^{1/q}, \nu^q \right)$
4. $A^\lambda = \left(\mu^\lambda, \left(1 - (1 - \nu^q)^\lambda \right)^{1/q} \right)$

Where, λ is a positive real number.

Definition 3. [36] Let $A(\mu_A, \nu_A)$ and $B(\mu_B, \nu_B)$ be any two q-ROFNs, And their hesitancy degree are $\pi_A(x) = (1 - (\mu_A(x)^q + \nu_A(x)^q))^{1/q}$ and $\pi_B(x) = (1 - (\mu_B(x)^q + \nu_B(x)^q))^{1/q}$ respectively, then the hamming distance measure of two q-ROFNs A and B is defined as follows.

$$d(A, B) = \frac{1}{2}\left(\left|\mu_A(x)^q - \mu_B(x)^q\right| + \left|\nu_A(x)^q - \nu_B(x)^q\right| + \left|\pi_A(x)^q - \pi_B(x)^q\right|\right) \quad (2)$$

These definitions are augmented based on the original Pythagoras fuzzy set. By doing so, the fuzzy set theory and its application are being enriched. Meanwhile, this augmented distance formula transforms abstract data comparisons into metrics on vectors. Both of them provide a theoretical basis for the study of fuzzy decision-making.

3 A New Ranking Method for q-rung Orthopair Fuzzy Numbers

Based on the above study analysis, we learn that the existing scoring function still has some problems in dealing with the q-rung orthopair fuzzy ordering problem of order q-ROFNS. For example, 1) Some score functions contradict common sense, thus the accuracy is not high; 2) Some scoring functions are exactly the opposite of all other methods, thus reliability is insufficient; 3) Some scoring functions do not adapt to the change of q value in q-ROFN and its stability needs to be improved. As we can see, there are still some problems needs to be overcome. So, to address the above problems, we propose new scoring functions and ranking method.

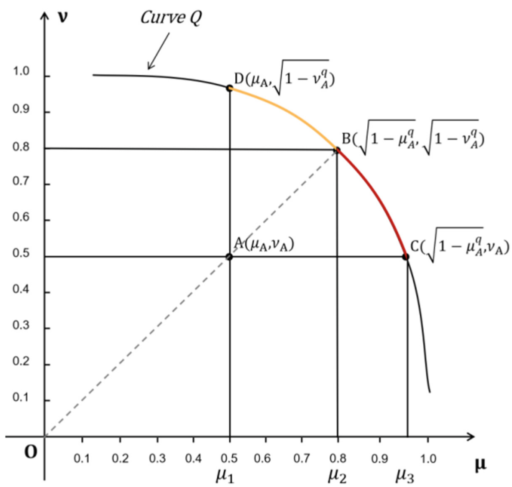

Fig. 1. Geometric illustration of the proposed ranking method

As shown in Fig. 1, The values of the q-rung orthopair fuzzy number of order q is presented on the right axis of the plane. Curve Q indicates the q-ROFNs with a hesitation degree of 0, and the value of q-ROFNs is $\mu^q + \nu^q - 1 \leqslant 0$. For any two points

$A = (\mu_A, \nu_A)$ and $F = (\mu_F, \nu_F)$ in the range of values, take point A as an example. It contains information of membership degree μ_A (μ axis) and ν_A (ν axis). This information is expanded to project the curve Q in the μ-axis direction to obtain point C, and point D is obtained by making the projection of curve Q in the axis direction of ν. In this way, this information is expanded to obtain the length of arc BD, arc BC, and arc CD. The advantage of this practice is that we can obtain more effective information by reasonably mining the potential value of existing data. By obtaining the length of line segment through arc differential equation, the comparison of abstract fuzzy number can be transformed into visual length comparison, which improves the readability and understandability of fuzzy number comparison. Membership (μ-axis) and non-membership (ν-axis) represent different meanings respectively. The greater the degree of membership (or non-membership), the better the value. Following this principle, this method fully extracts the information of q-order orthogonal fuzzy numbers, and makes some improvements (μ) and ($1/\nu$) Adjustment in direction, and according to this condition, reasonable comparison and ranking methods are proposed to obtain the score value $S(A)$ of point A. Similarly, we can obtain the score value $S(F)$ of point F. The final ranking is obtained by comparing the sizes of $S(A)$ and $S(A)$.

