# Price-Based Power Control in NOMA Based Cognitive Radio Networks Using Stackelberg Game

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**Abstract.** This paper studies the price-based power control strategies for non-orthogonal multiple access (NOMA) based cognitive radio networks. The primary user (PU) profits from the secondary users (SUs) by pricing the interference power made by them. Then, SUs cooperate to maximize their total revenue at the base station (BS) with successive interference cancellation (SIC) while considering their payoff to the primary user. The pricing and power control strategies between the PU and SUs are modeled as a Stackelberg game. The closed-form expression of the optimal price for the non-uniform pricing scheme is given. The computational complexity of the proposed uniform-pricing algorithm is only linear with respect to the number of SNs. Simulation results are presented to verify the effectiveness of our proposed pricing algorithm.

**Keywords:** Non-orthogonal multiple access · Cognitive radio network · Successive interference cancellation · Stackelberg game

## 1 Introduction

With the rapid development of wireless communications and the growing shortage of spectrum resources, cognitive radio has been proposed to improve spectrum and energy efficiency by sharing the spectrum of primary users (PUs) with secondary users (SUs) in future network [1–3]. Besides, non-orthogonal multiple access (NOMA) technology is another promising technique to improve spectrum efficiency and support the great traffic volume in the fifth generation (5G) Network [4–6]. In the underlay based CR network, SUs can access the spectrum

This work was partially supported by the National Natural Science Foundation of P. R. China (No. 61701064), Basic Research and Frontier Exploration Project of Chongqing (cstc2019jcyj-msxmX0264) and the China Scholarship Council (file No. 201808500024).


H. Gao et al. (Eds.): ChinaCom 2019, LNICST 312, pp. 589–606, 2020.
https://doi.org/10.1007/978-3-030-41114-5_44

owned by PUs if the interference power (IP) from the SUs to the PU's receiver under interference temperature power (ITP) limit. Furthermore, the NOMA technique can be used in underly CR networks to improve the system performance of SUs because the interference power from weak SU can be canceled at the based station.

There are many studies focus on CR-NOMA system [7–11]. In [7], the authors proposed cooperative relaying strategies to address inter-network and intra-network interference in cognitive NOMA system. Liu *et al.* in [8] studied the large-scale underlay CR-NOMA system with two different power constraints to characterize the performance. In [9], Liang *et al.* studied the spectrum sharing in an underlay CR-NOMA system, and presented a non-transferable utility (NTU) coalition formation game between the cognitive users (CUs) and the PU. Moreover, some studies are also concerned with resource allocation of underlay CR-NOMA system [10,11]. For instance, Song *et al.* in [10] considered NOMA-based cognitive radio network with SWIPT, joint power allocation and sensing time optimizing algorithm based on dichotomy method is proposed to maximize the system throughput. Considering a cognitive multiple-input single-output NOMA with SWIPT, Mao *et al.* in [11] proposed a penalty function-based algorithm to minimize system power consumption.

Price-based power control of CR networks was investigated in [12–17]. By using the non-cooperative game with pricing scheme, the authors in [12] proposed a payment-based power control scheme to ensure the fairness of power control among SUs in CR networks. Considering the system efficiency and user-fairness issues, Yang *et al.* in [13] investigated cooperative Nash bargaining power-control game (NBPCG) model based on distributed power control and gave a signal-to-interference-plus-noise ratio (SINR)-based utility function. In [14], Yu *et al.* studied the pricing-based power control problems in CR networks, and they considered the competition as a non-cooperative game between SUs, and model the pricing problem as a non-convex optimization problem. Using a Stackelberg game to model the competitive behavior in [15], BS can maximize its revenue by pricing and SUs can profit by controlling its transmit power. In order to gain more revenue for a general case based CR networks system model compared with [15], the authors in [16] proposed a novel algorithm to find the optimal price for the PU and SUs. In [17], considering the quality of service (QoS) of the SUs, the authors proposed an optimal pricing algorithm for the interaction between the PU and the SUs. Wang *et al.* in [18] proposed a novel price-based power allocation algorithm based on the Stackelberg game to improve the revenue of BS and the sum rate of the users. In [19], the authors proposed a branch and bound based price-based power control algorithm to solve the non-convex revenue maximization problem for CR networks.

In this paper, we model the pricing strategy between PU and SUs as a Stackelberg game under the ITP model. First, PU plays a leader who prices the SUs to control the interference power made by SUs under the ITP limit. Then, the PU will select a suitable price to gain higher revenue from SUs. Simultaneously, SUs will choose an optimal power to maximize their total revenue at BS. Finally, we use Stackelberg game with non-uniform pricing (N-UP) scheme and uniform pricing (UP) scheme to model the strategy between them.

The rest of this paper is organized as follows. In Sect. 2, we present the system model for NOMA system based cognitive radio networks. Section 3 introduces the optimal price for two pricing schemes, and a distributed algorithm is proposed for UP scheme. In Sect. 4, the performance of the proposed two pricing schemes are evaluated by simulations. Finally, conclusions are stated in Sect. 5.

## 2 System Model

Considering the NOMA based CR networks comprised of one base station (BS), $n$ SUs and one PU as shown in Fig. 1. The SUs transmit the signal to BS with NOMA technology and SIC is employed at BS. The channel coefficient of SU $i$ to BS and PU link is denoted by $h_i (i = 1, \ldots, n)$ and $h_{i0} (i = 1, \ldots, n)$, respectively. At first, the PU charges each SU with a proportional price according to its interference power $p_i (i = 1, \ldots, n)$, and $p_i$ needs satisfy $\sum_{i=1}^{n} h_{j0} p_i \leq T$, where $T$ is the maximal interference power of PU. Then, SUs will pay PU to access the spectrum, and they will form a group to maximize the total utility.

