Cross-Sections Between Geometric Patterns of the Past and the Generative Arts of Today

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Abstract. Geometric Patterns have a long tradition dating back to early Antiquity. The heyday of the use of geometric patterns to create advanced compositions was during the era of the Islamic Enlightenment. The widespread use of everyday mathematics in the period also facilitated the increase of the competence of artists with high skills in abstracting visual content. Today, various methods are used to create such complex shapes faithful to their original representations. While focusing on the historical stance of geometric patterns and their visual integrity, this paper proposes an alternative system to implement trigonometric and computational methods to generate geometric patterns out of simple constructive shapes. Furthermore, this paper also employs the use of computer programming to generate dynamic geometric patterns to build a bridge between the geometric patterns of the past and the generative arts of today.

Keywords: Geometric Patterns · Generative Arts · Computation · Trigonometry

1 Introduction

Abstraction is a phenomenon frequently encountered in science, art, and philosophy. We resort to abstraction as a tool to describe an obvious or non-obvious phenomenon through our way of thinking and to enable us to move between the relations that appear before us on philosophical grounds. Due to its ontologically distant stance from the physical state of the thing or things it is related to, the act of abstraction inevitably brings with it the relations of the signifier and the signified that emerge at different levels between things. Abstraction does not exist physically in the world we live in, but as a product of thought in our minds. Abstract relations that art forms through representation with thoughts have been handled in different ways in movements such as Cubism, Fauvism, Abstract Expressionism, Rayonism, Suprematism, Futurism, Opt Art, and Constructivism throughout the historical evolution of art.

Imaginary depictions of various animals, human figures, and signs found in different caves in various parts of the world, estimated to date back tens of thousands of years, can be thought of as abstract representations created on a physical surface. Located in southeastern France, Chauvet Cave contains one of the best-preserved cave paintings in the world. A distinctive image of large red dots found in the first section of this cave is

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highly speculative in appearance. The researchers found that these red dots were made by a single person by coating the palm of the right hand with red paint and then transferring this color to the wall. It is impossible to reach a clear conclusion about what the resulting work represents due to its ambiguous appearance, which is far from being a figurative work. This is one of the earliest abstract works we come across because what it points to is incomprehensible and unknown. According to Waldman [1], this abstraction has a connection with hallucinations. A view that this relationship might be true emerged through interviews with San tribe members in South Africa in the 19th century. The walls of the Drakensburg Mountains were painted by the San Indians just 200 years ago in abstract patterns very similar to those in France and Spain. In the interviews, the people of the tribe said that they went into a kind of trance and had various hallucinations before painting the walls. Based on what was told, the idea that all the abstract descriptions in the Paleolithic caves were created from the snapshots seen by the ancient people living in the cave away from daylight under similar conditions began to make sense.

Abstraction consists of things that do not belong to the world of realities that we live together among people. However, when the expression of abstract things is considered, it becomes inevitable that they are depicted with signifiers that exist in the world at some point. Mathematics, which emerges for us to perceive the order and complexity that exists in the universe, has a very rich structure in terms of abstraction. Mathematics, which has a unique language structure with the symbols and abstract concepts it embodies and gains more and more depth by using this language, has a very important place in our understanding of the concrete world we live in. According to Flusser [2], when words are insufficient to understand the world, it is necessary to understand the world by calculating and numbers are pictures of thoughts. Numbers, as symbolic structures, enable the tangible things that exist in the world to be counted. In addition, mathematics also deals with abstract content within its own systematic. While the idea of a three-dimensional space is widely adopted in the visible universe, it is possible to accept the existence of a space with more dimensions, even infinite dimensions, thanks to mathematics. In this sense, mathematics has a structure that can provide a transition between abstract and concrete universes. Whether mathematics is a human invention or whether it already exists in nature is another controversial issue. Exactly how and when it emerged is not known, but according to the available information, the oldest existing mathematical texts were found in Mesopotamia and Egypt around 1900 BC. Nevertheless, the Pythagoreans initiated the systematic study of mathematics within a discipline in the 6th century BC. When we look at the Ancient Greek Civilization, Euclid's book "Elements", one of the studies in the field of mathematics, has a very effective place today. Based on the works of ancient Greek mathematicians, this work is considered to be the backbone of geometry, especially since it covers the subject of geometry comprehensively.