Take point A as an example, the specific calculation steps and calculation methods are shown below.

As Fig. 1 shows, Q is a curve $\mu^q + \nu^q - 1 = 0, A = (\mu_A, \nu_A)$ is a q-ROFN, μ_A is membership degree of A, ν_A is non-membership degree of A.

1) Over point A draw a parallel line of axis X which meets curve Q at point C. The coordinate of point C is,

$$\left(\sqrt[q]{(1-\nu_A^q)}, \nu_A\right) \tag{3}$$

2) Similarly, over point A draw a parallel line of axis X, meeting curve Q at point D. The coordinate of point D is,

$$\left(\mu_A, \sqrt[q]{(1-\mu_A^q)}\right) \tag{4}$$

3) Connect point O and point A, then line OA means $L : \nu = \nu_A/\mu_A \times \mu$ (A straight line over the origin). Draw the extension line of OA and meet curve Q at point B, the coordinate of point B is,

$$\left(\frac{\mu_A}{\sqrt[q]{\mu_A^q + \nu_A^q}}, \frac{\nu_A}{\sqrt[q]{\mu_A^q + \nu_A^q}}\right) \tag{5}$$

When point A is located on curve Q, the hesitancy degree of q-ROFN A is 0. Then there is no intersection point B, C, D. Therefore, the distance of point A from curve Q is 0. Point A has the highest degree of information reliability, and the larger of μ_A, the better the q-ROFN A.

When point A is not on the curve Q, according to the principle of arc length, the farther point A is from curve Q, the lower the reliability and the longer the arc length CD. Based on this, we use arc-length CD as a measure to represent the information reliability of q-ROFN A.

$$L_{CD} = L_{BC} + L_{BD} \tag{6}$$

where, L_{CD} is the length of arc CD, L_{BD} is the length of arc BD, L_{BC} is the length of arc BC. Use the arc differential formula,

$$L_{BD} = \int_{\mu_s}^{\mu_v} \sqrt{1 + (f'(\mu))^2} d\mu \tag{7}$$

$$L_{BC} = \int_{\mu_c}^{\mu_s} \sqrt{1 + (f'(\mu))^2} d\mu \tag{8}$$

where, $f'(\mu)$ indicates the log μ for the derivative, and $\int_{\mu_B}^{\mu_D} X$, $\int_{\mu_C}^{\mu_B} X$ express the Integrals of arcBD and arcBC in the direction. The equation $L = \int_{x_2}^{x_1} \sqrt{1 + (f'(x))^2} dx$ is the arc differential equation.

And, the smaller of non-membership degree, the better the q-ROFN is. Considering this factor, we proposed a new score function.

Definition 4. A new ranking method is proposed.

$$S = \frac{\mu + 1}{\frac{\nu^* L_{CD}}{L} + 1} \tag{9}$$

where L is the total length of the curve Q. For two q-ROFNs $q_1\left(\mu_{q_1}, \nu_{q_1}\right)$, $q_2\left(\mu_{q_2}, \nu_{q_2}\right)$, the ranking methods are as follows.

$$\begin{aligned} &IF\ S(q_1) > S(q_2), THEN\ q_1 \succ q_2 \\ &IF\ S(q_1) < S(q_2), THEN\ q_1 \prec q_2 \\ &IF\ S(q_1) = S(q_2), THEN\ q_1\ q_2 \end{aligned} \tag{10}$$

4 A New Distance Measure for q-rung Orthopair Fuzzy Numbers

The study of the existing distance formulas is based on the general fuzzy numbers, while the q-ROFNs expand the expression range of the data infinitely, and the existing distance formulas have some limitations to both the accuracy of the data and the completeness of the information. Therefore, it is very necessary to propose a distance formula adapted to q-ROFNs environment.

