Formulate Full View Camera Sensor Coverage by Using Group Set Coverage

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Abstract. Full view is an important subject in the study of wireless camera sensor coverage. It involves three issues, camera direction selection, camera location selection, and moving-directional target coverage. In this paper, we study them step by step, from one issue, two issue, to three issues. In each step, we formulate a group set cover- age problem, and study its computational complexity and approximation solutions. Especially, we employ a new type of approximations, the global approximation of local optimality for nonsubmodular optimization.

1 Introduction

Nowadays, the camera sensor plays an important role in our daily life. It has been installed in almost everywhere, such as supermarkets, home, street intersections, cars, etc. The camera sensor is widely used on tracing criminal activities. Currently, it is also used for tracing COVID-19 virus. In study of camera sensors, the coverage is an important issue.

The camera sensor is a type of directional sensor. The covering region of a camera sensor is a sector with center at the camera sensor (Fig. 1). The radius and the angle of the sector are called *sensing radius* and *sensing angle*, respectively. A target point is said to be *covered* by a camera sensor if it lies in the covering region of the camera sensor. The *sensing direction* is represented by the

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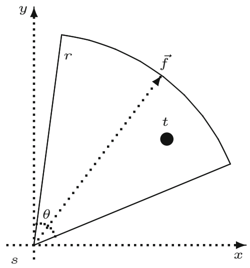

Fig. 1. Camera sensor s with sensing radius r, angle θ, direction $\boldsymbol{f}$ covers a target t.

ray dividing the sensing angle evenly, which is usually adjustable continuously or discretely.

Consider a set of target points and a camera sensor. When the sensing direction is rotated to different positions, the camera sensor would cover different subsets of target points. Therefore, each camera sensor would generate a *group* of subsets of target points. Using a group of subsets to represent a camera sensor, each camera sensor coverage problem may be formulated as a group set cover problem.

Among various coverage requirements, the full view is a tough one. However, it is very important since it enable the sensor system to monitor moving targets.

In this paper, we are going to study the full-view camera sensor coverage problem, which involves three issues:

– Camera sensing direction selection.
– Camera location selection.
– Coverage of moving-directional targets.

We organize our study into three steps based on consideration of one issue, two issues, until all three issues. In each step, we will formulate considered camera coverage problem into a group set coverage problem. Since in all three steps, formulated group set coverage problems are NP-hard, we design and analysis of its approximation solution.

Related Works. The sensor coverage was studied initially on wireless sensors with omni antennas and a set of target points or a target area. For example, Cardei *et al.* [1] studied the maximum lifetime coverage problem on a set of homogeneous wireless sensors with omni antennas and a set of target points and raised a long-standing open problem on the existence of polynomial-time constant-approximation algorithm. Ding *et al.* [2] gave the first positive solution for the open problem by presenting a polynomial-time 4-approximation, and Li and Jin [3] found a PTAS (polynomial-time approximation scheme). Wu *et al.* [4] studied the connected sensor cover problem on homogeneous wireless sensors with omni antennas and a target area, and raised an open problem on the existence of polynomial-time constant-approximation algorithm. Actually, they showed the existence of a polynomial-time $O(r)$-approximation and a polynomial-time $O(\log^2 n \log\log n)$-approximation where r and n are two unrelated parameters, which strongly suggests the existence of a polynomial-time constant-approximation. However, this problem is still open today.

Meanwhile, many efforts [5–10] have been made on extending research work from wireless sensors with omni antennas to directional sensors, i.e., wireless sensors with directional antennas so that the covering region is as shown in Fig. 1. Such extension may meet surprising trouble. For example, for wireless sensors with omni antennas, the minimum sensor cover problem has PTAS [10]. However, for directional sensors covering target points, the corresponding minimum sensor cover problem has not been found to have a polynomial-time constant-approximation so far [5].