In this book, Euclid, with his assumptions on plane geometry, revealed that mathematics cannot have an absolute equivalent in the world. For example, according to the information expressed in ten items known as Euclid's axioms, only one line passes through two points. However, although it is possible in theory to obtain an absolute truth beyond the physical equivalent of the point concept, it is not valid in practice.

According to the same axioms, it is possible to define a circle with any center and any radius, but a perfect circle cannot be observed in nature or produced out of nothing. The speculative thoughts that developed on the abstract and concrete states that emerged between the perfect forms and their representations in the world were previously discussed by the famous philosopher Plato. Plato's Theory of Forms rejects the reality of the material world, accepting what exists in materiality as only an image or copy of the real world. According to the theory of forms, there are two different worlds: the visible world of concrete objects grasped by the senses and constantly changing, and the world of abstract objects grasped by the pure mind. When considered from this point of view, the discipline of geometry can only create its perfect forms through pure reason, while the order of formal relations existing in this world becomes calculable through some axioms. It is possible to find similar reflections of the perfect forms predicted by geometry in nature. For example, while hexagonal shapes form the basic structure of beehives, surprisingly, the symmetrical order is naturally formed by bees. However, it should be noted that the hexagons we encounter never have the perfection to form the perfect hexagonal form. According to Platonic thought, perfection is not an attainable phenomenon in this world. Art is seen as worthless by Plato because it cannot go beyond creating a copy of the copy that appears in the world. Real beauty is possible when it reflects the perfection of geometry. However, Plato's effort to create perfect geometries, which he considered impossible, formed the basis of the Art of Geometry years later.

2 Geometry and Art

While the world we reside in provides us with an environment that we think is trivial to experience, our curiosity about existence reinforces the questioning of the realities behind the order and chaos in the world. These inquiries try to identify the building blocks of possible orders or irregularities within a systematic structure created by mathematics, which is an abstract language we use to describe the universe. Although the realm of mathematics has a very effective role in understanding the world, it is never sufficient on its own. The connection of mathematics with other fields and the emerging collaborations are sometimes strong and sometimes weak. The relations between art and mathematics have been discussed in-depth for a very long time. Despite Plato's trying to keep art away from mathematics, art has never been completely separated from mathematics and mathematics from art. Observing the most obvious commonalities in this subject in the art of geometry is possible.

Throughout history, we see geometric patterns as visual content in many different forms in architecture and art. Geometric patterns sometimes appear as the reflection of an order we observe in nature, and sometimes as the abstract results of an intuitive process that emerges with different techniques applied. Geometric patterns obtained with the help of a simple compass and ruler reveal a very rich world with their complex structures that emerge at different levels in the visual content they present to us, and at the same time with their flawless layouts. In the art of geometry, which takes its inspiration from in-depth abstractions, we encounter some mysterious bridges that are tried to be established between the perceived and the designed. When these bridges are masterfully constructed, they have a simple structure that allows us to look through a

window extending to absoluteness, repetition, infinity, simplicity, complexity, order, and chaos.

The use of geometry in art inevitably leads to the emergence of an abstract narrative due to the nature of the material. Movements such as Cubism, Fauvism, Abstract Expressionism, Rayonism, Suprematism, Futurism, Opt Art, and Constructivism that emerged in the twentieth century contain geometry as a formal content material when considered purely from a visual point of view. Since the works of art that appeared in the movements mentioned above emerged in the light of factors such as the discourse of the artwork and the identity of the creative artist, they were able to stay away from the criticisms of decorative workmanship that the geometric patterns produced anonymously in the past. However, geometric patterns have undergone significant developments during their historical evolution. They have gained an interesting structure in terms of the ideas they represent, as well as the emergence of unique and non-figurative visual content.

When we try to examine the first examples of geometric patterns, the mosaics found on the floors of the Gordion Palace in Phrygia in the 9th century BC emerge as a remarkable example of non-figurative art [3]. Although there is no figurative narrative in the motifs used in the mosaics in the Megaron II region, various shapes such as checkerboard patterns, meanders, and interlocking triangles are observed [4]. These shapes take place in a way to form different pattern groups among themselves, but these pattern groups do not contain any harmonious relationship between them. Even if we accept that they are arranged randomly, it is impossible to deny the existence of an abstract narrative revealed by the geometric patterns in this example.