Definition 5. Because of the development of fuzzy numbers, previous experts and scholars have extended the expression of q-ROFNs. To fit the expression of the q-ROFNs, we also extend the distance formula to order q. The proposed distance formula is as follows.

$$d(q_1, q_2) = \sqrt[q]{\frac{1}{2}\left(\left|\sqrt[q]{\mu_{q_1}^q - \mu_{q_2}^q}\right| + \left|\sqrt[q]{\nu_{q_1}^q - \nu_{q_2}^q \ominus}\right. + \left|\sqrt[q]{\pi_{q_1}^q - \pi_{q_2}^q \ominus}\right)\right.} \tag{11}$$

The proposed distance formula satisfies the following three basic theorems,
Let $A_1 = (\mu_1, \nu_1)$ and $A_2 = (\mu_2, \nu_2)$ be two q-ROFNs, then
Property 1 $d(A_1, A_2) = d(A_2, A_1)$
Property 2 $d(A_1, A_2) = 0$ only if $A_1 = A_2$
Property 3 $0 \leqslant d(A_1, A_2) \leqslant 1$
Proof. For two q-ROFNs $A_1 = \left(\mu_{A_1}, \nu_{A_1}\right)$ and $A_2 = \left(\mu_{A_2}, \nu_{A_2}\right).\mu_{A_1}, \mu_{A_2}, \nu_{A_1}, \nu_{A_2} \in [0, 1][0, 1]$, and $0 \leqslant \mu_{A_1}^q + \nu_{A_1}^q \leqslant 1, 0 \leqslant \mu_{A_2}^q + \nu_{A_2}^q \leqslant 1$. Then,

$$d(A_1, A_2) = \sqrt[q]{\frac{1}{2}\left(\left|\sqrt[q]{\mu_{A_1}^q - \mu_{A_2}^q}\right| + \left|\sqrt[q]{\nu_{A_1}^q - \nu_{A_2}^q}\right| + \left|\sqrt[q]{\pi_{A_1}^q - \pi_{A_2}^q}\right|\right)} = \sqrt[q]{\frac{1}{2}\left(\left|\sqrt[q]{\mu_{A_2}^q - \mu_{A_1}^q}\right| + \left|\sqrt[q]{\nu_{A_2}^q - \nu_{A_1}^q}\right| + \left|\sqrt[q]{\pi_{A_2}^q - \pi_{A_1}^q}\right|\right)} = 0$$

Let $d(A_1, A_2) = \sqrt[q]{\frac{1}{2}\left(\left|\sqrt[q]{\mu_{A_1}^q - \mu_{A_2}^q}\right| + \left|\sqrt[q]{\nu_{A_1}^q - \nu_{A_2}^q}\right| + \left|\sqrt[q]{\pi_{A_1}^q - \pi_{A_2}^q}\right|\right)} = 0$, it must be $\left|\sqrt[q]{\mu_{A_1}^q - \mu_{A_2}^q}\right| = \left|\sqrt[q]{\nu_{A_1}^q - \nu_{A_2}^q}\right| = \left|\sqrt[q]{\pi_{A_1}^q - \pi_{A_2}^q}\right| = 0$, only when $\mu_{A_1} = \mu_{A_2}, \nu_{A_4} = \nu_{A_2}, \pi_{A_4} = \pi_{A_2}$ then the equations will only be established.

$$\begin{aligned} d(A_1, A_2) &= \sqrt[q]{\frac{1}{2}\left(\left|\sqrt[q]{\mu_{A_1}^q - \mu_{A_2}^q}\right| + \left|\sqrt[q]{\nu_{A_1}^q - \nu_{A_2}^q}\right| + \left|\sqrt[q]{\pi_{A_1}^q - \pi_{A_2}^q}\right|\right)} \\ &= \sqrt[q]{\frac{1}{2}\left(\left|\sqrt[q]{\mu_{A_1}^q - \mu_{A_2}^q}\right| + \sqrt[q]{\nu_{A_1}^q - \nu_{A_2}^q}|| + \left|\sqrt[q]{\left(1 - \mu_{A_1}^q - \nu_{A_1}^q\right) - \left(1 - \mu_{A_2}^q - \nu_{A_2}^q\right)}\right|\right)} \\ &= \sqrt[q]{\frac{1}{2}\left(\left|\sqrt[q]{\mu_{A_1}^q - \mu_{A_2}^q}\right| + \sqrt[q]{\nu_{A_1}^q - \nu_{A_2}^q}|| + \left|\sqrt[q]{\left(\mu_{A_2}^q - \nu_{A_1}^q\right) + \left(\mu_{A_2}^q - \nu_{A_1}^q\right)}\right|\right)} \\ &\leqslant \sqrt[q]{\frac{1}{2}\left(\left|\sqrt[q]{\mu_{A_1}^q}\right| + \left|\sqrt[q]{\nu_{A_1^q}^q}\right| + \left|\sqrt[q]{\mu_{A_2}^q + \nu_{A_2}^q}\right|\right)} \leqslant \sqrt[q]{\frac{q}{2}\left(\mu_{A_1}^q + \nu_{A_1}^q + \mu_{A_2}^q + \nu_{A_2}^q\right)} \leqslant 1 \end{aligned}$$