The camera sensor is a type of directional sensors. In the study of camera sensors, the popular target object is the directional target, such as the human face [11] which can be identified only from certain range of direction. A directional target u has a facing angle at u and a facing direction which bisects the facing angle evenly (Fig. 2). A directional target u is *covered* by a camera sensor a if u lies in the covering region of s and segment su lies in both the sensing angle of s and the facing angle of u as shown in the middle of Fig. 3. In the study of the camera sensor coverage on directional targets, many efforts [12–14] have been made in the literature. Among them, it may be worth mentioning the work of Wang and Cao [15] who proposed an important concept "full view". A directional target is *full-view* covered by a set of camera sensors if for the target in any facing direction, there exists a camera sensor covering the target. An area is *full view* covered by a set of camera sensors if a directional sensor lying at any point in the area can be full view covered by the set of camera sensors.

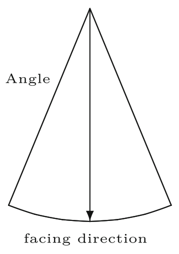

Fig. 2. Directional target.

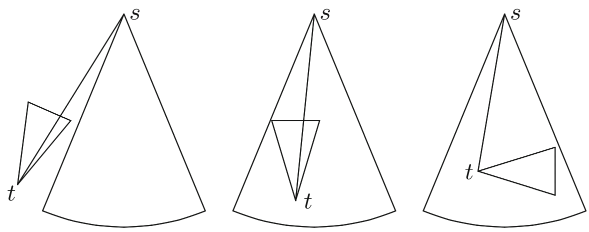

Fig. 3. A directional target t is covered by a camera sensor s (in middle case).

Many issues on full view have been studied in the literature, such as full view barrier cover [16,17], full view area coverage [18,19], lifetime of full view coverage [20], orientation scheduling [21]. However, none of them have touched the maximum coverage problem studied in this paper, especially for full view coverage.

Our Contribution. In this paper, we study full-view coverage in camera sensor networks, in which every camera's direction is adjustable and every target has a moving direction. Our study consists of three steps.

1. In the first step, we study how to select the camera direction and formulate it into the maximum group set coverage problem. This problem is proved to be NP-hard and it has a polynomial-time $(1 - 1/e)$-approximation. In this step, considered targets are target points or fixed-directional targets.
2. In the second step, we transform each moving-directional target into a set of fixed-directional targets, so that the full-view camera coverage problem can be formulated into the maximum group set coverage with composed targets. This is also NP-hard. We will present an approximation which has a global performance ratio $(1 - 1/e)^\alpha(1 + \varepsilon)$ for local optimality.
3. In the third step, we give a constraint on the number of cameras and face a camera location selection problem. Our formulation is the maximum group set coverage with size constraint and composed targets. We show that this is an NP-hard problem which also has an approximation with a global performance ratio $(1 - 1/e)^\alpha(1 + \varepsilon)$ for local optimality.

The global approximation for local optimality is a new type of approximation algorithms for nonsubmodular optimizations [22]. The study of such a type of approximations is in the background that classic approximation performance measure, the ratio of objective function values between approximation solution and (global) optimal solution does not work well. In this paper, we are going to employ the technique of nonconvex relaxation to obtain the global approximation for local optimality.

2 Problem Formulation

Consider a heterogeneous camera sensor network. We will formula various camera sensor coverage problems into group set cover problems.

Note that each camera can rotate its sensing direction, and at each orientation, it covers a subset of target points or a subset of fixed-directional targets. Therefore, selecting a sensing direction corresponds to selecting one subset from a group. Hence, the sensing direction selection problem for covering the maximum number of target points or fixed-directional targets can be formulated as the following problem.

Problem 1 (Maximum Group Set Coverage). *Given m groups $\mathcal{G}_1$, $\mathcal{G}_2$, ..., $\mathcal{G}_m$ of subsets of a finite set X, select one subset from each group to cover the maximum total number of elements.*

To reduce the maximum full-view coverage problem to a group set coverage problem, we replace each full-view directional target by a finite number of fixed-directional targets as follows.

