



Passive Target Detection Based on Rao Using Multi-satellite Illumination

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Abstract. In this paper, hypotheses of target detection features is proposed when constructing the signal model. Under this condition, the Rao detection method is extended to weak echo signal detection using multi-satellite illumination. Furthermore, a multi-satellite weak echo signal detection algorithm is proposed based on Rao detection. When the noise variance is unknown, the Rao detector is designed according to the existence of an interfering target. Moreover, the detector' detection thresholds are obtained given the false alarm probability, respectively. Finally, we evaluate the Rao detection performance in different situations with extensive simulations.

Keywords: Multi-satellite illumination · Passive detection · Rao detection · Target detection

1 Introduction

Target detection is significant in the ground dual receiver system. The system is divided into a reference channel and a monitoring channel. The reference channel receives the direct wave signals from multiple external emitters, while the monitoring channel receives the echo signals reflected by the target. The traditional passive detection method is susceptible to the noise in the reference channel based on the reference channel's direct wave signal and the monitoring channel's echo signal [4, 5]. As such, the detection performance is easily affected by the noise of the reference channel [7–11]. At present, most of them assume that

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the signal-to-noise ratio of the reference channel is high, and on this evidence, the echo signal is detected, which is inconsistent with the actual scene. The existing method of achieving weak echo detection is based on Rao detection. However, this detection method is in the single emitter scene modeling analysis.

To effectively detect the weak echo of multiple satellites, a multi-satellite joint target detection method based on the Rao detection method is proposed in this paper, which is not easily affected by the reference channel noise. Simulation results show that the detection method proposed in this paper is feasible and can detect weak echo signals.

2 System Model

In the actual receiving scene, the ground receiving system consists of two antennas. One is called the monitoring channel antenna, and the other is the reference channel antenna. The reference antenna is placed in the satellite direction, and the monitoring antenna is an antenna with a wide beam pointing to the monitoring area. When the satellite emitter radiates signals, we expect to receive a direct wave signal and an echo signal reflected by the target. The time delay of the echo signal is τ , and the Doppler frequency shift is f_d . In this process, some clutter will inevitably be received. If there are multiple satellites as external radiation sources, the signal received by the monitoring channel is represented by $x(t)$, which can be expressed as

$$\begin{aligned}
 x(t) = & \sum_{\eta=1}^M a_{\eta} s_{\eta}(t - \tau_{\eta}) e^{j2\pi f_{d\eta} t} \\
 & + \sum_{\eta=1}^M \phi_{\eta,0} s_{\eta}(t) + \sum_{\eta=1}^M \sum_{k=1}^{N_c} \phi_{\eta,k}(t) s_{\eta}(t - \tau_{\eta,k}^{(c)}) \\
 & + \sum_{m=1}^K \gamma_m^{(t)} s_m(t - \tau_m^{(t)}) e^{j2\pi f_{dm}^{(t)} t} + n_s(t),
 \end{aligned} \tag{1}$$

where M is the number of satellite radiation sources; $f_{d\eta}$, τ_{η} and a_{η} are Doppler frequency shift, delay and amplitude of target echo respectively; $n_s(t)$ is Gaussian white noise; K is the number of jamming targets; $\beta_m^{(t)}$, $\tau_m^{(t)}$, $f_{dm}^{(t)}$ are amplitude, delay and Doppler frequency shift of the target echo; $\tau_{\eta,k}^{(c)}$ and $\phi_{\eta,k}(t)$ are the delay and amplitude of multipath signal respectively; T is the observation time; N_c is the number of multipath of a single satellite signal; the amplitude $\phi_{\eta,k}(t)$ of multipath is a random process with special power spectral density [1]. In the signal model, we assume that the amplitude of multipath signal has special power spectral density, which can be expressed by multiple frequencies $f_i^{(c)}$. We can obtain the following results

$$\phi_{\eta,k}(t) = \sum_{i=-Q}^Q \phi_{\eta,k,i} e^{j2\pi f_i^{(c)} t}, \tag{2}$$

where $f_i^{(c)} = (i - 1 - Q)\Delta f$, $i = 1, 2, \dots, 2Q + 1$, and Q and Δf are two suitable parameters selected according to the actual scene. In this case, $\phi_{\eta,k,i}$ is the amplitude of the multipath signal at frequency $f_i^{(c)}$ and time delay $\tau_{\eta,k}^{(c)}$ at $K = 1, 2, \dots, N_c$.