And it is obviously that $d(A_1, A_2) \geqslant 0$.

The distance formula proposed in this paper expands the data table widely based on the traditional distance, and it makes full use of precision of the data and the completeness of the information, So the calculation of the orthogonal fuzzy number distances of the two q orders can be more accurate.

5 The Extended TODIM Method Under q-rung Orthopair Fuzzy Environment

The TODIM method is the method proposed by Gomes [37]. It does not need to determine the reference point and it fully considers expert mindset. This method ranks candidates by calculating their relative advantages. The approach is defined as follows.

Let candidate set $A = \{A_1, A_2, \cdots, A_m\}$ as a collection of m options; C is a collection of n evaluation criteria. It is presented as $C = \{C_1, C_2, \cdots, C_n\}$ v. And the evaluation of the criteria weight vectors is s$\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}^T$, where $\sum_{j=1}^{n} \omega_j = 1, 0 \leqslant \omega_j \leqslant 1$. we define $\omega_{jr} = \omega_j/\omega_r$ as the relative weight of the criterion C_j to C_r, and $\omega_r = \max\left\{\omega_j | j = 1, 2, \cdots, n\right\}$.

The evaluation value of candidate A on criteria C is expressed by $a_{ij} = (\mu_A, \nu_A)$, $(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$. Where $0 \leqslant \mu_A^q \leqslant 1, 0 \leqslant \nu_A^q \leqslant 1, 0 \leqslant \mu_A^q + \nu_A^q \leqslant 1$. Then the classical TODIM approach involves the following steps.

Step 1. Construct the decision making matrix $X = \left[a_{ij}\right]_{m \times n}$.
Step 2. Consider experts' psychological behavior (i.e., the reference dependence and loss aversion). Calculate the relative dominance degree $\phi_j = (A_i, A_r)$ of scheme A_i to scheme A_r under each criterion C_j.

$$\phi_j(A_i, A_r) = \begin{cases} \sqrt{\omega_{jr} * \left(a_{ij} - a_{rj}\right) / \sum_{j=1}^{n} \omega_{jr}} & IF\ a_{ij} - a_{rj} > 0 \\ 0 & IF\ a_{ij} - a_{rj} = 0 \\ -\frac{1}{\theta}\sqrt{\left(\sum_{j=1}^{n} \omega_{jr}\right) * \left(a_{ij} - a_{rj}\right)/\omega_{jr}} & IF\ a_{ij} - a_{rj} < 0 \end{cases} \tag{12}$$

In the above dominance degree formula, θ is an attenuation factor of losses with a range of $0 < \theta < \sum_{j=1}^{n} \omega_j / \omega_r$ [38]. The smaller the value θ, the higher the experts' aversion to loss.