Lemma 1. *Consider a directional target t with facing angle α. Then t can be replaced by $\lceil 4\pi/\alpha \rceil$ fixed-directional targets with facing angle at most $\alpha/2$ such that if these fixed-directional targets are covered by a set of camera sensors, $\mathcal{S}$, then t is full-view covered by $\mathcal{S}$.*

Proof. Divide angle 2π at t evenly into $\lceil 4\pi/\alpha \rceil (= h)$ angles α_1, ..., α_h each of at most $\alpha/2$. Replace target t by h targets at location of t with fixed facing angles α_1, ..., α_h, respectively (Fig. 4).

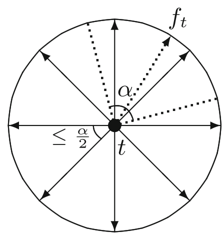

Fig. 4. Proof of Lemma 1.

Then for any orientation of target t, its facing direction must be contained in an angle α_i. This angle α_i must be entirely contained in the facing angle of t in current orientation. Since the target with facing direction α_i is covered by the set of camera sensors, $\mathcal{S}$, we can immediately see that target t in current orientation is covered by $\mathcal{S}$. □

Now, the full-view coverage of a directional target is represented by a composition of several fixed-directional targets. Hence, the sensing direction selection problem for full-view directional targets can be formulated as follows.

Problem 2 (Maximum Group Set Coverage with Composed Targets). *Consider m groups $\mathcal{G}_1$, $\mathcal{G}_2$,..., $\mathcal{G}_m$ of subsets of a finite set X. All elements in X are partitioned into subsets X_1, X_2,... X_r, each called a* composed target. *The problem is to select one subset from each group to cover the maximum total number of composed targets, where a composed target is said to be covered if all elements in the composed target are covered.*

Note that each composed target X_t corresponds to a full-view directional target t and all elements in X_t correspond to fixed-directional targets obtained from decomposing the full-view directional target t by Lemma 1. We may assume that for any camera sensor s, the line segment ts does not overlap with any boundary of the decomposition (otherwise the decomposition can be rotated a little to avoid overlap). This property implies the following observation.

Lemma 2. *For every composed target X_t, no subset in any group $\mathcal{G}_i$ covers two elements in X_t.*

Now, consider the camera sensor allocation problem. Since each possible location for camera sensor may also be represented by a group of subsets, selecting k locations for camera sensors corresponds to selectiing k groups. Therefore, the camera sensor allocation problem for full-view directional targets can be formulated as the following problem.

Problem 3 (Maximum Group Set Coverage with Size Constraint). *Consider m groups $\mathcal{G}_1$, $\mathcal{G}_2$,..., $\mathcal{G}_m$ of subsets of a finite set X. All elements in X are partitioned into subsets X_1, X_2,..., X_r, each called a* composed target*, and they satisfy property sated in Lemma 2. Given an integer $k > 0$, the problem is to select k groups and then select one subset from each selected group to cover the maximum total number of composed targets.*

The camera sensor allocation for target points and fixed-directional targets can be formulated into a special case of Problem 3; in such a case, every composed target contains only one element.

3 Maximum Group Set Coverage

3.1 Complexity

A *group set cover* is a collection of subset groups $\mathcal{G}_1$, $\mathcal{G}_2$, ..., $\mathcal{G}_m$ such that all elements can be covered by $\cup_{i=1}^{m} S_i$ where S_i is selected from group $\mathcal{G}_i$. The group set cover problem can be seen as an extension of the set cover problem, since the latter can be viewed as a special case of the former in which each group has only one subset.

Problem 4 (Group Set Cover). *Given a collection of subset groups $\mathcal{G}_1$, $\mathcal{G}_2$, ..., $\mathcal{G}_m$, does this collection form a group set cover?*

Theorem 1. *The group set cover problem is NP-complete.*

Proof. The problem belongs to NP since we can one subset from each group and check if their union cover all elements in polynomial-time. To show NP-hardness of the group set cover problem, consider a well-known P-hard problem, the minimum set cover problem. Its decision version is as follows: Given a collection $\mathcal{C}$ of subsets of a finite set X, and an integer $k > 0$, find out whether there are at most k subsets in $\mathcal{C}$ such that their union contains X.