Fig. 1. The mosaics found on the floors of the Gordion Palace in Phrygia

According to Kiilerich [5] some of the most intricate abstractions can be seen in the bright, colorful walls and vaults of the Basilica of San Vitale, inaugurated in the year 547. We encounter wavy zigzag stripes in the patterns applied on the curved inner surfaces of the triple arches. On the front face of the arch, we see a duplicated version of a long

and thin drop motif surrounded by the center in a semicircular area. In these examples, we notice how the geometric patterns are in harmony with the architectural spaces. This relationship between pattern and architecture continues to be examined as a subject that has continued throughout history. It is possible to say that Islamic Art was the most comprehensive period in which geometric shapes were handled by placing them in the center as the basic formal elements of art. For centuries, geometric patterns have been used frequently and skillfully in Islamic Art, especially in mosque architecture.

3 Islamic Geometric Patterns

In many societies from ancient times to the present day, the use of geometric elements in works of art has been at the center of the attention of artists. According to Wichmann and Wade [6], Geometric Designs have long attracted the attention of Muslim designers and craftsmen. They have a spiritual or otherworldly aura without being associated with specific doctrinal propositions. Likewise, they lack any symbolic value in an Islamic setting. First, they allow craftsmen to showcase their expertise and the complexity of their craft, and at the same time amaze and fascinate with their pure complexity. Nevertheless, it should be noted that it is not a coincidence that the art of Geometry developed under the roof of Islam.

When in the 6th century, Islam emerged in the Arabian Peninsula, which is geographically defined as the Near East, the pagan belief was quite common in the region. There were many idols representing the belief of paganism in the Kaaba in Mecca, where the Prophet Muhammad was born. Nonetheless, the message that the Prophet Muhammad received in the revelations was a warning that people can attain salvation in the hereafter only through devotion to the one and only God [7]. Being a monotheistic religion, Islam avoided all kinds of idol worship. Following his appointment as a religious leader by the people of Mecca, Muhammad's first act was to remove the 360 idols surrounding the Ka'ba. Mecca has previously been a major pagan pilgrimage site [8]. The fact that Islam forbade the worship of idols was already a practice that existed in earlier monotheistic religions. Unlike what was practiced in the past, the religion of Islam did not see the need for any intercession between God and the individual. It has always been thought that the relationship between man and the divine God must occur with a degree of purity, independence, and transparency. Considering this aspect, the place of the art of geometry in the religion of Islam is better understood. Islam assumes that any figurative image that will be presented in places of worship can be perceived as iconoclastic and thinks that such images will interfere with the direct devotion that will occur between the individual and God. For this reason, since geometric patterns do not have a dogmatic structure that will create a directed judgment in the individual, they have been much more accepted in Islamic Art with the abstract content they offer.

Another reason for the widespread use of geometry in Islamic Arts is the intense interest in science and technology, which gained momentum, especially in the Middle East in the 8th and 9th centuries. In this Islamic Enlightenment period, also known as the Abbasid period, Classical Greek and Roman teachings were blended with Persian and Hindu teachings in Baghdad, which was by then the capital of the Abbasids. Baghdad lived its heyday during the reign of Caliph Harun al-Rashid. During this period, ancient

Greek texts were extensively translated into Arabic. During the reign of the Abbasids, the "House of Wisdom" (Beyt al-Hikmah) was established as a leading intellectual center. This was a crucial component of the Islamic Golden Age that developed with the ongoing translation movement.

Over time, there have been very eminent scientists who have grown up in the House of Wisdom. For example, al-Khwarizmi made significant contributions to the science of mathematics with his studies in the field of algebra. The emergence of the word algorithm is also based on the work of al-Khwarizmi. The works of Ibn Al-Haytham using geometry extensively made significant contributions to optics and visual perception principles. With the dominance of mathematics and geometry in many areas of the Islamic world, the art of geometry has also undergone structurally significant developments.

The most distinctive feature of Islamic Geometry Art is the symmetrical and proportional arrangements established between the shapes. This art, which has a technical structure that enables extremely complex structures to be obtained with a simple compass and ruler, has detailed content consisting of systematic methods in which the delicate balances between shapes and the resulting order must be carefully constructed. Regrettably, we have very little information and documents about how this art was performed in its time. Topkapi, Tashkent, and Mirza Akbar Scrolls, among the documents in hand, are important sources for understanding Islamic geometric ornament works. Created by master builders in the late medieval Iranian world, the Topkapi parchment contains a rich repertoire of geometric drawings for wall surfaces and vaults. The scroll, which was created by skilled builders in the late medieval Iranian civilization, has a vast repertory of geometric drawings for wall surfaces and vaults [9].