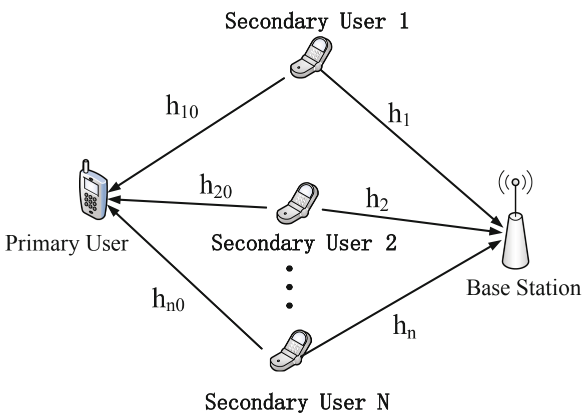


**Fig. 1.** System model.

We model the strategy between SUs and PU as a Stackelberg game. PU plays the leader that chooses a price for each SU to maximize its total revenue under ITP limit. Then, SUs can be viewed as the followers to obtain the best power for optimal revenue while considering their payoff to PU. Let $u$ denotes the PU's revenue, and the optimization problem of PU can be written as

$$\begin{aligned} &\text{maximize } u\left(c_1, \cdots, c_n\right) = \sum_{i=1}^{n} c_i h_{i0} p_i \\ &\text{subject to } p_i \geq 0, \ \sum_{j=1}^{n} h_{j0} p_j \leq T, \end{aligned} \tag{1}$$

where $p_i$ is interference power of the $i$-th SU under the given price of $c_i$, and $T$ denotes the maximal interference power at PU.

Let $\tilde{u}$ denotes the utility of the $i$th SU, and it contains two parts: $wR$ is the income from the total rate achieved at BS when SUs transmit the given power $p_i(i = 1, \ldots, n)$, and $\sum_{i=1}^{n} c_i h_{i0} p_i$ is the payment to the PU. So the revenue optimization problem of $i$th SU can be expressed as

$$\begin{aligned} &\text{maximize } \tilde{u}\left(p_1, \cdots, p_n\right) = wR - \sum_{i=1}^{n} c_i h_{i0} p_i \\ &\text{subject to } 0 \leq p_i \leq p_i^{max}, \end{aligned} \tag{2}$$

where $w$ is the preference of SUs for the gain of unit rate. $c_i$ is the price of $i$th user per unit of interference power. Let the SINR of $PU$ to $BS$ link is $n_0$, thus the sum rate of SUs $R$ can be written as:

$$R = \log\left(1 + \frac{\sum_i^n h_i p_i}{n_0}\right). \tag{3}$$

## 3 Solution to Stackelberg Game

In this section, we give the solution to Stackelberg game under the N-UP scheme and UP scheme. First, some lemmas are introduced to solve the above game. And the lemmas show the relationship between $p_i$ and $c_i(i = 1, \ldots, n)$ for the lower-level problem for SUs.

### 3.1 Non-uniform Pricing

**Lemma 1.** *Given a fixed price $c_i(i = 1, \ldots, n)$, let $P = (p_1, \ldots, p_n)$ be the optimal transmit power of problem (2). For all $i \in \{1, \cdots, n\}$, there exists a set of Lagrange multiplier $\lambda_1, \cdots, \lambda_n$, and $\mu_1, \cdots, \mu_n$, which satisfy the following equations:*

$$\begin{aligned} &\frac{wh_i}{n_0 + \sum_{i=1}^{n} h_i p_i} - c_i h_{i0} + \mu_i - \lambda_i = 0, \\ &\mu_i p_i = 0, \\ &\lambda_i(p_i^{max} - p_i) = 0, \\ &\lambda_i \geq 0, \mu_i \geq 0, (i = 1, \ldots, n). \end{aligned} \tag{4}$$

Proof: Let $L(p, \lambda, \mu) = w\log\left(1 + \frac{\sum_i^n h_i p_i}{n_0}\right) - \sum_{i=1}^{n} c_i h_{i0} p_i + \sum_{i=1}^{n} \mu_i p_i + \sum_{i=1}^{n} \lambda_i (p_i^{max} - p_i)$, then by using Karush-Kuhn-Tucker (KKT) conditions [20], we can obtain

$$\begin{aligned} \frac{\partial L(p, \lambda, \mu)}{\partial p_i} &= 0, \\ \mu_i p_i &= 0, \\ \lambda_i (p_i^{max} - p_i) &= 0, \\ \lambda_i \geq 0, \mu_i \geq 0, (i &= 1, \ldots, n). \end{aligned} \tag{5}$$

Then, substituting $L(p, \lambda, \mu)$ into the first equation of (5), the proof of the Lemma 1 is completed.

Next, multiplying $p_i$ in the first equations of (4), we can get

$$\begin{aligned} &\frac{w h_i p_i}{n_0 + \sum_{i=1}^{n} h_i p_i} - c_i h_{i0} p_i + \mu_i p_i - \lambda_i p_i \\ &= \frac{w h_i p_i}{n_0 + \sum_{i=1}^{n} h_i p_i} - c_i h_{i0} p_i - \lambda_i p_i^{max} = 0 \end{aligned} \tag{6}$$

so the revenue of PU from SU $i$ can be written as:

$$c_i h_{i0} p_i = \frac{w h_i p_i}{n_0 + \sum_{i=1}^{n} h_i p_i} - \lambda_i p_i^{max}, \tag{7}$$

However, PU wants to choose a price $c_i (i = 1, \cdots, n)$ to maximize the total revenue $u$, that means $\lambda_i = 0$, $(i = 1, \cdots, n)$. Then, put (7) to (4), we can get following lemma:

**Lemma 2.** *When PU gets the maximal utility with the optimal price, the optimal transmit power for each SU can be obtained by solving the following problem:*

$$\begin{aligned} &\text{maximize} \quad u(p_1, \cdots, p_n) = \sum_{i=1}^{n} c_i h_{i0} p_i = \frac{w \sum_{i=1}^{n} h_i p_i}{n_0 + \sum_{i=1}^{n} h_i p_i} \\ &\text{subject to} \quad 0 \leq p_i \leq p_i^{max}, \ \sum_{j=1}^{n} h_{j0} p_j \leq T. \end{aligned} \tag{8}$$

Because (8) is increased with $\sum_{i=1}^{n} h_i p_i$, it can be equivalent to the following optimization problem:

$$\begin{aligned} &\text{maximize } \overline{u}\left(p_1, \cdots, p_n\right) = \sum_{i=1}^{n} h_i p_i \\ &\text{subject to } 0 \le p_i \le p_i^{max}, \ \sum_{j=1}^{n} h_{j0} p_j \le T. \end{aligned} \tag{9}$$