The reference channel's signal may also be affected by multipath and target echo in the actual scene. Therefore, it will affect the performance of subsequent target detection. Therefore, before detecting the target in the monitoring channel, it is necessary to use some signal processing algorithms such as beamforming or channel equalization to suppress multipath and echo signals in the reference channel. The space-time constant modulus algorithm [2] can recover the degradation of detection performance caused by the presence of echo and multipath signals in the reference channel. Therefore, after a proper signal processing algorithm and band-pass filter separation, the signal in the reference channel can be expressed as

$$y_\eta(t) = b_\eta s_\eta(t) + n_\eta(t) \quad 0 \leq t < T \quad \eta = 1, 2, \dots, M, \tag{3}$$

where b_η is the amplitude of the direct wave signal received by the reference antenna; $n_\eta(t)$ is the sum of the noise of the reference channel receiver and the interference signal after signal processing.

The signals $x(t)$ and $y_\eta(t)$ are sampled at time $t_n = nT_s$, $n = 1, 2, \dots, N$. At the n -th sampling interval, the sampling signals of the received signals in the monitoring channel and the reference channel are represented as $x(t)$ and $y_\eta(t)$, respectively, as follows

$$\begin{aligned} x[n] = & \sum_{\eta=1}^M a_\eta s_\eta [n - n_\eta] e^{j2\pi\Omega_n} + \sum_{\eta=1}^M \phi_{\eta,0} s_\eta [n] \\ & + \sum_{\eta=1}^M \sum_{k=1}^{N_c} \sum_{i=-Q}^Q \phi_{\eta,k,i} s_\eta [n - n_{\eta,k}^{(c)}] e^{j2\pi\Omega_m^{(t)}} \end{aligned} \tag{4}$$

$$+ \sum_{m=1}^K \gamma_m^{(t)} s_\eta [n - n_m^{(t)}] e^{j2\pi\Omega_m^{(t)}} + n_s [n],$$

$$y_\eta(n) = b_\eta s_\eta(n) + n_\eta(n) \quad n = 1, 2, \dots, N \tag{5}$$

where $\Omega_\eta = 2\pi f_{d\eta} T_s$ and $\Omega_m^{(t)} = 2\pi f_{dm}^{(t)} T_s$ represent the normalized Doppler frequency of target echo and jamming target echo respectively. And $\tau_\eta = n_\eta T_s$, $\tau_m^{(t)} = n_m^{(t)} T_s$, $\tau_{\eta,k}^{(c)} = n_{\eta,k}^{(c)} T_s$, where n_η , $n_m^{(t)}$ and $n_{\eta,k}^{(c)}$ represent the echo signal of received signal in the monitoring channel, jamming target echo and multipath delay respectively. Substituting Eq. (5) into Eq. (4), we can get the following results

$$\begin{aligned}
 x(n) &= \sum_{\eta=1}^M \sum_{k=1}^{N_c} \sum_{i=-Q}^Q \frac{\phi_{\eta,k,i}}{b_\eta} y_\eta \left[n - n_{\eta,k}^{(c)} \right] e^{j2\pi f_i^{(c)} T_s n} \\
 &+ \sum_{\eta=1}^M \frac{a_\eta}{b_\eta} y_\eta [n - n_\eta] e^{jn\Omega_\eta} + \sum_{\eta=1}^M \frac{a_\eta}{b_\eta} n_\eta [n - n_\eta] e^{jn\Omega_\eta} \\
 &+ \sum_{m=1}^K \frac{\gamma_m^{(t)}}{b_\eta} y_m \left[n - n_m^{(t)} \right] e^{jn\Omega_m^{(t)}} + \sum_{\eta=1}^M \frac{\phi_{\eta,0}}{b_\eta} y_\eta [n] \quad (6) \\
 &+ \sum_{\eta=1}^M \sum_{k=1}^{N_c} \sum_{i=-Q}^Q \frac{\phi_{\eta,k,i}}{b_\eta} y_\eta \left[n - n_{\eta,k}^{(c)} \right] e^{j2\pi f_i^{(c)} T_s n} + n_s(n) \\
 &+ \sum_{m=1}^K \frac{\gamma_m^{(t)}}{b_\eta} n_m \left[n - n_m^{(t)} \right] e^{jn\Omega_m^{(t)}} + \sum_{\eta=1}^M \frac{\phi_{\eta,0}}{b_\eta} n_\eta [n].
 \end{aligned}$$