To construct a polynomial-time reduction, we set $\mathcal{G}_1 = \mathcal{G}_2 = \cdots = \mathcal{G}_k = \mathcal{C}$. Then $\mathcal{C}$ has a subcollection of at most k subsets (say $\{S_{i_1}, S_{i_2}, \ldots, S_{i_\ell}\}$ with $\ell \leq k$) covering X if and only if $\{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_\ell\}$ and $\{S_{i_1}, S_{i_2}, \ldots, S_{i_\ell}\}$ form a group set cover, where S_{i_j} is viewed to be a set selected from group $\mathcal{G}_j$.

Corollary 1. *The maximum group set coverage problem is NP-hard.*

3.2 Approximation Solution

The maximum group set coverage problem can be formulated into a 0–1 linear programming as follows: Consider m groups $\mathcal{G}_1, ..., \mathcal{G}_m$ of subsets of a finite set X. Note that an indicator for an event is a 0–1 variable to indicate whether the event occurs or not. Let x_{iS} be the indicator for event that subset S is selected from group $\mathcal{G}_i$. Denote $n = |X|$ and $\mathcal{G} = \mathcal{G}_1 \cup \cdots \cup \mathcal{G}_k$. Let y_j be the indicator for event that element j is covered by some selected subset.

$$\begin{aligned} \max\ & y_1 + y_2 + \cdots + y_n & (ILP1) \\ \text{s.t.}\ & y_j \le \sum_{i=1}^{m} \sum_{S:j\in S\in\mathcal{G}_i} y_{iS} \quad \forall j = 1, ..., n, \\ & \sum_{S:S\in\mathcal{G}_\rangle} y_{iS} \le 1 \quad \text{for all } i = 1, ..., m, \\ & y_j \in \{0, 1\} \quad \text{for all } j = 1, ..., n, \\ & x_{iS} \in \{0, 1\} \quad \text{for all } S \in \mathcal{G} \text{ and } i = 1, 2, ..., m. \end{aligned}$$

By LP-relaxation and randomized rounding technique, we can compute an approximation (x_{iS}, y_j) such that following result holds. The computation time is $O((n + mg)^{3.5})$ where $n = |X|$, m is the number of groups and g is the maximum number of subsets in a group. The detail can be found in [26].

Theorem 2.

$$E\left[\sum_{j=1}^{n} y_j\right] \ge (1 - e^{-1})opt_1$$

where opt_1 is the objective value of an optimal solution for the group set coverage problem (ILP1).

4 Maximum Group Set Coverage with Composed Targets

Note that the maximum group set coverage problem can be considered as a special case of the maximum group set coverage problem with composed targets, in which every composed target contains only one element. Since the former is NP-hard, so is the latter. Therefore, we only focus on algorithm design and analysis in this section.

Consider m groups $\mathcal{G}_1, ..., \mathcal{G}_m$ of subsets of a finite set X which is partitioned into τ composed targets X_1, X_2, ..., X_τ, i.e., $X = X_1 \cup X_2 \cup \cdots X_\tau$ and $X_t \cap X_{t'} = \emptyset$ for $1 \le t < t' \le \tau$. Recall the observation made in Lemma 2. Let x_{iS} be the indicator for event that subset S is selected from group $\mathcal{G}_i$. Denote $n = |X|$ and $\mathcal{G} = \mathcal{G}_1 \cup \cdots \cup \mathcal{G}_k$. Let y_j be the indicator for event that element j is covered by some selected subset. The following is a 0–1 multi-linear programming.

$$\begin{aligned}
\max\ & \sum_{t=1}^{\tau} \prod_{j \in X_t} y_j && (INLP2)\\
\text{s.t. } & y_j \le \sum_{i=1}^{m} \sum_{S: j \in S \in \mathcal{G}_i} x_{iS} \quad \text{for all } j = 1, ..., n,\\
& \sum_{S: S \in \mathcal{G}_i} x_{iS} \le 1 \quad \text{for all } i = 1, ..., m,\\
& y_j \in \{0, 1\} \quad \text{for all } j = 1, ..., n,\\
& x_{iS} \in \{0, 1\} \quad \text{for all } S \in \mathcal{G}_i \text{ and } i = 1, 2, ..., m.
\end{aligned}$$