**Theorem 1.** *Sorting the index number of SUs in descending order by $h_i/h_{i0}$, the optimal transmit power $p_i$ of the ith SU is given by:*

$$p_i = \begin{cases} p_i^{\max}, \ if \sum_{j=1}^{i} h_{j0} p_j^{\max} \le T, \\ T - \sum_{j=1}^{i-1} h_{i0} p_i^{\max}, \ if \ \sum_{j=1}^{i-1} h_{j0} p_j < T \le \sum_{j=1}^{i} h_{j0} p_j, \\ 0, \ else. \end{cases} \tag{10}$$

*Proof:* We prove the theorem with different case of active constraint. First, we consider the case of the ITP constraint is satisfied under the maximum transmit power. That means $\sum_{i=1}^{n} h_{i0} p_i^{max} \le T$, then the problem (9) is equal to maximize the linear combination of each SU's transmit power with the weighting factor $h_i$ under the power constraint of $p_i^{max}$. Since the object function is increasing with $p_i$, then the optimal power is $p_i = p_i^{max}$. Next, we consider the case that $\sum_{i=1}^{n} h_{i0} p_i^{max} > T$ is valid. Since $\bar{u}$ increases with $p_i$, we have $\sum_{i=1}^{n} h_{i0} \tilde{p}_i = T$ at the optimal power $\tilde{p} = (\tilde{p}_1, \ldots, \tilde{p}_n)$. It means the ITL constraint is an active constraint. Moreover, as the object function is a linear function, the optimal power must exist at the extreme point of the constraint. So the optimal power must have an expression as (10). Furthermore, if the index number of two SU is not sorted in descending order by $h_i/h_{i0}$, we can change their power to optimize the objection function. From the above discussion, we have completed the proof of Theorem 1.

From Theorem 1 and the relationship between the optimal price and power in Lemma 1, we give the optimal price for PU as follows:

**Theorem 2.** *Let $p_i$ be the expression as (10), then the optimal price $c_i$ that PU charge for the ith SU can be written as*

$$c_i = \frac{w h_i}{h_{i0} \left( n_0 + \sum_{j=1}^{n} h_j p_j \right)}. \tag{11}$$

From Theorems 1 and 2, the SU has a better effective channel gain $h_i/h_{i0}$, it will have more opportunities to transmit. The optimal price for each SU is proportional to $\frac{h_i}{h_{i0}}$, which means the SU who has a better effective channel gain $\frac{h_i}{h_{i0}}$ will pay a higher price than others. Since PU charges a better SU with a higher price, the profit of PU will be higher. If the interference power is an active constraint, then the interference power to the PU will be equal to the ITP at the optimal price. The utility of PU is bounded by the effective channel gain of SU. If the effective channel gain is larger, PU will get more benefit. This is because the SU with the larger effective channel gain will prefer to pay the PU to access the spectrum. Moreover, if the $p_i^{max} \geq T/h_{i0}$, then the optimal price of the PU just allowed one SU who have the largest $\frac{h_i}{h_{i0}}$ to transmit with the power $T/h_{i0}$.

Since the optimal prices is expressed as a closed-form, PU can set them if it has all the channel information of the CR networks. Assuming that the user is indexed by the descending order of $h_i/h_{i0}$, PU can get the optimal price even if it doesn't know the channel information between the SUs and the BS by following distributed algorithm:

(1) PU chooses a uniform price $c_0$ large enough for all SU which can admit only one SU and the interference power is less than $T$.

(2) While the interference power is less than $T$, PU decreases the price $c_0$ until the new SU is admit or the interference power is equal to $T$, set the pricing for the $1-th$ SU by $c_1 = c_0$. If the former admit user $i$ decreases its power, PU reduces the maximal price $c_i$ to let the power of user $i$ to be $p_i^{max}$

(3) Repeated (2) for the new SU until interference power is equal to $T$ or all the SUs are admitted to transmit with their maximum power.

The proposed algorithm above is easy to implement in a distributed way. Moreover, the algorithm can find the optimal price for each SU as described by the following Lemma.

**Lemma 3.** *The distributed algorithm will be converged to the optimal price.*

*Proof:* Without of loss generality, we index the number of SUs such that $h_1/h_{10} > h_2/h_{20} > \ldots > h_n/h_{n0}$. From the distributed algorithm, all SUs will sequentially access the spectrum of PU with a uniform price $c_0$. As the price $c_0$ decreases, the SUs will admit in the spectrum in ascending order. PU decreases $c_0$ until ITL will meet with equal. Then, PU increases $c_i$ for those admitted users if the power of those users are unchanged. We only need to prove that the utility of PU is decreasing function of the price $c_i$ when the power used by the $i-th$ SU is less than its maximal power. Case one: $\sum_i^n h_{i0}p_i^{max} > T$. First, all SUs decrease to such that the interference power at the primary user is $T$. Set the price large enough for those not admission user and not admit them. The PU increases the price $c_i$ for the user $i$ if the power of user $i$ remains unchanged. Case two: $\sum_i^n h_{i0}p_i^{max} \leq T$. Then all the users are admitted to access the spectrum of the PU, the price $c_i$ updates as the case one.

Since the N-UP needs PU to measure each SUs interference power, this will be complex at the PU's receiver. Then we consider the UP case that the PU charges the total interference power from SUs by using the same price.

### 3.2 Uniform Pricing

In this section, PU sets a uniform interference power price $c = c_i, (i = 1, \cdots, n)$ for SUs. Thus, the optimal transmit power control strategy for SUs is denoted as:

$$\begin{aligned} &\text{maximize } \tilde{u}\left(p_1, \cdots, p_n\right) = w\log\left(1 + \frac{\sum_i^n h_i p_i}{n_0}\right) - c\sum_{i=1}^{n} h_{i0}p_i \\ &\text{subject to } 0 \leq p_i \leq p_i^{max}. \end{aligned} \tag{12}$$

And the PUs revenue optimization problem is:

$$\begin{aligned} &\text{maximize } u(c) = c\sum_{i=1}^{n} h_{i0}p_i \\ &\text{subject to } \sum_{i=1}^{n} h_{i0}p_i \leq T. \end{aligned} \tag{13}$$

**Theorem 3.** *Assuming that the ratio of channel coefficient between $SUs - BS$ link and $SUs - PU$ link denotes as $h_i/h_{i0}, (i = 1, \cdots, n)$ and the ratio order is decreasing as $h_1/h_{10} > h_2/h_{20} > \ldots > h_n/h_{n0}$. Given a uniform price $c$, the optimal transmit power from (12) can be expressed as:*

$$p_i = \max\left\{\min\{p_i^{\max}, w/\left(ch_{i0}\right) - \sum_{j=1}^{i-1} h_j p_j^{\max}/h_i - n_0/h_i\}, 0\right\}. \tag{14}$$

Before proving Theorem 3, we first give Lemma 4 to show the optimal power for each SU with uniform price $c$.