By integrating the last four noise terms in Eq. (6) into $n_s[n]$, the above equation can be rewritten as follows

$$\begin{aligned}
 x(n) &= \sum_{\eta=1}^M \alpha_\eta y_\eta [n - n_\eta] e^{jn\Omega_\eta} + \sum_{\eta=1}^M c_{\eta,0} y_\eta [n] \\
 &+ \sum_{\eta=1}^M \sum_{k=1}^{N_c} \sum_{i=-Q}^Q c_{\eta,k,i} y_\eta \left[n - n_{\eta,k}^{(c)} \right] e^{j2\pi t_i^{(c)} T_s n} \quad (7) \\
 &+ \sum_{m=1}^K \beta_m^{(t)} y_m \left[n - n_m^{(t)} \right] e^{jn\Omega_m^{(t)}} + n_s[n],
 \end{aligned}$$

where $\alpha_\eta = \Delta a_\eta / b_\eta$, $c_{\eta,0} = \Delta \phi_{\eta,0} / b_\eta$, $c_{\eta,k,i} = \Delta \phi_{\eta,k,i} / b_\eta$, $\beta_m^{(t)} = \Delta \gamma_m^{(t)} / b_\eta$. By constructing the matrix Λ and matrix Γ as follows

$$[\Lambda^{(c)}]_{kj} = 2\pi f_k^{(c)} T_s, \quad (8)$$

$$[\Gamma^{(c)}]_{kj} = n_j, \quad (9)$$

where $k = 1, 2, \dots, 2Q + 1; j = 1, 2, \dots, N_c$.

By defining vectors $\Omega \triangleq [0, \text{vec}(\Lambda)]^T$ and $n \triangleq [0, \text{vec}(D)]^T$, Eq. (6) can be reexpressed as follows

$$\begin{aligned}
 x[n] &= \sum_{\eta=1}^M a_\eta y_\eta [n - n_\eta] e^{j2\pi \Omega_\eta n} + \sum_{\eta=1}^M \phi_{\eta,0} y_\eta [n] \\
 &+ \sum_{\eta=1}^M \sum_{k=1}^{N_c} c_{\eta,k} y_\eta \left[n - n_{\eta,k}^{(c)} \right] e^{jn\Omega_{\eta,k}^{(c)}} + n_s[n] \quad (10) \\
 &+ \sum_{m=1}^K \beta_m^{(t)} y_m \left[n - n_m^{(t)} \right] e^{j2\pi \Omega_m^{(t)} n},
 \end{aligned}$$

where $n = 1, 2, \dots, N$; $P = (2Q + 1)N_c + 1$; $\Omega_{\eta,k}^{(c)}$ and $n_{\eta,k}^{(c)}$ are the k -th elements of $\Omega^{(c)}$ and $n^{(c)}$, respectively. As shown in the above formula, $\Omega_{\eta,k}^{(c)}$ and $n_{\eta,k}^{(c)}$ at $k = 2, 3, \dots, P$ correspond to the Doppler frequency shift and time delay of multipath signal in the monitoring channel; c_1 at $k = 1$ corresponds to the amplitude of direct wave signal.

3 Multi Satellite Weak Echo Signal Detection Based on Rao Detection

The general problem of target detection is to detect the target when there are jamming target echo, direct wave and multipath with unknown amplitude a . The problem can be described by the following assumptions

$$\begin{aligned}
 H_0 : x(n) = & \sum_{\eta=1}^M \sum_{k=1}^P c_{\eta,k} y_{\eta} [n - n_{\eta,k}^{(c)}] e^{jn\Omega_{\eta,k}^{(c)}} \\
 & + \sum_{m=1}^K \beta_m^{(t)} y_m [n - n_m^{(t)}] e^{jn\Omega_m^{(t)}} + n_s[n],
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 H_1 : x(n) = & \sum_{\eta=1}^M \alpha_{\eta} y_{\eta} [n - n_{\eta}] e^{jn\Omega_{\eta}} \\
 & + \sum_{\eta=1}^M \sum_{k=1}^P c_{\eta,k} y_{\eta} [n - n_{\eta,k}^{(c)}] e^{jn\Omega_{\eta,k}^{(c)}} \\
 & + \sum_{m=1}^K \beta_m^{(t)} y_m [n - n_m^{(t)}] e^{jn\Omega_m^{(t)}} + n_s[n],
 \end{aligned} \tag{12}$$