Its relaxation is as follows.

$$\begin{aligned}
\max\ & \sum_{t=1}^{\tau} \prod_{j \in X_t} y_j && (NLP2)\\
\text{s.t. } & y_j \le \sum_{i=1}^{m} \sum_{S: j \in S \in \mathcal{G}_i} x_{iS} \quad \text{for all } j = 1, ..., n,\\
& \sum_{S: S \in \mathcal{G}_i} x_{iS} \le 1 \quad \text{for all } i = 1, ..., m,\\
& 0 \le y_j \le 1 \quad \text{for all } j = 1, ..., n,\\
& 0 \le x_{iS} \le 1 \quad \text{for all } S \in \mathcal{G}_i \text{ and } i = 1, 2, ..., m.
\end{aligned}$$

Let (y_j^*, x_{iS}^*) be a $(1 - \varepsilon)$-approximate solution for local optimality of the multilinear programming (NLP2). A randomized rounding is executed as follows.

Randomized Rounding: With probability x_{iS}^*, randomly select one subset S from each group $\mathcal{G}_i$ (i.e., set $x_{iS} = 1$ and $x_{iS'} = 0$ for $S' \ne S$). Set

$$y_j = \begin{cases} 1 & \text{if element } j \text{ is covered by a selected subset,} \\ 0 & \text{otherwise.} \end{cases}$$

Let (x_{iS}, y_j) be a solution obtained from this randomized rounding. Similar to the proof of Lemma 3, we have the following properties.

Lemma 3. $E[y_j] \ge (1 - e^{-1}) y_j^*$.

Proof. For each $j = 1, ..., n$,

$$\begin{aligned}
\text{Prob}[y_j = 1] &= 1 - \prod_{i=1}^{m} \prod_{S: j \in S \in \mathcal{G}_i} (1 - x_{iS}^*) \ge 1 - \left(\frac{\sum_{i=1}^{m} \sum_{S: j \in S \in \mathcal{G}_i} (1 - x_{iS}^*)}{K_j} \right)^{K_j}\\
&= 1 - \left(1 - \frac{\sum_{i=1}^{m} \sum_{S: j \in S \in \mathcal{G}_i} x_{iS}^*}{K_j} \right)^{K_j} \ge 1 - \left(1 - \frac{y_j^*}{K_j} \right)^{K_j}.
\end{aligned}$$

where $K_j = |\{(i, S) \mid j \in S \in \mathcal{G}_i\}|)$.

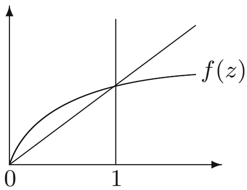

Fig. 5. Function $f(z)$.

Since the function $f(z) = 1 - (1 - \frac{z}{K_j})^{K_j}$ is monotone increasing and concave in $[0, 1]$, the function curve is above the line segment between two points $(0, f(0))$ and $(1, f(1)$ (Fig. 5). Note that $f(0) = 0$ and $f(z) = 1 - (1 - \frac{1}{K_j})^{K_j}$. Therefore,

$$f(z) \geq \left(1 - \left(1 - \frac{1}{K_j}\right)^{K_j}\right) z \geq (1 - e^{-1})z.$$

Thus,

$$\text{Prob}[y_j = 1] \geq (1 - e^{-1})y_j^*.$$

Hence, $E[y_j] = \text{Prob}[y_j = 1] \geq (1 - e^{-1})y_j^*$. □

Lemma 4. $E[\prod_{j \in X_t} y_j] = \prod_{j \in X_t} E[y_j]$.

Proof. By Lemma 2, for a fixed composed target t, events $\{y_j = 1\}_{j \in X_t}$ are independent. Therefore, this lemma holds. □

Theorem 3. *Let (y_j, x_{iS}) be an approximate solution obtained by the above randomized rounding. Then*

$$E\left[\sum_{t=1}^{\tau} \prod_{j \in X_t} y_j\right] \geq (1 - e^{-1})^{\alpha}(1 - \varepsilon)lopt_2$$

where $lopt_2$ is the objective value of a local optimal solution of (NLP2) and $\alpha = \max_{1 \leq t \leq \tau} |X_t|$.