**Lemma 4:** *Assuming that $h_1/h_{10} > h_2/h_{20} > \ldots > h_n/h_{n0}$ and let $p^* = (p_1^*, \ldots, p_n^*)$ be the optimal solution of (13) for a fixed price $c > 0$. If $p_i^* < p_i^{\max}$, then $p_j^* = 0 \ (j = i+1, \ldots, n)$.*

Proof: We prove it by using the contradiction method. Let $p^* = (p_1^*, \ldots, p_n^*)$ be the optimal solution of (13). If $p_i^* < p_i^{\max}$, there exists $j > i$ such that $p_j^* > 0$. And the following must be satisfied.

$$\frac{\partial u}{\partial p_i}(p_1, \ldots, p_n)|_{p_i = p_i^*} = \frac{wh_i}{n_0 + \sum_{i=1}^{n} h_i p_i^*} - ch_{i0} = 0 \tag{15}$$

and

$$\frac{\partial u}{\partial p_j}(p_1, \ldots, p_n)|_{p_j = p_j^*} = \frac{wh_j}{n_0 + \sum_{i=1}^{n} h_i p_i^*} - ch_{j0} \geq 0. \tag{16}$$

From (15), $\frac{1}{n_0 + \sum_{i=1}^{n} h_i p_i^*} = \frac{ch_{i0}}{wh_i}$ is obtained, then substituting it into (16), we get

$$\frac{ch_{i0}h_j}{h_i} - ch_{j0} \geq 0,$$

that means $\frac{h_i}{h_{i0}} \leq \frac{h_j}{h_{j0}}$.

We can see that the above process contradicts the fact that $\frac{h_i}{h_{i0}} > \frac{h_j}{h_{j0}}\,(j > i)$. Thus, the proof is completed.

Lemma 4 shows that for the optimal solution to (13), if $\frac{h_i}{h_{i0}} > \frac{h_j}{h_{j0}}$, the user $j$ can't transmit when user $i$ transmit power is less than its maximum power.

Let $p^* = (p_1^*, \ldots, p_n^*)$ be the optimal solution of (13). Next, we prove Theorem 3 by considering different value of c.

Let $\overrightarrow{0} = (0, \ldots, 0)$, and $\overrightarrow{p_i^{max}} = (p_1^{max}, \ldots, p_i^{max}, \ldots, 0)$, that means the elements of $\overrightarrow{p_i^{max}}$ is zero while $j \geq i$,

Case 1: $c \geq \frac{wh_1}{n_0 h_{10}}$, $\frac{\partial u}{\partial p_i}(p_1, \ldots, p_n) = \frac{wh_i}{n_0 + \sum_{i=1}^{n} h_i p_i} - ch_{i0} \leq 0$, so the optimal power is $p^* = (p_1^*, \ldots, p_n^*) = (0, \ldots, 0)$.

Case 2: $\frac{wh_1}{(n_0 + h_1 p_1^{max})h_{10}} \leq c < \frac{wh_1}{n_0 h_{10}}$,

$$\frac{\partial u}{\partial p_1}(p_1, \ldots, p_n)|_{p=\overrightarrow{0}} = \frac{wh_1}{n_0} - ch_{10} > 0, \tag{17}$$

$$\frac{\partial u}{\partial p_1}(p_1, \ldots, p_n)|_{p=\overrightarrow{p_1^{max}}} = \frac{wh_1}{n_0\, +\, h_1 p_1^{max}} - ch_{10} \leq 0, \tag{18}$$

From (17), (18) and Lemma 4, we know when $\frac{wh_1}{(n_0 h_{10} + h_1 p_1^{max})} \leq c < \frac{wh_1}{n_0 h_{10}}$, only $p_1$ is not zero. From

$$\frac{\partial u}{\partial p_1}(p_1, \ldots, p_n)|_{(p_1, 0, \ldots, 0)} = \frac{wh_i}{n_0\, +\, h_1 p_1} - ch_{i0} = 0, \tag{19}$$

then $p_1$ is obtained as follows:

$$p_1 = \frac{w}{ch_{10}} - \frac{n_0}{h_{10}}, \tag{20}$$

So if $c \in \left(\frac{wh_1}{\left(n_0 + h_1 p_1^{\max}\right)h_{10}}, \frac{wh_1}{n_0 h_{10}}\right)$, the optimal solution is

$$p_1^* = \frac{1}{h_1}\left(\frac{wh_1}{ch_{10}} - n_0\right) = \frac{w}{ch_{10}} - \frac{n_0}{h_{10}}, p_i^* = 0\,(i = 2, \ldots, n)\,.$$

Case 3: $\frac{wh_2}{h_{20}(n_0 + h_1 p_1^{max})} \leq c \leq \frac{wh_1}{(n_0 + h_1 p_1^{max})h_{10}}$

$$\frac{\partial u}{\partial p_1}(p_1, \ldots, p_n)|_{p=\overrightarrow{p_1^{max}}} = \frac{wh_1}{n_0\, +\, h_1 p_1^{max}} - ch_{10} \geq 0, \tag{21}$$

$$\frac{\partial u}{\partial p_2}(p_1, \ldots, p_n)|_{p=\overrightarrow{p_1^{max}}} = \frac{wh_2}{n_0\, +\, h_1 p_1^{max}} - ch_{20} \leq 0, \tag{22}$$

From (21), (22) and Lemma 4, the optimal solution is

$$p_1^* = p_1^{max}, p_i^* = 0\,(i = 2, \ldots, n)\,,$$

when $c \in [\frac{wh_2}{h_{20}(n_0 + h_1 p_1^{max})}, \frac{wh_1}{(n_0 + h_1 p_1^{max})h_{10}}]$, Using the same argument, when $c \in \left(\frac{wh_i}{h_{i0}\left(n_0+\sum_{j=1}^{i} h_j p_j^{\max}\right)}, \frac{wh_i}{h_{i0}\left(n_0+\sum_{j=1}^{i-1} h_j p_j^{\max}\right)}\right)$ $(i=2,\ldots,n)$, For $j=1,\ldots,i-1$

$$\frac{\partial u}{\partial p_j}(p_1,\ldots,p_n)|_{p=\overrightarrow{p_j^{max}}} = \frac{wh_j}{n_0 + \sum_{k=1}^{j} h_k p_k^{max}} - ch_{j0} > 0, \tag{23}$$

$$\frac{\partial u}{\partial p_i}(p_1,\ldots,p_n)|_{p=\overrightarrow{p_i^{max}}} = \frac{wh_j}{n_0 + \sum_{k=1}^{i} h_k p_k^{max}} - ch_{i0} \leq 0, \tag{24}$$