where $n = 0, 1, \dots, N - 1$; the first item is target echo; the second item is multipath and direct wave; the third item is jamming echo; the fourth item is Gaussian noise; M is the number of satellites; P is the number of multipath paths; n_{η} , Ω_{η} , α_{η} are the delay, Doppler frequency shift and amplitude of echo signal; when $k = 1$, $c_{\eta,k}$ is the amplitude of direct wave; when $K = 2, 3, \dots, P$, $\Omega_{\eta,k}^{(c)}$, $n_{\eta,k}^{(c)}$, $c_{\eta,k}$ are Doppler frequency shift, delay and amplitude of multipath signal; K is the number of jamming targets; $\Omega_m^{(t)}$, $n_m^{(t)}$, $\beta_m^{(t)}$ are Doppler frequency shift, delay and amplitude of jamming target echo signal; n_s is Gaussian white noise. To solve this problem, the following assumptions are made:

- (1) When $m = 1, 2, \dots, M$, the amplitude $\beta_m^{(t)}$ of jamming target echo signal is a certain unknown variable;
- (2) The number of jamming targets K is unknown. When $m = 1, 2, \dots, K$, the time delay $n_m^{(t)}$ and Doppler frequency shift $\Omega_m^{(t)}$ of each jamming echo are known;

- (3) When $\eta = 1, 2, \dots, M$ and $k = 2, \dots, P$, the amplitude $c_{\eta,k}$ of multipath signal is an unknown variable. When $k = 1$, $c_{\eta,1}$ is the amplitude of the direct wave signal and is a certain unknown variable;
- (4) When $\eta = 1, 2, \dots, M$ and $k = 2, \dots, P$, the delay $n_{\eta,k}^{(c)}$ and Doppler frequency shift $\Omega_{\eta,k}^{(c)}$ of multipath signal are known variables;
- (5) n_s is additive white Gaussian noise with unknown variance σ^2 ;
- (6) The subsequent detection needs to construct the probability density function of sampling signal $\mathbf{x} = [x(0), x(1), \dots, x(N - 1)]$ under two assumptions.

Rao detection can be regarded as an approximate generalized likelihood ratio detection. Thus the two have comparable detection performance. Since Rao detection only needs to estimate unknown parameters under the hypothesis H_0 instead of estimating target signal parameters, the calculation amount and complexity can be effectively reduced. Therefore, Rao detection is relatively easy to implement and has good weak signal detection performance. The unknown parameters of the hypothesis H_0 can be estimated by the maximum likelihood estimation method. In the following, a detector based on Rao detection is deduced in two cases: the variance σ^2 of noise is unknown and there is no jamming target, i.e. $K = 0$; the variance σ^2 of noise is unknown and there are jamming targets, i.e. $K \neq 0$.

This part contains the derivation of Rao detection when the noise variance σ^2 is unknown, but there is no jamming target. Rao detection only needs to estimate unknown parameters under the hypothesis H_0 , and the unknown parameters can be estimated with maximum likelihood. Rao detection can be expressed as [3]

$$T(x) = \frac{\partial \ln p(\mathbf{x}; \alpha_\eta, \sigma^2, c_{\eta,k}, H_1)}{\partial \alpha_\eta} \Bigg|_{\substack{\alpha_\eta=0, \sigma^2=\hat{\sigma}_0^2 \\ c_{\eta,k}=\hat{c}_{\eta,k0}}} \left[\mathbf{I}^{-1}(\tilde{\theta}) \right]_{\alpha_\eta \alpha_\eta} \underset{H_0}{\stackrel{H_1}{\geq}} \lambda, \tag{13}$$