Necipoğlu [10] makes crucial inferences about how the patterns on the parchments can be drawn. Two- and three-dimensional drawings of the Topkapi Scroll were created with underlying geometric grid systems consisting of inked sketch lines or non-inked, incised lines. These complex drawings, which are based on a certain library of geometric shapes and obtained by shifting, reflection, shift reflection, and rotation operations in symmetrical states, realize the structuring of design thinking.

Today, there are various methods and techniques applied to draw Islamic Geometric Patterns [8, 11–14]. Generally, the initial geometric shape positioned at the center of the drawing acts as the main determining factor in shaping the geometric structure that will emerge as a result. According to Abdullahi & Embi [14], these constructive polygons can have 6,8,10 folded convex or concave structures. Compass and straight edge were the only tools available for generations to create polygons and the necessary angles. As a result, all these polygons are based on templates made from grids of circles and come from harmonic subdivisions of circles. For this reason, it is one of the most common methods to generate these geometric basic determinants using a compass and a ruler by applying a few procedural operations. Apart from the drawing method based on the mentioned procedures, there are also methods that deal with the subject mathematically. In the next section, after determining the main constructive component of the basic image, an alternative system will be proposed for generating the shape because of calculating the relevant vertex points.

4 Trigonometric and Computational Methods of Generating Geometric Patterns

In the proposed method, the first step is to examine the pattern and reveal the motif, which is the pattern's basic component, in its simplest form. Then, the second step is to determine the vertex points that make up this motif by applying various trigonometric methods. After the motif is obtained, the relations between the motifs are examined for the tessellation processes. To explain this proposed method with practice, two sample designs will be examined. Let's start with the following example.

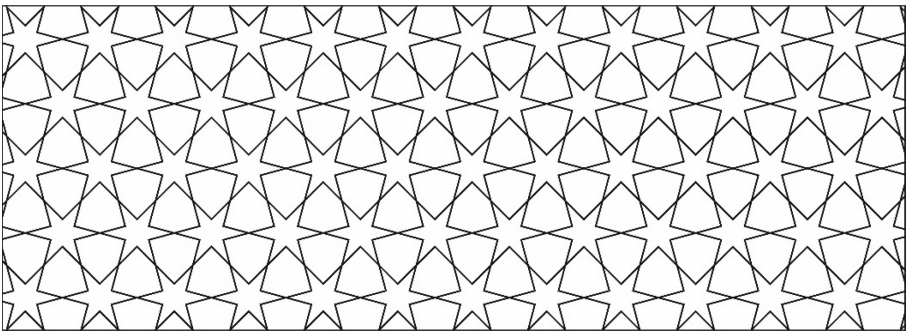

Fig. 2. Geometric pattern example

Example 1:

When the pattern structure in this example is analyzed, it is possible to make different visual inferences. Assuming that the main recurring visual component is a star, the main motif to be produced would be the following figure.

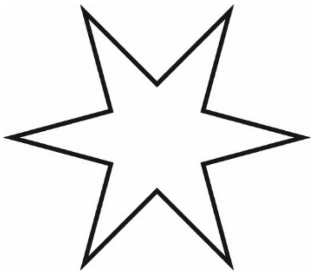

Fig. 3. The Motif

To determine the vertex points of the star determined as the basic motif above, a trigonometric analysis should be performed. The process of determining the coordinates of the inner and outer points to form the six-pointed star can be performed as follows.

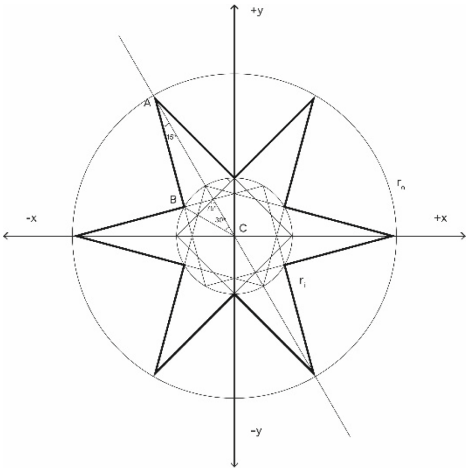

Fig. 4. Trigonometric Analysis

With the help of the trigonometric formula to be applied over the triangle ABC, it is possible to obtain the following relations between the radius lengths forming the inner and outer circles.

