An Off-Grid Sparse Representation Based Localization Method for Near-Field Sources

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Abstract. Near-field source localization is a potential research topic in next-generation wireless communications. Most existing methods focus on traditional subspace based methods or on-grid sparse methods. In this paper, we propose an off-grid sparse representation localization method. First, by obtaining a high order cumulant matrix we construct an angle based off-grid signal model and then employ the alternatively iterating optimization method to estimate the angles. For range estimation, a range based off-grid signal model is constructed by using the angle estimations and solved by alternatively iterating method. Simulation results reveal that, the proposed method not only enjoys high estimation accuracy, but also can realize auto-pairing of angles and ranges.

Keywords: Near-field localization · Sparse representation · Off-grid signal model · Alternative iteration

1 Introduction

In the traditional wireless communication, the distance between the user and the base station is generally much larger than the antenna size of the base station, so the traditional array receiving model is based on the far-field hypothesis, that is, the signal sent by the user incident to the base station antenna can be regarded as a plane wave. In this case, the channel information is determined by the channel attenuation coefficient and the arrival angle of the incident signal relative to the base station antenna (Direction-of-Arrival: DOA) In order to obtain DOA information, researchers proposed a number of estimation algorithms under the far-field hypothesis. The most famous algorithms are MUSIC (Multiple Signal Classification: MUSIC) [1–4], ESPRIT (Estimation of Parameters by Rotational Invariant Techniques) [2] and L1-SVD (L_1 reconstruction after Singular Value Decomposition) [3], where the subspace class methods represented by MUSIC and ESPRIT are known for their high resolution. In a good environment with known signal number and large snapshot, the method can realize superresolution estimation and has approximate optimal estimzation performance. However, the subspace class method relies on the orthogonality of Signal subspace and Noise subspace to achieve direction finding. In some harsh scenarios, such as multi-path, small snapshot and low

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(Signal-to-Noise Ratio: SNR) scenarios, the orthogonality between two subspaces will be destroyed, which will seriously affect its direction finding performance. The sparse direction-finding method that is based on the hypothesis of spatial angle sparsity represented by L1-SVD has high scene adaptability, able to fit the above harsh scenarios so as to achieve the correct positioning. However, this method is based on the spatial Angle division and assumes that the incident signal will fall on the divided grid without errors. When the number of grids is small, this method is difficult to achieve the required estimation accuracy, otherwise, when the number of grids is large, it will be restricted by RIP(Restricted Isometry Property). At the same time, a large number of grids will bring high computation, thus greatly reducing the computing efficiency. For this reason, the researcher proposed an off-grid class direction finding method [5, 6], which is independent of the above assumptions, thus greatly improving the application scope of the sparse representation direction-finding method. The signal is no longer assumed to fall on the prearranged grid, but can be distributed in the whole angle space in the off-grid method. The array guidance vector is approximated by the first-order Taylor expansion formula, so a signal model based on sparse signal and agger deviation as joint variables is established. Based on this model, the researchers put forward multi-factor to solve the sparse signal and the offset, and then get the signal direction. Zhu etc. studied the error of the reconstructed matrix in the compressed sensing algorithm and proposed a new method called Sparsity-cognizant total least-squares method [7]. Yang etc. proposed a Basis Pursuit Denoising (BPDN) model to jointly solve the signal and offset [8]. Based on Sparse Bayesian Learning theory, literature [6] proposed an efficient direction finding method. In literature [9], a direction finding method based on two-step iterative optimization is proposed, which is accomplished alternately optimizing sparse signals and offset.