Step 3. According to the Dominance Degree Matrix, the Overall Dominance $\phi_j = (A_i, A_r)$ of the Scheme A_i for A_r Can Be Calculated.

$$\phi(A_i, A_r) = \sum_{j=1}^{n} \phi_j(A_i, A_r), j \in N, r \in N \tag{13}$$

Step 4. Comprehensively calculate the overall advantage of candidate A_i and other candidates $\Phi(A_i)$, and rank the candidates according to the size of $\Phi(A_i)$.

$$\Phi(A_i) = \frac{\left(\sum_{r=1}^{n} \phi(A_i, A_r) - \min\left\{\sum_{r=1}^{n} \phi(A_i, A_r)\right\}\right)}{\max\left\{\sum_{r=1}^{n} \phi(A_i, A_r)\right\} - \min\left\{\sum_{r=1}^{n} \phi(A_i, A_r)\right\}} \tag{14}$$

To accommodate the complexity of the data when solving practical problems, the proposed method extends every link of the TODIM decision making method process to order q. Firstly, it proposes a ranking comparison of q-rung orthopair fuzzy numbers of order q, then it proposes distance formulas that can accommodate the q-ROFS. Based on this, this paper integrates this approach with the traditional TODIM methods, and it proposes an extended TODIM decision making method based on q-rung orthopair fuzzy decision making of q-ROFS.

Step 1. Based on the decision making scheme, the main evaluation criteria, decision making experts evaluate different candidates A according to criterion C. Then we get the decision making matrix $X_q = \left[a_{ij}\right]_{m \times n}$, $a_{ij} = (\mu_A, \nu_A)(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$. It is defined as follows.

$$X_q = \left(a_{ij}\right)_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array} \tag{15}$$

Step 2. The relative weights of C_j were calculated based on the different weight values for each evaluation criterion.

$$\omega_{jr} = \omega_j/\omega_r, j, r = 1, 2 \ldots\ldots n \tag{16}$$

Where ω_j is the weight of the criterion C_j, $\omega_r = \max\{\omega_j \ominus \mathrm{j} = 1, 2, \cdots, \mathrm{n}\}$, and $0 \leqslant \omega_r \leqslant 1$

Step 3. Standardize the evaluation criteria and assign scores to each data metric using the proposed score function, then get the score matrix $S_q = [S_{ij}]_{m\times n}$, S_{ij} is a score value.

$$S_q = (s_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mn} \end{bmatrix} \end{array} \tag{17}$$

Step 4. Calculate the dominance degree under each criterion, and use the scoring matrix to make judgment and get a dominance degree matrix.

$$\phi_j(A_i, A_r) = \begin{cases} \sqrt{\omega_{jr} * d(a_{ij} - a_{rj}) / \sum_{j=1}^{n} \omega_{jr}} & IF\ s_{ij} - s_{rj} > 0 \\ 0 & IF\ s_{ij} - s_{rj} = 0 \\ -\frac{1}{\theta}\sqrt{\left(\sum_{j=1}^{n} \omega_{jr}\right) * d(a_{ij} - a_{rj}) / \omega_{jr}} & IF\ s_{ij} - s_{rj} < 0 \end{cases} \tag{18}$$

Consider the two matrices, X_q and S_q. If $s_{ij} > s_{rj}$, then the dominance degree of $\phi_j = (A_i, A_r)$ will be represented as $\sqrt{\omega_{jr} \times d(a_{ij}, a_{rj}) / \sum_{j=1}^{n} \omega_{jr}}$, where $d(a_{ij}, a_{rj})$ is the distance formula for the expansion mentioned above. And if $s_{ij} = s_{rj}$, the dominance degree between a_{ij} and a_{rj} is zero.

From this, derive a dominance matrix of the j^{th} criterion. And it is shown as follows.

$$\phi_j = \left[\phi_j(A_i, A_r)\right]_{(m\times m)} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} A_1 \; A_2 \; \cdots \; A_n \\ \begin{bmatrix} 0 & \phi_j(A_1, A_2) & \ldots & \phi_j(A_1, A_m) \\ \phi_j(A_2, A_1) & 0 & \ldots & \phi_j(A_2, A_m) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_j(A_n, A_1) & \phi_j(A_n, A_2) & \ldots & 0 \end{bmatrix} \end{array} \tag{19}$$

Step 5. Calculate the overall dominance $\varphi(A_i)$ of each candidate A_i over the candidate A_i, the calculation method is as follows.

$$\varphi(A_i, A_t) = \sum_{j=1}^{m} \phi_j(A_i, A_t), i = 1, 2 \ldots . m \tag{20}$$

The overall dominance matrix is obtained as follows.