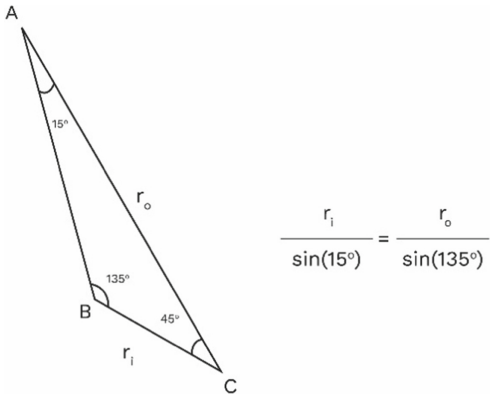

Fig. 5. Ratio relation between the inner and outer radius

In this way, the twelve vertex points that make up the six-pointed star can be easily determined.

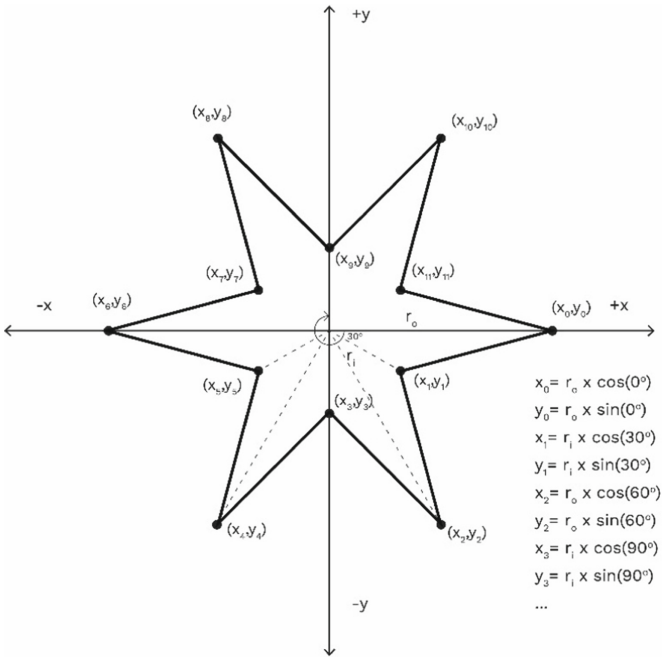

Fig. 6. Determining the vertex points

The next thing to do is to calculate the distances that will carry out the tessellation of the motif.

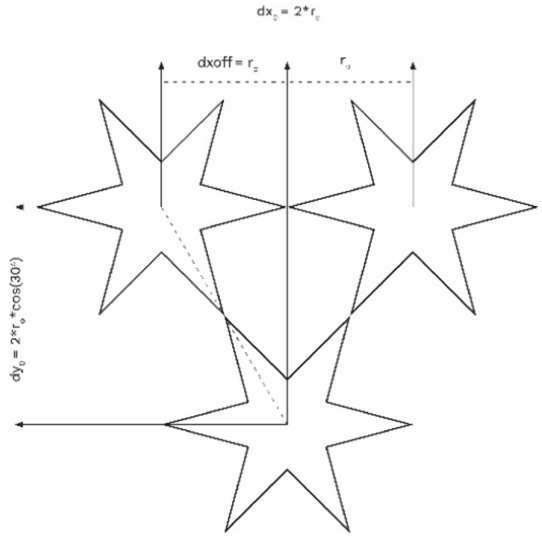

Fig. 7. Tessellating the motif

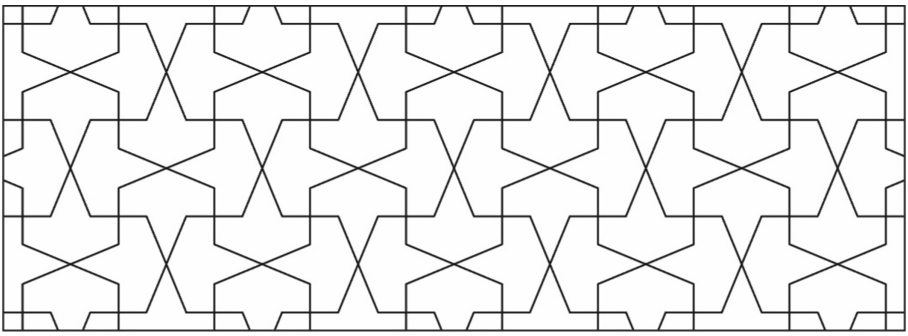

Fig. 8. Geometric Pattern Example

Example 2:

It is possible to show different approaches to what the recurring motif is in the above geometric pattern. Since geometric patterns usually have structures that are divided into 4, 5, or 6 sections to form a circular symmetry, we can start by defining the basic motif of this pattern as in Fig. 9.1 on the left. However, the interesting side of this analysis is that this motif also appears to be composed of four different sub-figures. When we want to simplify the obtained four-legged shape, even more, we see that there are four separate sub-parts (see Fig. 9.2).

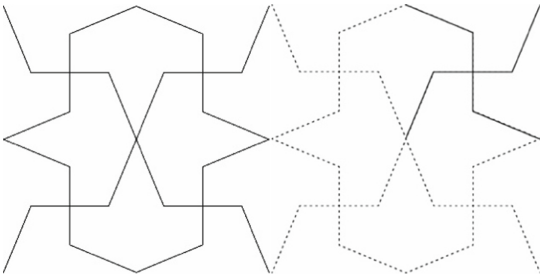

Fig. 9. 1 on the left, 2 on the right

In this way, it turns out that a partially complex pattern actually consists of only two lines. As can be seen in the two suggested examples, looking at a geometric design with a correct approach allows an effective solution to be found. It is possible to produce more complex patterns in similar ways with this method. As a result of all analyzes, the vertex points obtained can be converted into shapes with graphic programming methods. In this study, p5.js, an open-source JavaScript library, was used to perform this operation.

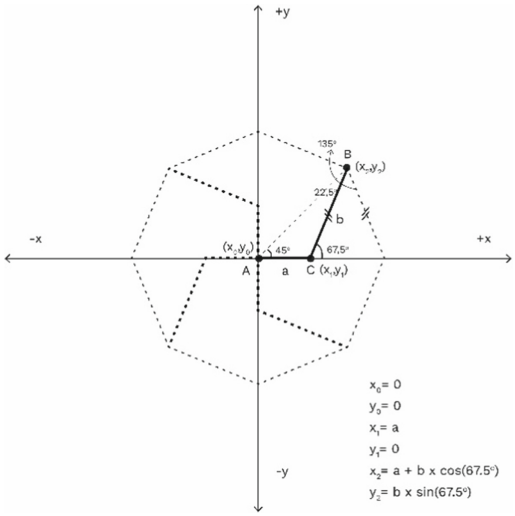

Fig. 10. Determining the vertex points

5 Conclusion

With the widespread use of screens in daily life and turning them into an indispensable consumption element, the reign of images declared its dominance in the 2000s. It has become impossible to spend a day without ever looking at a screen of varying dimensions. Not only that we are enslaved to illuminating surfaces, but with the light emitting projection devices all surfaces can become a frame for reflecting imagery. In addition, the intensive process of digitalization in the quest to transform all our previous habits has facilitated the proliferation of "digital arts" in recent years. We are increasingly seeing that various digital art events are happening around us. In addition to this, it is possible to say that an aesthetic language that we started to see in common in many of the works we encountered in these exhibitions was formed. The use of a multitude of abstract shapes with varying degrees of colors and scalar transformations is widely observed in this genre of artworks that rely on computational powers. Nevertheless, there is also a term attached to this kind of genre which is Generative Arts. In generative arts, the content is either entirely or partially produced by an autonomous system that is recently considered a computational system. Generative arts use various techniques such as creative coding, implementing external data, biomimicry, randomization, interactivity, etc. to create artistic audio-visual experiences. As these experiences were built, it's surprising to see similar methods being used to create geometric patterns long before today. Both systems of thought apply a procedural approach to generate complex contents with the use of simple geometric shapes such as dots, lines, circles, rectangles, etc. This procedural approach, better called "algorithms", is the fundamental character shared between these two stylistic methods.

As it was discussed in the previous section, the use of trigonometric and computational methods to generate geometric patterns enables an artist to use coding to animate these patterns with multiple degrees of freedom. Creating dynamic geometric patterns

with animating characteristics uncovers the hidden path between geometric patterns and generative arts that was far beyond our attention for a long time.

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