where λ is the detection threshold; $[\mathbf{I}(\theta)]$ is the Fisher information matrix; the unknown parameters under the hypothesis H_0 are expressed as vector form, i.e. $\theta = [a_\eta, c_{\eta,k}, \sigma^2]$, and $\tilde{\theta}$ is the maximum likelihood estimation of θ under the hypothesis H_0 . Under the hypothesis H_0 , $a_\eta = 0$, then $\tilde{\theta} = \left[0, \hat{c}_{\eta,k0}, \hat{\sigma}_0^2 \right]$; $\hat{c}_{\eta,k0}$ and $\hat{\sigma}_0^2$ are the maximum likelihood estimates of $c_{\eta,k}$ and σ^2 under the hypothesis H_0 . The probability density function under the hypothesis H_0 differentiates the unknown parameters $c_{\eta,k}$ and σ^2 . Then set these two derivative values equal to zero. The maximum likelihood estimation of these two unknown parameters can be solved, consistent with the estimated value of *GLRT* under the same situation.

Under the hypothesis H_1 , the probability density of $x(n)$ can be expressed as

$$\begin{aligned}
 & p(\mathbf{x}; \alpha_\eta, \sigma^2, c_{\eta,k}, H_1) \\
 &= \frac{1}{(\pi\sigma^2)^N} \exp \left[\left(-\frac{1}{\sigma^2} \sum_{n=0}^{N-1} |x[n] - \sum_{\eta=1}^M \alpha_\eta y_\eta [n - n_\eta] e^{jn\Omega_\eta} \right. \right. \\
 &\quad \left. \left. - \sum_{\eta=1}^M \sum_{k=1}^P c_{\eta,k} y_\eta \left[n - n_{\eta,k}^{(c)} \right] e^{jn\Omega_{n,k}^{(c)}} \right|^2 \right) \right]. \tag{14}
 \end{aligned}$$

After taking the logarithm of Eq. (14), differentiate a_η . Then, the maximum likelihood estimation of $\alpha_\eta, c_{\eta,\kappa}, \sigma^2$ under the hypothesis H_0 is substituted into the formula. We can get the following results

$$\begin{aligned}
 & \left. \frac{\partial \ln p(\mathbf{x}; \alpha_\eta, \sigma^2, c_{\eta,\kappa}, H_1)}{\partial \alpha_\eta} \right|_{\substack{\alpha_\eta = 0, \sigma^2 = \hat{\sigma}_0^2 \\ c_{\eta,k} = \hat{c}_{\eta,k}}} \\
 &= \left(-\frac{2}{\hat{\sigma}_0^2} \sum_{n=0}^{N-1} \left| x[n] - \sum_{\eta=1}^M \alpha_\eta y_\eta [n - n_\eta] e^{jn\Omega_\eta} \right. \right. \\
 &\quad \left. \left. - \sum_{\eta=1}^M \sum_{k=1}^P c_{\eta,k} y_\eta [n - n_{\eta,k}^{(c)}] e^{jn\Omega_{n,k}^{(c)}} \right| y_\xi^* [n - n_\xi] e^{-jn\Omega_\xi} \right), \tag{15}
 \end{aligned}$$

where $\xi = 1, 2, \dots, M$; $c(\cdot, \cdot)$ is the maximum likelihood estimation of the amplitude of all satellite direct wave multipath signals; $c(\cdot, \eta)$ is the element of column η of $c(\cdot, \cdot)$, that is, the amplitude of the η -th satellite signal in terms of multipath signal and direct wave.

After the Fisher information matrix is expressed, the elements of the matrix except diagonal are all zero elements, so the Fisher information matrix can be simplified as

$$\mathbf{I}(\theta) = \begin{bmatrix} E \left[\frac{\partial^2 \ln p(\mathbf{x}; \alpha_\eta, \sigma^2, c_{\eta,k}, H_1)}{\partial \alpha_\eta \alpha_\eta} \right] & 0 & 0 \\ 0 & E \left[\frac{\partial^2 \ln p(\mathbf{x}; \alpha_\eta, \sigma^2, c_{\eta,k}, H_1)}{\partial \sigma^2 \sigma^2} \right] & 0 \\ 0 & 0 & E \left[\frac{\partial^2 \ln p(\mathbf{x}; \alpha_\eta, \sigma^2, c_{\eta,k}, H_1)}{\partial c_{\eta,k} c_{\eta,k}} \right]. \end{bmatrix} \tag{16}$$