In the next generation wireless communication system, it is an important research direction to enhance the spatial resolution of the base station and improve the spatial reuse capability. For this reason, researchers proposed the concept of Extremely Large Aperture Array (ELAA) [10]. The aperture size of ELAA ranges from several meters to tens of meters, and its near-field area can reach several kilometers. Therefore, the traditional signal model which relies on the far-field hypothesis cannot be applied to the ELAA scenario. So it is necessary to study the localization method based on the near-field signal model. When the user is located in the near-field area of the base station antenna array, the base station receives the signal in the form of spherical wave. The positioning information of the user is determined by the DOA and the distance between the user and the base station. Therefore, the two-dimensional parameters of the signal source, DOA and distance, need to be solved simultaneously for the near-field positioning. Because of the complexity of the guidance vector in the near-field signal model, the spherical wave signal model is generally simplified by using the second-order Taylor expansion. Based On this model, the researchers put forward a series of near field positioning methods, such as Oblique Projection MUSIC (Oblique the Projection MUSIC: OPMUSIC) method [11]. Two step MUSIC (Two - Stage MUSIC: TSMUSIC) method [12], and the Sparse (On - grid point Approach: OSA) [13], etc. Among them, OPMUSIC and TSMUSIC are traditional subspace class methods and can simultaneously realize the positioning of near and far field hybrid source. These two methods inherit the advantage of high precision of subspace class method, but lose the array aperture. OSA is a sparse representation class method, The DOA and distance parameters are obtained by dividing the grid in Angle space and distance space respectively and using the weighted norm minimization model. At the same time, OSA can realize the automatic matching of DOA and distance. However, this method is also restricted by meshing effect. That is, the mesh needs to be subdivided to improve the estimation accuracy, and is limited by the RIP criterion and high computational efficiency. Therefore, a positioning method with high estimation accuracy without increasing mesh density is urgently needed.

In this paper, we propose an off-grid sparse representation based localization method for near-field sources. First, This method establishes an off-grid signal model based on angle parameters by obtaining a high-order cumulant matrix and then realizes the estimation of Angle by means of alternating iterative optimization method. Then an off-grid signal model based on distance parameter is established according to the Angle estimation and solved by alternatively iterating method.

The symbols used in this paper are as follows: for A matrices, A^T and A^H represent the transpose matrix and conjugate transpose matrix of the matrix A respectively. $\|A\|_2$ and $\|A\|_F$ represent the two norm and Frobenius norm of the matrix A respectively. $\odot$ represents Hadamard product. diag means taking the diagonal elements of a matrix or converting a vector to a diagonal matrix. $\Re$ represents a real part operation.

2 Signal Model

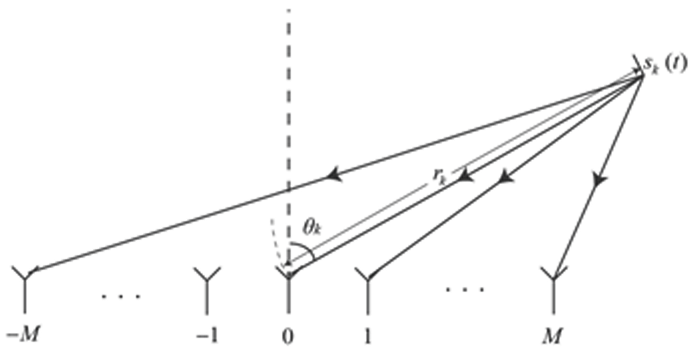


Fig. 1. Illustration of NF source localization

Suppose there are k narrow band near-field signals incident on a Uniform Linear Array (ULA) with N array elements. The index value of the array element is $\Omega = \{-M, \cdots, M\}$. As shown in Fig. 1, the incident Angle of the k-th incident signal relative to the array normal is θ_k. If the middle array element is taken as the reference array element, the distance between the k-th source and the reference array element is r_k. The output of the entire array is,

$$x(t) = As(t) + n(t) \tag{1}$$

Where, x(t) is the array output at time t, $A = [a(\theta_1, r_1), \cdots a(\theta_K, r_K)]$ is the array manifold matrix, $a(\theta_k, r_k) = \left[e^{j[-M\omega_k + (-M)^2\phi_k]}, \ldots, e^{j[M\omega_k + M^2\phi_k]}\right]^T$ is the guide vector of the k-th signal, s(t) is the incident signal at time t, n(t) is the zero mean additive gaussian white noise at time t, and