Proof. By Lemmas 3 and 4, we have

$$E\left[\sum_{t=1}^{\tau} \prod_{j \in X_t} y_j\right] = \sum_{t=1}^{\tau} E\left[\prod_{j \in X_t} y_j\right] = \sum_{t=1}^{\tau} \prod_{j \in X_t} E[y_j] \geq (1 - e^{-1})^{\alpha} \sum_{t=1}^{\tau} \prod_{j \in X_t} y_j^*$$
$$= (1 - e^{-1})^{\alpha}(1 - \varepsilon) \cdot lopt_2.$$

The theorem is proved. □

We summarize the above randomized algorithm into Algorithm 2.

Algorithm 1 Approximation for maximum group set coverage with composed targets

input m groups $\mathcal{G}_1$, $\mathcal{G}_2$, ..., $\mathcal{G}_m$ of subsets of a finite set X and a partition of X into composed targets $X = X_1 \cup X_2 \cup \cdots \cup X_\tau$.
output a collection $\mathcal{S}$ of subsets of X.

```
0:  S ← ∅;
1:  solve (NLP2) to obtain a (1 − ε)-approximate solution (x*_iS, y*_j);
2:  for i = 1 to m do
3:      select one subset S from G_i with
         probability x*_iS;
4:      S ← S ∪ {S}
5:  end-for
6:  return S.
```

The running time of Algorithm 1 depends on the running time for computing local optimal solutions (Kuhn-Tucker points) of (NLP2), which is a multilinear maximization with linear constraints. By a sequence of linear programs [23,24], a sequence of feasible points can be generated to be globally convergent to those local optimal solutions. However, it is open to analyze the running time to obtain a $(1-\varepsilon)$-approximation. In fact, those algorithms have linear convergence rate, which means that under certain condition, the number of steps can be bounded by a polynomial with respect to $O(1/\varepsilon)$. However, in general case, such an upper bound is proved only for quadratic objective function [27].

5 Maximum Group Set Coverage with Size Constraint

Note that the element coverage problem can be viewed as a special case of the composed target coverage problem. In such a case, every composed target contains only one element. Therefore, with size constraint, we only study composed targets. Moreover, the problem is NP-hard since unconstrained case is. In the following, we design approximation algorithm for this size constrained case.

Consider m subset groups $\mathcal{G}_1, ..., \mathcal{G}_m$ on finite set X which is partitioned into τ composed targets X_1, X_2, ..., X_τ, i.e., $X = X_1 \cup X_2 \cup \cdots X_\tau$ and $X_t \cap X_{t'} = \emptyset$ for $1 \le t < t' \le \tau$. Note that Lemma 2 is assumed for these composed targets. Let x_{iS} be the indicator for event that subset S is selected from group $\mathcal{G}_i$. Let $|X| = n$ and suppose an integer $k \ge 1$ is given. Let y_j be the event that element j is covered by a selected subset and z_i the indicator for event that a subset is selected from group $\mathcal{G}_i$. The following is a 0–1 multi-linear programming.

$$\begin{aligned}
\max \ & \sum_{t=1}^{\tau} \prod_{j \in X_t} y_j & (INLP3)\\
\text{s.t.} \ & y_j \le \sum_{i=1}^{m} \sum_{S: j \in S \in \mathcal{G}_i} x_{iS} \quad \text{for all } j = 1, ..., n, \\
& \sum_{S: S \in \mathcal{G}_i} x_{iS} \le z_i \quad \text{for all } i = 1, ..., m, \\
& \sum_{i=1}^{m} z_i \le k \\
& y_j \in \{0,1\} \quad \text{for all } j = 1, ..., n, \\
& z_i \in \{0,1\} \quad \text{for all } i = 1, ..., m, \\
& x_{iS} \in \{0,1\} \quad \text{for all } S \in \mathcal{G} \text{ and } i = 1, 2, ..., m.
\end{aligned}$$