From (23), (24) and Lemma 4, then the optimal solution is $p_1^* = p_1^{\max}, \ldots, p_{i-1}^* = p_{i-1}^{\max}, p_i^* = \frac{w}{ch_{i0}} - \frac{\left(n_0+\sum_{j=1}^{i-1} h_j p_j^{\max}\right)}{h_i}, p_j^* = 0\,(j=i+1,\ldots,n)$, when $c \in \left(\frac{wh_i}{h_{i0}\left(n_0+\sum_{j=1}^{i} h_j p_j^{\max}\right)}, \frac{wh_i}{h_{i0}\left(n_0+\sum_{j=1}^{i-1} h_j p_j^{\max}\right)}\right)$ $(i=2,\ldots,n)$.

when $c \in \left(\frac{wh_i}{h_{i0}\left(n_0+\sum_{j=1}^{i-1} h_j p_j^{\max}\right)}, \frac{wh_{i-1}}{h_{i-10}\left(n_0+\sum_{j=1}^{i-1} h_j p_j^{\max}\right)}\right)$ $(i=2,\ldots,n)$, For $j=1,\ldots,i-1$, we have

$$\frac{\partial u}{\partial p_j}(p_1,\ldots,p_n)|_{p=\overrightarrow{p_j^{max}}} = \frac{wh_j}{n_0 + \sum_{k=1}^{j} h_k p_k^{max}} - ch_{j0} > 0, \tag{25}$$

$$\frac{\partial u}{\partial p_i}(p_1,\ldots,p_n)|_{p=\overrightarrow{p_{i-1}^{max}}} = \frac{wh_i}{n_0 + \sum_{k=1}^{i-1} h_k p_k^{max}} - ch_{i0} < 0, \tag{26}$$

from (25), (26) and Lemma 4, then the optimal solution is $p^* = (p_1^{max},\ldots, p_{i-1}^{max}, 0,\ldots,0)$, when $c \in \left(\frac{wh_i}{h_{i0}\left(n_0+\sum_{j=1}^{i-1} h_j p_j^{\max}\right)}, \frac{wh_{i-1}}{h_{i-10}\left(n_0+\sum_{j=1}^{i-1} h_j p_j^{\max}\right)}\right)$ $(i = 2,\ldots,n)$, when $c \leq \frac{wh_n}{h_{n0}(n_0+\sum_{i=1}^{n} h_i p_i^{max})}$,

$$\frac{\partial u}{\partial p_i}(p_1,\ldots,p_n)|_{p=\overrightarrow{p_n^{max}}} = \frac{wh_i}{n_0 + \sum_{k=1}^{n} h_k p_k^{max}} - ch_{i0} \geq 0, \tag{27}$$

From (27), then the optimal solution is $p^* = (p_1^{max},\ldots,p_n^{max})$, when $c \leq \frac{wh_n}{h_{n0}(n_0+\sum_{i=1}^{n} h_i p_i^{max})}$.

From the above discussion, the optimal solution $p^*$ for a fixed price $c$ can be concluded as:

$$p^* = \begin{cases} (0,\cdots,0), & \text{if } c \geq \frac{wh_1}{h_{10}n_0}, \\ (w - \frac{ch_{10}n_0}{h_1}, 0,\ldots,0), & \text{if } \frac{wh_1}{h_{10}n_0} \geq c \geq \frac{wh_1}{h_{10}(n_0+h_1p_1^{max})}, \\ (p_1^{max}, 0,\ldots,0), & \text{if } \frac{wh_1}{h_{10}(n_0+h_1p_1^{max})} \geq c \geq \frac{wh_2}{h_{20}(n_0+h_1p_1^{max})}, \\ (p_1^{max},\ldots,p_{i-1}^{max},0,\ldots,0), & \text{if } \frac{wh_{i-1}}{h_{i-10}(n_0+\sum\limits_{j=1}^{i-1} h_j p_j^{max})} > c \geq \frac{wh_i}{h_{i0}(n_0+\sum\limits_{j=1}^{i-1} h_j p_j^{max})}, \\ (p_1^{max},\cdots,p_{i-1}^{max},p_i^*,0,\cdots,0), & \text{if } \frac{wh_i}{h_{i0}(n_0+\sum\limits_{j=1}^{i-1} h_j p_j^{max})} > c \geq \frac{wh_i}{h_{i0}(n_0+\sum\limits_{j=1}^{i} h_j p_j^{max})}, \\ (p_1^{max},\ldots,p_n^{max}), & \text{if } \frac{wh_n}{h_{n0}(n_0+\sum_{i=1}^{n} h_i p_i^{max})} \geq c, \end{cases} \tag{28}$$

where $p_i^* = c\sum_{j=1}^{i-1} h_{j0}p_j^{\max} + w - \frac{ch_{i0}\sum_{j=1}^{i-1} h_j p_j^{\max}}{h_i}$. Because the expression (14) in Theorem 3 is equivalent to (28), the proof of Theorem 3 is completed.

Theorem 3 shows that when the price $c \in \left[\frac{wh_i}{h_{i0}(n+\sum_{j=1}^{i-1} h_j p_j^{max})}, \frac{wh_i}{h_{i0}(n_0+\sum_{j=1}^{i} h_j p_j^{max})}\right]$, user $j(j < i-1)$ will use the maximal power $p_j^{max}$, user $i$ will use power $\frac{w}{ch_{i0}} - \frac{\sum_{j=1}^{i-1} h_j p_j^{\max}}{h_j} - \frac{n_0}{h_j}$, and other users' power will be zero. Therefore, at most one user's power less than the maximal power while others will transmit signal with maximal power or not transmit for a fixed price $c$.