$$\omega_k = -2\pi\frac{d}{\lambda}\sin(\theta_k) \tag{2}$$

$$\phi_k = \pi\frac{d^2}{\lambda r_k}\cos^2(\theta_k) \tag{3}$$

Where, d represents the spacing between elements and λ represents the signal wavelength. Our purpose is to recover DOA parameters $\theta = \{\theta_1, \cdots, \theta_K\}$ and distance parameters $r = \{r_1, \cdots, r_K\}$ of k-th signals through the array reception model (1) and complete the pairing.

Before introducing the method proposed in this paper, we propose the following hypothesis:

(1) in order to avoid Angle ambiguity, the matrix interval d should satisfy: $d \leq \lambda/4$;
(2) the incident signal is a narrow-band zero-mean stationary random process, and its fourth-order cumulant is not zero;
(3) array received noise is independent of the signal and is additive zero-mean gaussian white noise.

3 Methods Mentioned

The proposed method consists of two steps: first, the fourth-order cumulant matrix is obtained according to the array output information, and the angle-based off-lattice signal model is established to solve the Angle information. Secondly, the off-grid signal model based on distance parameter is established to realize the solution of distance parameter.

3.1 Solution of Angle Parameters

The fourth-order cumulant of array output is defined as:

$$\begin{aligned} c(m, n, p, q) &= \mathrm{cum}\left\{x_m(t),\ x_n^*(t),\ x_p(t),\ x_q^*(t)\right\} \\ &= \sum_{k=1}^{K} c_{s_k} e^{j[(m-n)-(p-q)]\omega_k} e^{j[(m^2-n^2)-(p^2-q^2)]\phi_k} \end{aligned} \tag{4}$$

Where, c_{s_k} represents the fourth-order cumulant of the k-th signal. Let $\overline{m} = m + N + 1$, $\overline{n} = n + N + 1$, we get the following fourth-order cumulant matrix:

$$\begin{aligned} C(\overline{m}, \overline{n}) &= \mathrm{cum}\left\{x_m(t), x_{-n}^*(t), x_{-n}(t), x_n^*(t)\right\} \\ &= \sum_{k=1}^{K} c_{s_k} e^{j2(m-n)\omega_k} \end{aligned} \tag{5}$$

The matrix can be further expressed as follows:

$$C = \sum_{k=1}^{K} c_{s_k} \overline{a}(\theta_k)\overline{a}^H(\theta_k) = \overline{A}(\theta)C_s\overline{A}^H(\theta) \tag{6}$$

Where, $C_s = \mathrm{diag}([c_{s_1}, \cdots, c_{s_K}])$, $\overline{A}(\theta) = [\overline{a}(\theta_1), \cdots, \overline{a}(\theta_K)]$, $[e^{j2(-N)\omega_k}, \cdots, 1, \cdots, e^{j2N\omega_k}]^T$. From formula (6), it can be seen that the matrix C is only related to the Angle parameter, but not to the distance parameter. Let $\overline{S} = C_s\overline{A}^H(\theta)$ it be the input signal of virtual array, then model (6) can be regarded as the output of virtual array in a far-field source scenario, that is,

$$C = \overline{A}(\theta)\overline{S} \tag{7}$$

The Angle space is divided into Q grids, and the grid set is $\vartheta = \{\vartheta_1, \cdots, \vartheta_Q\}$, $\vartheta_{q_k}(q_k \in \{1, \cdots, Q\})$ is defined as the nearest grid point from θ_k. As θ_k is evenly distributed throughout the Angle space, the error between θ_k and ϑ_{q_k} will always exist, and is defined as $\delta_{q_k} = \theta_k - \vartheta_{q_k}$. Then the guidance vector of the k-th incident signal $a(\theta_k)$ can be expressed by the first-order Taylor expansion as,