Its relaxation is as follows.

$$\begin{aligned}
\max \ & \sum_{t=1}^{\tau} \prod_{j \in X_t} y_j & (NLP3)\\
\text{s.t.} \ & y_j \le \sum_{i=1}^{m} \sum_{S: j \in S \in \mathcal{G}_i} x_{iS} \quad \text{for all } j = 1, ..., n, \\
& \sum_{S: S \in \mathcal{G}_i} x_{iS} \le z_i \quad \text{for all } i = 1, ..., m, \\
& \sum_{i=1}^{m} z_i \le k, \\
& 0 \le y_j \le 1 \quad \text{for all } j = 1, ..., n, \\
& 0 \le z_i \le 1 \quad \text{for all } i = 1, ..., m, \\
& 0 \le x_{iS} \le 1 \quad \text{for all } S \in \mathcal{G} \text{ and } i = 1, 2, ..., m.
\end{aligned}$$

Let (y_j^*, x_{iS}^*, z_i^*) be a $(1-\varepsilon)$-approximate solution for local optimality of nonlinear programming (NLP3). A randomized rounding can be executed as follows.

Randomized Rounding: Consider m collections of k groups as follows:

$$\mathcal{C}_1 = \{\mathcal{G}_1, \mathcal{G}_2, ..., \mathcal{G}_k\}, \ \mathcal{C}_2 = \{\mathcal{G}_2, \mathcal{G}_3, ..., \mathcal{G}_{k+1}\}, \ ... \ \mathcal{C}_m = \{\mathcal{G}_m, \mathcal{G}_1, ..., \mathcal{G}_{k-1}\}.$$

First, select one collection $\mathcal{C}_\ell$ with probability p_ℓ, where $\{p_\ell\}_{\ell=1}^m$ is a solution to the following equation system.

$$(a_{i\ell}) \cdot \begin{pmatrix} p_1 \\ \vdots \\ p_m \end{pmatrix} = \begin{pmatrix} z_1^* \\ \vdots \\ z_m^* \end{pmatrix} \qquad (1)$$

and $(a_{i\ell})$ is an $m \times m$ matrix with

$$a_{i\ell} = \begin{cases} 1 & \text{if } \mathcal{G}_i \in \mathcal{C}_\ell, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Note that $(a_{i\ell})$ is a circulant matrix which is nonsingular. Hence equation system (1) has a unique solution. Since group $\mathcal{G}_i$ belongs to k collections $\mathcal{C}_i, \mathcal{C}_{i-1}, \ldots, \mathcal{C}_{i-k+1}$, equation system (1) can be rewritten as

$$\sum_{\ell=i-k+1}^{i} p_\ell = z_i^* \text{ for } i = 1, \ldots, m, \tag{3}$$

where "+" is in the sense of modular m.

Next, for each group $\mathcal{G}_i$ in the selected collection $\mathcal{C}_\ell$, randomly select one subset S from $\mathcal{G}_i$ with probability x_{iS}^*/z_i^* (and no subset of $\mathcal{G}_i$ is selected with probability $1 - \sum_{S \in \mathcal{G}_i} x_{iS}^*/z_i^*$.)

$$y_j = \begin{cases} 1 & \text{if element } j \text{ is covered by a selected subset,} \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 5. $E[y_j] \geq (1 - e^{-1}) y_j^*$.

Proof. For each $j = 1, ..., n$, let $I_j = \{(i, S) \mid j \in S \in \mathcal{G}_i\}$. Notice that for each $(i, S) \in I_j$,

$$\begin{aligned} &\text{Prob}[(i,S) \text{ is picked}] \\ =& \sum_{\ell=1}^{m} \text{Prob}[C_\ell \text{ is picked}] \times \text{Prob}[(i,S) \text{ is picked} \mid C_\ell \text{ is picked}] \\ =& \sum_{\ell=i-k+1}^{i} p_\ell \cdot \frac{x_{iS}^*}{z_i^*} = z_i^* \cdot \frac{x_{iS}^*}{z_i^*} = x_{iS}^*, \end{aligned}$$