$$\varphi = [\varphi(A_i, A_t)]_{(m\times m)} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} A_1 \; A_2 \; \cdots \; A_n \\ \begin{bmatrix} 0 & \varphi_j(A_1, A_2) & \ldots & \varphi_j(A_1, A_m) \\ \varphi_j(A_2, A_1) & 0 & \ldots & \varphi_j(A_2, A_m) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_j(A_n, A_1) & \varphi_j(A_n, A_2) & \ldots & 0 \end{bmatrix} \end{array} \tag{21}$$

Step 6. The Final Decision Making Value.

$$\Gamma(A_i) = \frac{\left(\sum_{r=1}^{n} \varphi(A_i, A_t) - \min\left\{\sum_{r=1}^{n} \varphi(A_i, A_t)\right\}\right)}{\max\left\{\sum_{r=1}^{n} \varphi(A_i, A_t)\right\} - \min\left\{\sum_{r=1}^{n} \varphi(A_i, A_t)\right\}} \tag{22}$$

To clearly illustrate the idea of the proposed method, we make a flow chart as shown in Fig. 2.

Compared to the traditional TODIM decision making method, the proposed method in this paper firstly innovates the score function in calculating the relative dominance degree, and it uses the geometric significance and connection between fuzzy numbers to propose a ranking method based on information reliability to compare the advantages and disadvantages of two fuzzy numbers; Secondly, this paper proposes a distance formula more adapted to the q-rung orthogonal fuzzy environments, and uses this distance formula to calculate the relative advantage matrix to increase the objectivity of the method; Finally, this paper extends the traditional TODIM decision method to the q-rung orthogonal fuzzy environment, and proposes the extended q-rung orthogonal fuzzy TODIM decision method by combining the proposed ranking method and the distance formula, and applies it to the decision making method.

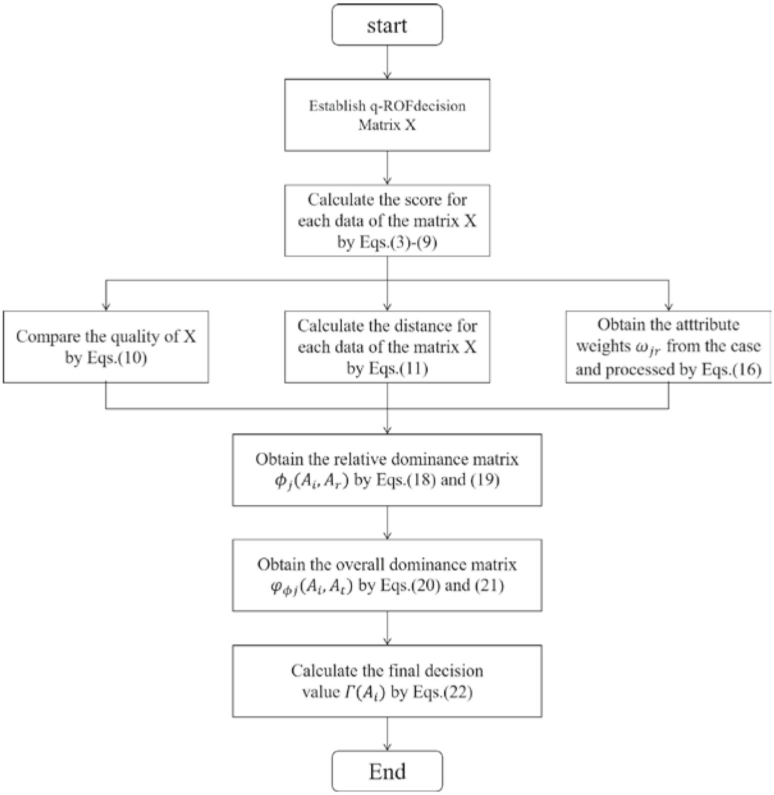

Fig. 2. Flow chart of decision making process

6 Conclusions

Based on the analysis of previous q-ROFNs ranking method, this paper proposes a new q-ROFNs ranking method by eliminating some drawbacks of the former. Besides, this paper proposes a new distance formula for q-ROFNs, and applies it to the decision making method.

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