Substitute the optimal power given by (14) and $c_i = c$ into (1), the optimization problem for the PU can be rewritten as:

$$\begin{aligned} &\text{maximize } u(c) = c\sum_{i=1}^{n} h_{i0}p_i \\ &\text{subject to } p_i \geq 0,\ \sum_{j=1}^{n} h_{j0}p_j \leq T, \end{aligned} \tag{29}$$

where $p_i = \max\left\{\min\{p_i^{\max}, w/(ch_{i0}) - \sum_{j=1}^{i-1} h_j p_j^{\max}/h_i - n_0/h_i\}, 0\right\}$.

We give the optimal solution to (29) and consider two different cases under the constraint of interference temperature limit:

Case One: $\sum_{i=1}^{n} h_{i0}p_i^{max} \leq T$, which means that the PU can tolerate the interference of all SUs with the maximum transmit power. When the price $c \in [b_i, a_i]$, $u(c)$ can be reduced to

$$u(c) = \begin{cases} 0, & \text{if } c \geq \frac{wh_1}{h_{10}n_0}, \\ w - \frac{ch_{10}n_0}{h_1}, & \text{if } \frac{wh_1}{h_{10}n_0} \geq c \geq \frac{wh_1}{h_{10}(n_0+p_1^{max})}, \\ ch_{10}p_1^{max}, & \text{if } \frac{wh_1}{h_{10}(n_0+p_1^{max})} \geq c \geq \frac{wh_2}{h_{20}(n_0+p_1^{max})}, \\ c\sum_{j=1}^{i-1} h_{j0}p_j^{\max} + w - \frac{ch_{i0}\sum_{j=1}^{i-1} h_j p_j^{\max}}{h_i}, & \text{if } b_i > c \geq a_i, \\ c\sum_{j=1}^{i-1} h_{j0}p_j^{max}, & \text{if } a_i > c \geq b_{i+1}, \\ c\sum_{j=1}^{n} h_{i0}p_i^{max}, & \text{if } \frac{wh_n}{h_{n0}(n_0+\sum_{i=1}^{n} h_i p_i^{max})} \geq c, \end{cases}$$

where $a_i = \frac{wh_i}{h_{i0}(n_0 + \sum_{j=1}^{i} h_j p_j^{max})}$, $b_i = \frac{wh_i}{h_{i0}(n_0 + \sum_{j=1}^{i-1} h_j p_j^{max})}$$(i = 1, \cdots, n)$. $u(c)$ is a piecewise linear function and $u(a_i) > u(b_{i+1})$, so the optimal price $c^*$ can be written as:

$$c^* = \underset{a \in \{a_1, \ldots, a_n\}}{\arg\max} \; u(a) \tag{30}$$

Case Two: When $\sum_{j=1}^{n} h_{i0} p_i^{max} > T$, there must exist $k \in (1, \cdots, n)$ such that

$$\sum_{i}^{k-1} h_i p_i^{max} < T, \tag{31}$$

and

$$\sum_{i}^{k} h_i p_i^{max} \geq T, \tag{32}$$

Using Lemma 4, only the first $k$ users can be admitted to transmit when the interference power constraint is satisfied. From (12), the power of each user deceases with $c$, there exists a maximal $c_{max}$ such that the transmit power of the user will be $p_{max} = (p_1^{max}, \cdots, p_{k-1}^{max}, (T - \sum_i^k p_i^{max})/h_{k0}, 0, \cdots, 0)$.

$$c_{max} = \frac{wh_k}{h_{k0}(n_0 + \sum_i^{k-1} h_i p_i^{max}) + h_k(T - \sum_i^k p_i^{max})} \tag{33}$$

Since the $u(c)$ is piecewise linear function in $[c_{max}, \infty)$, using the same argument as case one, the optimal price $c^*$ can be written as:

$$c^* = \underset{a \in \{a_1, \ldots, a_{k-1}, c_{max}\}}{\arg\max} \; u(a) \tag{34}$$

From (30) and (34), the optimal price for PU is given. It can be seen that the complexity to find the optimal price is at most $O(n)$, where $n$ is the number of SUs.

We give the closed-form optimal price for the N-UP scheme and propose the best power control strategies for CR networks to admit one user access the spectrum if the transmit power of each user is large enough. Then, the simulations compare the influence of N-UP and UP scheme on the performance of PU and SUs in the next section.

## 4 Simulation Results

In this section, we evaluate the performance of the proposed pricing scheme. The channel gains of all links experience Rayleigh fading with the variance of 1. We set $w = 1$, $T = 1$, $p_i^{max} = 10$, $\forall i$, the variance of the noise is 1. The channel gain $h$ and $h_0$ is randomly generated $10^4$ times in our simulations. Figures 2 and 3 show the utility of PU and SUs versus the number of SUs. It can be seen that

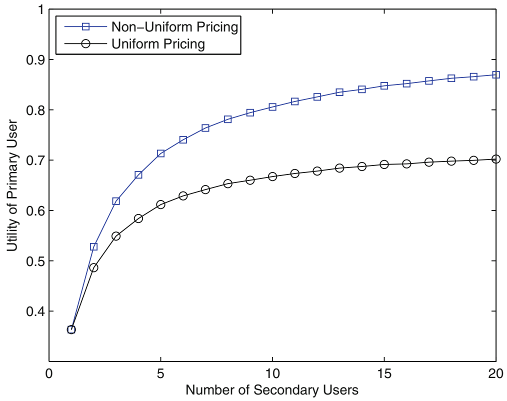


**Fig. 2.** Utility of primary user versus the number of secondary users.

the utilities of PU and SUs increases with the number of SUs. This is because the probability of the channel gains of SUs is better when the number of SUs is larger. Therefore, SUs need to pay more for PU in order to gain more profit.