where the third equality is because of (3). Then

$$\begin{aligned} \text{Prob}[y_j = 0] &= \prod_{(i,S) \in I_j} \text{Prob}[(i,S) \text{ is not picked}] = \prod_{(i,S) \in I_j} (1 - x_{iS}^*) \\ &\leq \left(\frac{\sum_{(i,S) \in I_j} (1 - x_{iS}^*)}{K_j} \right)^{K_j} = \left(1 - \frac{\sum_{(i,S) \in I_j} x_{iS}^*}{K_j} \right)^{K_j} \\ &\leq \left(1 - \frac{y_j^*}{K_j} \right)^{K_j}, \end{aligned}$$

where $K_j = |I_j|$. Hence,

$$E[y_j] = \text{Prob}[y_j = 1] \geq 1 - \left(1 - \frac{y_i^*}{K_j} \right)^{K_j} \geq (1 - e^{-1}) y_j^*$$

The lemma is proved. □

Lemma 6. $E[\prod_{j\in X_t} y_j] = \prod_{j\in X_t} E[y_j]$.

Proof. It is the same as the proof of Lemma 4. □

Theorem 4. *Let* (y_j, x_{iS}) *be the approximate solution obtained by the above randomized rounding. Then*

$$E\left[\sum_{t=1}^{\tau}\prod_{j\in X_t} y_j\right] \geq (1-e^{-1})^{\alpha}(1-\varepsilon)lopt_3$$

where $lopt_3$ *is the objective value of a local optimal solution for (NLP3) and* $\alpha = \max_{1\leq t\leq\tau}|X_t|$.

Proof. It is similar as the proof of Theorem 3. □

We summarize the above algorithm into Algorithm 3.

Algorithm 2 Approximation for maximum group set coverage with size constraint and composed targets

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input m groups $\mathcal{G}_1$, $\mathcal{G}_2$, ..., $\mathcal{G}_m$ of subsets of a finite set X, a partition of X into composed targets $X = X_1 \cup X_2 \cup \cdots \cup X_\tau$, and an integer $k > 0$.
output a collection $\mathcal{S}$ of subsets of X.
0: $\mathcal{S} \leftarrow \emptyset$;
1: Solve (NLP3) to obtain a $(1-\varepsilon)$-approximate solution (x^*_{iS}, y^*_j, z^*_i);
2: Solve equation system (1) to obtain p_ℓ for $\ell = 1, 2, ..., m$;
3: Select one collection $\mathcal{C}_\ell$ with probability p_ℓ;
4: **for** each $i \in \mathcal{C}_\ell$ **do**
5: select one subset S from $\mathcal{G}_i$ with probability x^*_{iS}/z^*_i;
6: $\mathcal{S} \leftarrow \mathcal{S} \cup \{S\}$
7: **end-for**
8: **return** $\mathcal{S}$.

The running time of Algorithm 2 depends on computing a local optimal solution of (NLP3), which is an open problem in the situation stated at the end of last section.

6 Conclusion

In this paper, the group set coverage problem is proposed as a mathematical formulation for the study of camera sensor coverage problems. Through such a formulation, approximation algorithms are successfully designed for several maximization problems on camera sensor coverage.

A challenge is to study the minimization problem for group set coverage. Given a collection of subset groups on a finite set, find a minimum subcollection which is a group set cover. This may not be a proper research problem since it is NP-complete to determine whether a collection of subset groups is a group set cover, that is, it is NP-hard to determine whether the input has a feasible solution or not. According to Theorem 2, a proper research problem may be stated as follows: Given a collection of subset groups on a finite set, find a minimum subcollection covering at least $1 - 1/e$ percentage of all elements (i.e., for which there is an efficient selection that select one subset from each group such that at least $1 - 1/e$ portion of all elements are covered by selected subsets).

The above mentioned challenge may suggest us to study geometric version of group set coverage. Actually, the camera sensor coverage has some geometric structure which has been lost when it is formulated into the group set coverage problem. How to formulate a geometric version of group set coverage in order to reflect the geometric structure of camera sensor coverage? It may be an interesting problem in our future research.

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