$$a(\theta_k) = a(\vartheta_{q_k}) + b(\vartheta_{q_k})\delta_{q_k} \tag{8}$$

Where, $b(\vartheta_{q_k})$ represents the derivative of $a(\vartheta_{q_k})$ at ϑ_{q_k}, Definition$B^{\circ} = [b(\vartheta_1), \cdots, b(\vartheta_Q)]$, $\Delta = \mathrm{diag}(\delta)$, $\delta = [\delta_1, \cdots, \delta_Q]^T$ then model (7) can be extended to the following off-grid signal model,

$$C = \left(\overline{A}^{\circ} + B^{\circ}\Delta\right)\overline{S}^{\circ} \tag{9}$$

Where, $\overline{A}^{\circ} = [\overline{a}(\vartheta_1), \cdots, \overline{a}(\vartheta_Q)]$, $\overline{S}^{\circ}$ is the sparse signal after expansion, and its non-zero value position represents the size ϑ_{q_k}. According to the sparse representation theory, we propose the following model based on ℓ_1 norm minimization:

$$\min_{\overline{S}^{\circ},\delta} \beta\left\|\overline{S}^{\circ}\right\|_{2,1} + \frac{1}{2}\left\|C - \left(\overline{A}^{\circ} + B^{\circ}\Delta\right)\overline{S}^{\circ}\right\|_F^2 \tag{10}$$

Where, $\beta > 0$ is a user-defined parameter. It can be seen from model (10) that the model is a non-convex problem due to the existence of bilinear variables $\Delta\overline{S}^{\circ}$, so it is difficult to solve in polynomial time. In order to solve the problem effectively. We do this by iterating between variables $\overline{S}^{\circ}$ and Δ.

In the Q-th iteration, fix Δ first and update $\overline{S}^{\circ}$ by solving the following optimization problem,

$$\overline{S}^{\circ(q+1)} = \min_{\overline{S}^{\circ}} \beta\left\|\overline{S}^{\circ}\right\|_{2,1} + \frac{1}{2}\left\|C - \left(\overline{A}^{\circ} + B^{\circ}\Delta^{(q)}\right)\overline{S}^{\circ}\right\|_F^2 \tag{11}$$

Since model (11) is a convex optimization problem, it can be solved by CVX and other optimization toolkits. After the optimal solution $\overline{S}^{\circ(q+1)}$ is obtained, Δ is updated according to the following optimization problems

$$\delta^{(q+1)} = \min_{\delta} \beta \left\| \overline{S}^{\circ(q+1)} \right\|_{2,1} + \frac{1}{2} \left\| C - \left(\overline{A}^{\circ} + B^{\circ} \Delta \right) \overline{S}^{\circ(q+1)} \right\|_F^2 \tag{12}$$

It is noted that model (12) is essentially a weighted least squares estimation about δ, so its closed solution can be obtained through a series of derivations. According to literature [9], the solution of model (12) can be obtained as

$$\hat{\delta}^{(q+1)} = \Re\left\{ D^{-1} f \right\} \tag{13}$$

Among them,

$$D = (B^{\circ H} B^{\circ}) \odot \left(\overline{S}^{\circ(q+1)} \overline{S}^{\circ(q+1)H} \right) \tag{14}$$

$$f = \mathrm{diag}\left(\overline{S}^{\circ(q+1)} \left(C - \overline{A}^{\circ} \overline{S}^{\circ(q+1)} \right)^H B^{\circ} \right) \tag{15}$$

When the iteration ends, we can obtain the spatial power spectrum P^{final} and the new grid set in the case of off-grid,

$$\vartheta^{\circ} = \vartheta + \hat{\delta}^{final} \tag{16}$$

Where P^{final} is generated by the optimal solution of model (11) at the end of iteration, and $\hat{\delta}^{final}$ is the size at the end of iteration $\hat{\delta}^{(q+1)}$. Then the estimated value $\hat{\theta}$ of DOA can be obtained by searching for P^{final} the corresponding size of the first K peak index values in ϑ°.