Figure 4 shows the interference power of PU versus the number of SUs. We can see that the interference power under the N-UP scheme is larger than UP scheme when the number of SUs is more than one. Moreover, the interference power of PU under the N-UP scheme increases as the number of SUs first, then they meet the interference power when the number of SUs is more than five. However, the interference power of PU under the UP scheme decreases as the number of SUs increases. The difference between these two schemes is that the interference power limit is always attained for N-UP scheme when the maximal interference power made by SUs is larger than the interference power limit. For

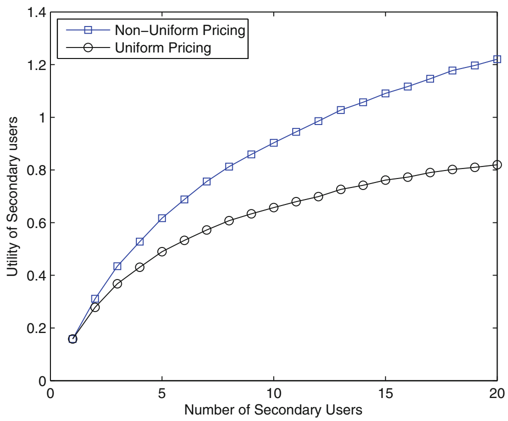


**Fig. 3.** Utility of secondary users versus the number of secondary users.

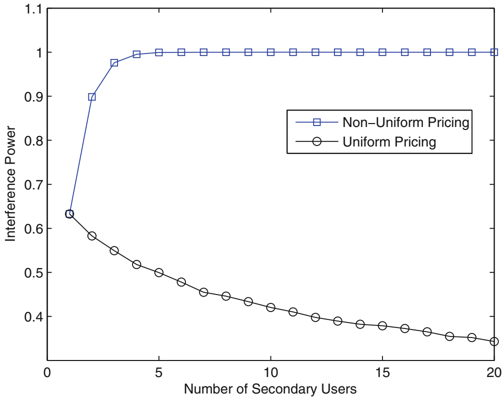


**Fig. 4.** Total interference power versus the number of secondary users.

UP scheme, when the utility of PU is determined by the channel gain of SUs other than the interference power limit, so the interference power limit is not meet as equality at the optimal price.

Figure 5 shows the sum rate of SUs versus the number of SUs. The sum rate under both UP and N-UP scheme increases as the number of SUs increases. This is because the N-UP scheme also allows SUs to transmit more power than UP scheme, which can be seen from Fig. 6.

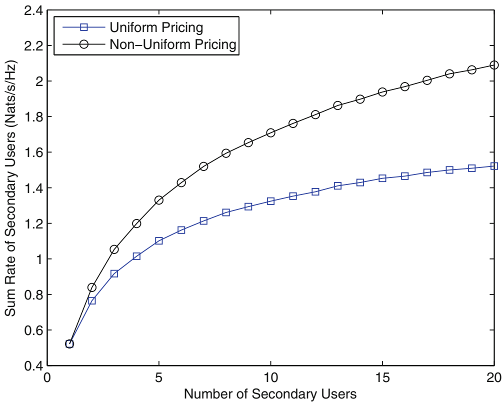


**Fig. 5.** Sum rate of secondary users versus the number of secondary users.

Figures 7 and 8 shows the utility of the PU and SUs versus the ITP when $p_i^{max} = 10\,\text{dB}$ and $n = 8$ is given in the CR networks. We can see the utility of SU of two schemes increases with ITP. While ITP is less than −5 dB, two schemes have the same utility. This is because the optimal two pricing scheme

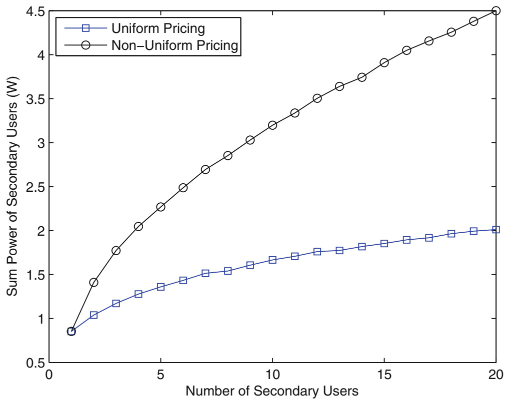


**Fig. 6.** Sum power of secondary users versus the number of secondary users.

allows only one user has the best channel condition to access the spectrum. As ITP increases from −5 dB to 30 dB, the utility of PU under N-UP scheme is larger than UP scheme. The reason is that N-UP scheme allows more SUs to access the spectrum. And the utilities of SUs of two schemes begin to saturate when ITP reaches 20 dB.

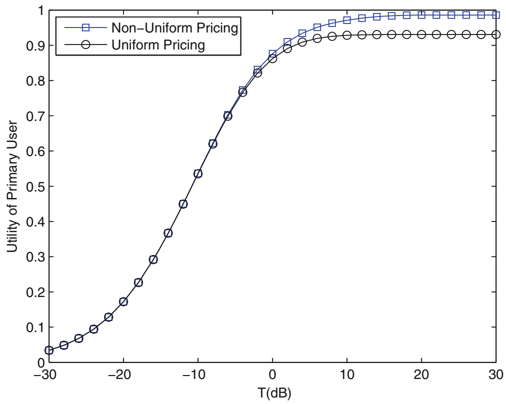


**Fig. 7.** Utility of primary user versus the interference temperature power.

Figure 9 shows the interference power versus ITP. The interference power of two schemes are the same when ITP is less than −5 dB, because the optimal price for two schemes are the same when ITP is small. As ITP increases from −5 dB to 30 dB, the interference power of N-UP scheme is larger than NU scheme. This is because the N-UP scheme always allows more users to transmit at their maximal power until the interference power constraint is satisfied.