The definition of each variable is as follows

$$\begin{aligned}
 & \frac{\partial^2 \ln p(\mathbf{x}; \alpha_\eta, \sigma^2, c_{\eta,k}, H_1)}{\partial \alpha_\eta \alpha_\eta} \\
 &= \sum_{\eta=1}^M \sum_{n=0}^{N-1} y_\eta [n - n_\eta] y_\xi^* [n - n_\xi] e^{-jn(\Omega_\xi - \Omega_\eta)}, \tag{17}
 \end{aligned}$$

$$\frac{\partial^2 \ln p(\mathbf{x}; \alpha_\eta, \sigma^2, c_{\eta,k}, H_1)}{\partial \sigma^2 \sigma^2} = \frac{N}{\sigma^4}, \tag{18}$$

$$\begin{aligned}
 & \frac{\partial^2 \ln p(\mathbf{x}; \alpha_\eta, \sigma^2, c_{\eta,k}, H_1)}{\partial c_{\eta,k} c_{\eta,k}} \\
 &= \sum_{\eta=1}^M \sum_{k=1}^P \sum_{n=0}^{N-1} y_\eta [n - n_{\eta,k}^{(c)}] y_\eta^* [n - n_{\eta,k}^{(c)}] e^{-jn(\Omega_{q,s}^{(c)} - \Omega_{\eta,k}^{(c)})}.
 \end{aligned} \tag{19}$$

The Fisher information matrix is

$$\begin{aligned}
 \left[\mathbf{I}^{-1}(\tilde{\theta}) \right]_{\alpha_\eta \alpha_\eta \alpha_\eta} &= \left[\mathbf{I}_{\alpha_\eta \alpha_\eta}(\tilde{\theta}) - \mathbf{I}_{\alpha_\eta \sigma^2}(\tilde{\theta}) \mathbf{I}_{\sigma^2 \sigma^2}^{-1}(\tilde{\theta}) \mathbf{I}_{\sigma^2 \alpha_\eta}(\tilde{\theta}) \right. \\
 &\quad \left. - \mathbf{I}_{\alpha_\eta c_{\eta,k}}(\tilde{\theta}) \mathbf{I}_{c_{\eta,k} c_{\eta,k}}^{-1}(\tilde{\theta}) \mathbf{I}_{c_{\eta,k} \alpha_\eta}(\tilde{\theta}) \right]^{-1}.
 \end{aligned} \tag{20}$$

Since $\mathbf{I}_{\alpha_\eta \sigma^2}(\tilde{\theta}) = \mathbf{I}_{\alpha_\eta c_{\eta,k}}(\tilde{\theta}) = 0$, it can be reduced to

$$\begin{aligned}
 & \left[\mathbf{I}^{-1}(\tilde{\theta}) \right]_{\alpha_\eta \alpha_\eta \alpha_\eta} \\
 &= \left[\mathbf{I}_{\alpha_\eta \alpha_\eta}^{-1}(\tilde{\theta}) \right] \\
 &= \frac{1}{\sum_{\eta=1}^M \sum_{n=0}^{N-1} y_\eta [n - n_\eta] y_\eta^* [n - n_\eta] e^{-jn(\Omega_\xi - \Omega_\eta)}}.
 \end{aligned} \tag{21}$$

By substituting the above formula into $T(x)$, the detection statistics of Rao detection can be obtained as follows

$$T(x) = \frac{\frac{1}{N} \left(\|x\|^2 - \hat{\mathbf{c}}_0^H \mathbf{R}_{ss} \hat{\mathbf{c}}_0 \right)}{\sum_{\eta=1}^M \sum_{n=0}^{N-1} y_\eta [n - n_\eta] y_\eta^* [n - n_\eta] e^{-jn(\Omega_\xi - \Omega_\eta)}}. \tag{22}$$

4 Simulation Results and Discussion

In order to verify the influence of direct wave and multipath on the detection performance based on Rao. Three satellite signals, GPS, DVB-S, and INMARSAT, are used for the simulation experiment. The carrier frequencies of the three signals are $f_G = 1.57$ GHz, $f_D = 12.38$ GHz and $f_i = 4.2$ GHz; The following assumptions are made: the time delay of the three echo signals is $1 \mu\text{s}$, $2 \mu\text{s}$ and $3 \mu\text{s}$; the Doppler frequency shift of the three echo signals is 100 Hz, 200 Hz and 300 Hz; the direct wave intensity of the three signals is -130.1 dBw, -111.83 dBw and -120.6 dBw; the power difference between the direct wave and its corresponding echo is 40 dB; the sampling points are 10^5 . The simulation results are shown in Fig. 1.