3.2 Solve the Distance Parameter

By taking the angle estimated value $\hat{\theta}$ Angle into model (1),

$$x(t) = A\left(\hat{\theta}, r\right) s(t) + n(t) \tag{17}$$

Where $A\left(\hat{\theta}, r\right)$ is the array manifold matrix with respect to distance r. The distribution space of the source distance r is $\left[0.62\left(\frac{D^3}{\lambda}\right)^{\frac{1}{2}}, \frac{2D^2}{\lambda}\right]$, where D is the aperture size of the array. Referring to the DOA solving process, we grid the spatial range of r to obtain the grid point set $r^{\circ} = \{r_1, \cdots, r_V\}$, and then obtain the sparse expansion model in the off-grid scenario.

$$x(t) = [A^{\circ}\left(\hat{\theta}, r^{\circ}\right) + B^{\circ}\left(\hat{\theta}, r^{\circ}\right) \Delta_r] s^{\circ}(t) + n(t) \tag{18}$$

Where, $A^{\circ}\left(\hat{\theta}, r^{\circ}\right) = \left[A^{\circ}\left(\hat{\theta}_1, r^{\circ}\right), \cdots, A^{\circ}\left(\hat{\theta}_K, r^{\circ}\right)\right]$, $B^{\circ}\left(\hat{\theta}, r^{\circ}\right)$ represents the derivative of r with respect to $A^{\circ}\left(\hat{\theta}, r^{\circ}\right)$, $s^{\circ}(t)$ represents the extended equivalent sparse signal, and Δ_r is similarly defined as Δ. We establish the following sparse reconstruction model

$$\min_{s^{\circ}(t), \Delta_r} \eta \sum\nolimits_t \left|s^{\circ}(t)\right| + \frac{1}{2} \sum\nolimits_t \left\| x(t) - [A^{\circ}\left(\hat{\theta}, r^{\circ}\right) + B^{\circ}\left(\hat{\theta}, r^{\circ}\right)\Delta_r] s^{\circ}(t) \right\|_2^2 \tag{19}$$

Where $\eta > 0$ is a user-defined parameter. This model is also a non-convex problem, and the solution method is similar to formula (11)–(16). At the same time, the model (19) is used to solve the distance parameters, which can realize the automatic pairing of Angle and distance parameters.

4 Simulation Experiment

We will verify the effectiveness of the proposed algorithm through some simulation experiments. In the simulation experiment, we selected OPMUSIC [11], TSMUSIC [12] and OSA [13] which are representative in the field of near-field positioning. The array adopted is a 7-element uniform linear array with an interval of $d = \lambda/4$. The incident signal is $e^{j\psi}$, where the phase ψ is uniformly distributed in the interval of $[0, 2\pi]$.The index to measure the performance of the algorithm is the RMSE (Root Mean Square Error) of the estimated results, which is defined as,

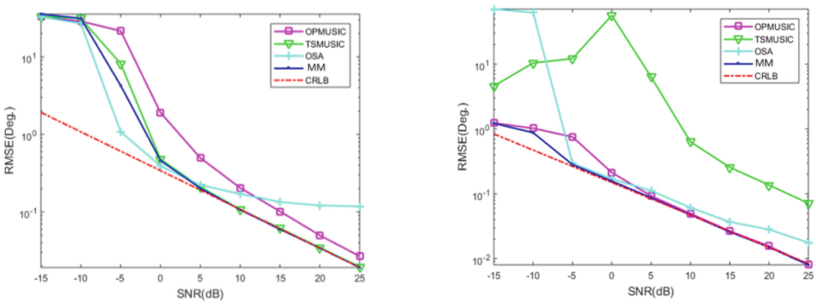
$$RMSE = \sqrt{\frac{1}{MoK} \sum\nolimits_{n=1}^{Mo} \left\| \theta_n^{Est} - \theta_n^{True} \right\|_2^2} \tag{20}$$

Where Mo represents the number of simulation, θ_n^{True} and θ_n^{Est} corresponds to the real value and estimated value of angle in the n-th simulation experiment respectively. Meanwhile, we used CRLB (Crammer-Rao Lower Bound) as a reference to measure the estimated performance.