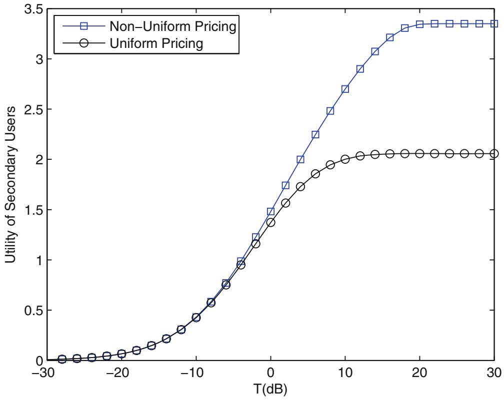


**Fig. 8.** Utility of secondary users versus the interference temperature power.

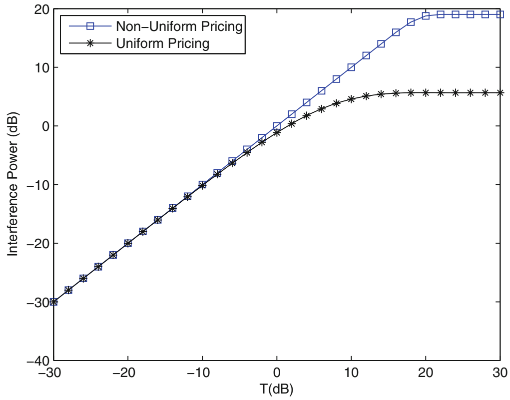


**Fig. 9.** Interference power versus the interference temperature power.

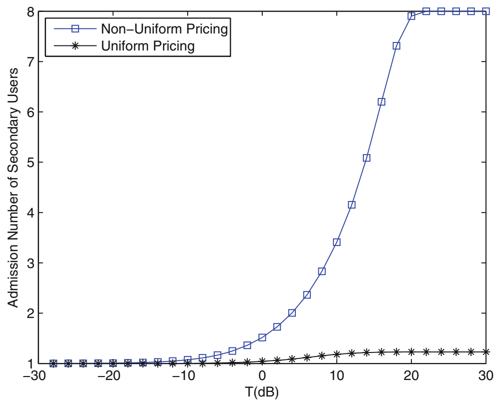


**Fig. 10.** Admission number of secondary users versus the interference temperature power.

Figure 10 shows the average admission SUs versus the ITP. When the ITP is less than −5 dB, both schemes admit one user. As the ITP increases from −5 dB to 30 dB, the admission number of SUs of N-UP scheme increases, that means all SUs will be allowed to access the spectrum when the ITP is large. However, the admission number of SUs of UP scheme is less than two even the ITP is large enough. This is because the interference power is always not equal to ITL at the optimal price for uniform scheme.

## 5 Conclusion

In this paper, we consider the price-based power control problem for CR-NOMA networks which contains one base station (BS), multiple SUs and one PU, and SIC is employed at receiver. We first model the pricing and power control strategies between PU and SUs as a Stackelberg game based on the interference temperature power. Then, PU plays a leader in the game and chooses a price for SUs in order to obtain maximum revenue under ITP limit. Moreover, SUs act as followers to select the optimal power while considering their payoff to PU. Furthermore, the non-uniform pricing scheme and uniform pricing scheme are proposed to evaluate the revenue of PU and SUs. Simulation results compare the different performance indexes of two schemes.

## References

1. Hong, X., Wang, J., Wang, C., Shi, J.: Cognitive radio in 5G: a perspective on energy-spectral efficiency trade-off. IEEE Commun. Mag. **52**(7), 46–53 (2014)
2. Liang, Y., Zeng, Y., et al.: Sensing-throughput tradeoff for cognitive radio networks. IEEE Trans. Wirel. Commun. **7**(4), 1326–1337 (2008)
3. Hasegawa, M., Hirai, H., et al.: Optimization for centralized and decentralized cognitive radio networks. Proc. IEEE **102**(4), 574–584 (2014)
4. Islam, S.M.R., Avazov, N., Dobre, O.A., Kwak, K.: Power-domain non-orthogonal multiple access (NOMA) in 5G systems: potentials and challenges. IEEE Commun. Surv. Tutor. **19**(2), 721–742 (2017)
5. Higuchi, K., Benjebbou, A.: Non-orthogonal multiple access (NOMA) with successive interference cancellation for future radio access. IEICE Trans. Commun. **98**(3), 403–414 (2015)
6. Ding, Z., et al.: Application of non-orthogonal multiple access in LTE and 5G networks. IEEE Commun. Mag. **55**(2), 185–191 (2017)
7. Lv, L., Chen, J., Ni, Q., Ding, Z., Jiang, H.: Cognitive non-orthogonal multiple access with cooperative relaying: a new wireless frontier for 5G spectrum sharing. IEEE Commun. Mag. **56**(4), 188–195 (2018)
8. Liu, Y., Ding, Z., Elkashlan, M., Yuan, J.: Nonorthogonal multiple access in large-scale underlay cognitive radio networks. IEEE Trans. Veh. Technol. **65**(12), 10152–10157 (2016)
9. Liang, W., Li, L., Shi, J., Fang, F., Ding, Z.: Cooperative game aided spectrum sharing in underlay cognitive radio networks employing NOMA schemes. In: Proceedings 2018 IEEE Globecom Workshops (GC Wkshps), Abu Dhabi, pp. 1–6 (2018)

10. Song, Z., Wang, X., Liu, Y., Zhang, Z.: Joint spectrum resource allocation in NOMA-based cognitive radio network with SWIPT. IEEE Access **7**, 89594–89603 (2019)
11. Mao, S., Leng, S., Hu, J., Yang, K.: Power minimization resource allocation for underlay MISO-NOMA SWIPT systems. IEEE Access **7**, 17247–17255 (2019)
12. Xie, X., Yang, H., Vasilakos, A.V., He, L.: Fair power control using game theory with pricing scheme in cognitive radio networks. J. Commun. Netw **16**(2), 183–192 (2014)
13. Yang, C., Li, J., Tian, Z.: Optimal power control for cognitive radio networks under coupled interference constraints: a cooperative game-theoretic perspective. IEEE Trans. Veh. Technol. **59**(4), 1696–1706 (2010)
14. Yu, H., Gao, L., Li, Z., Wang, X., Hossain, E.: Pricing for uplink power control in cognitive radio networks. IEEE Trans. Veh. Technol. **59**(4), 1769–1778 (2010)
15. Wang, Z., Jiang, L., He, C.: A novel price-based power control algorithm in cognitive radio networks. IEEE Commun. Lett. **17**(1), 43–46 (2013)
16. Wang, Z., Jiang, L., He, C.: Optimal price-based power control algorithm in cognitive radio networks. IEEE Trans. Wirel. Commun. **13**(11), 5909–5920 (2014)
17. Kang, X., Zhang, R., Motani, M.: Price-based resource allocation for spectrum-sharing femtocell networks: a stackelberg game approach. IEEE J. Sel. Areas Commun. **30**(3), 538–549 (2012)
18. Wang, Z., Wen, C., Fan, Z., Wan, X.: A novel price-based power allocation algorithm in non-orthogonal multiple access networks. IEEE Wirel. Commun. **7**(2), 230–233 (2018)
19. Wang, Z., Xiao, W., Wan, X., Fan, Z.: Price-based power control algorithm in cognitive radio networks via branch and bound. IEICE Trans. Inf. Syst. **102**(3), 505–511 (2019)
20. Luo, Z.-Q., Yu, W.: An introduction to convex optimization for communications and signal processing. IEEE J. Sel. Areas Commun. **24**(8), 1426–1438 (2006)