4.1 Changes of RMSE with SNR

In the first simulation experiment, it is assumed that two narrowband near-field signals are incident on the array at the position of $\{0^{\circ}, 1.3\lambda\}$ and $\{20^{\circ}, 3\lambda\}$. The number of quick beats collected is 600. Let the SNR range from −15 dB to 25 dB. The variation of RMSE estimated by each method with SNR is shown in Fig. 2.

It can be seen from Fig. 2(a) that off-grid signal method proposed in this paper (MM) can approach CRLB at a fast speed and continue to decline with the improvement of SNR and always keep close to CRLB. In contrast, although OSA can approach CRLB first, but its RMSE cannot continue to decrease with the increase of SNR, which shows a performance "saturation" phenomenon. OPMUSIC has low performance due to the loss of array aperture caused by the introduction of smoothing method. TSMUSIC exhibits similar estimation performance. It can be seen from Fig. 2(b) that the method in this paper is always close to CRLB when the SNR is greater than −5 dB. OPMUSIC can only approach CRLB when the SNR is greater than 5 dB. OSA deviated from CRLB even when the SNR was relatively high. TSMUSIC, on the other hand, performed poorly.

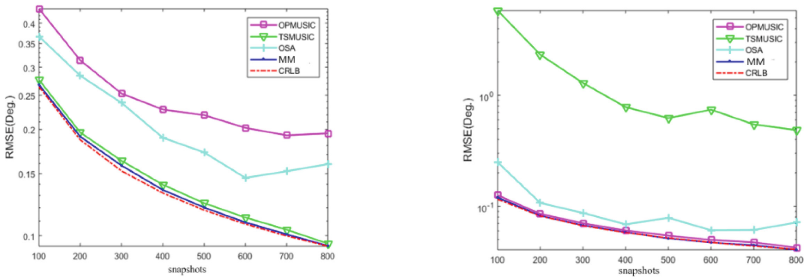


(a)comparison of angle estimation results (b)comparison of distance estimation results

Fig. 2. Performance analysis of RMSE with varying SNR.

4.2 RMSE Changes with the Number of Quick Beats

In the second simulation experiment, we compared RMSE of each method under different number of snapshots. The experimental parameters are basically the same as the first simulation experiment, except that the number of snapshots varies from 100 to 800 and the SNR is set to 10 dB. The experimental results are shown in Fig. 3.



(a)comparison of angle estimation results (b)comparison of distance estimation results

Fig. 3. Performance analysis of RMSE with varying number of snapshots.

It can be seen from Fig. 3(a) that the method in this paper can approach CRLB well in the whole range of snapshots.TSMUSIC shows similar estimated performance to the method in this paper. OPMUSIC and OSA have underperformed. It can be seen from Fig. 3(b) that the method mentioned still provides excellent estimation capability in the aspect of distance estimation. OPMUSIC performs similarly, while the other two methods perform less well. It can be seen from the above two simulation experiments that the method presented in this paper has a stable and excellent near-field source location capability.

5 Conclusion

This paper presents a localization method for near-field sources. Firstly, by obtaining a high order cumulant matrix we construct an angle based off-grid signal model and then employ the alternatively iterating optimization method to estimate the angles. Then, by taking advantage of the Angle estimation information, the off-grid extension model based on the distance parameter is constructed and the method similar to the Angle estimation is used to obtain the distance estimation value. This method not only has high estimation accuracy, but also can realize the automatic pairing of Angle and distance parameters.

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