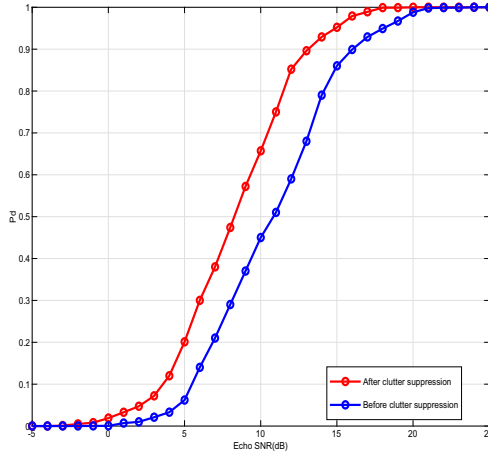


Fig. 1. Detection probability before and after suppressing direct wave and multipath

Figure 1 shows that the detection performance is improved by 4 dB after the direct wave and multipath are suppressed in the monitoring channel. This is because the amplitude of the direct wave and multipath must be estimated by maximum likelihood estimation when there are direct waves and multipath in the echo signal. Simultaneously, the existence of direct wave and multipath is a substantial interference to the target echo signal. The method of suppressing the direct wave and multipath can reduce their interference to the echo signal. However, the existence of the interference target echo still affects the detection of the echo signal.

In order to verify the influence of the number of satellite signals on the detection performance based on Rao. GPS, GPS + DVB-S, GPS + DVB-S + INMARSAT satellite signals are used for simulation experiments. The carrier frequencies of the three signals are $f_G = 1.57$ GHz, $f_D = 12.38$ GHz and $f_i = 4.2$ GHz; The following assumptions are made: the time delay of the three echo signals is 1 μ s, 2 μ s and 3 μ s; the Doppler frequency shift of the three echo signals is 100 Hz, 150 Hz and 200 Hz; the direct wave intensity of the three signals is -130.1 dBw, -111.83 dBw and -120.6 dBw; the power difference between the direct wave and its corresponding echo is 40 dB; the sampling points are 10^5 . The simulation results are shown in Fig. 2.

Figure 2 shows that the detection performance decreases with the increase of the number of signals at the same SNR. This is because with a rise in the number of satellite signals, the number of direct waves and multipath interference in the monitoring channel will also increase, which is equivalent to the increase of interference in the monitoring channel. In addition, multiple echo signals will affect each other, which will also affect the detection performance of echo signals. However, the joint detection of echoes by multiple satellites will improve the detection results' reliability, which is equivalent to improving the reliability of

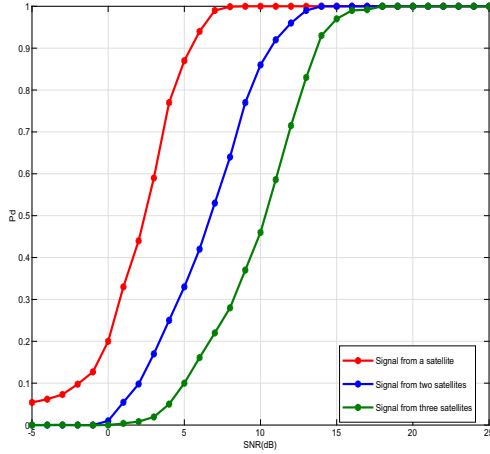


Fig. 2. Rao detection probability with different number of satellites

the detection results by reducing the detection performance, and it is of great significance in practical applications.

In order to verify the influence of the distance from the target to the receiver and the speed of the target on the RAO detection performance, three satellite signals, GPS, DVB-S, and INMARSAT, are used to simulate the performance of Rao detection. The following assumptions are made: the distances between the target and the receiver are 10 km, 15 km and 20 km, and the velocities are 300 km/h (83 m/s), 350 km/h (97 m/s) and 800 km/h (222 m/s) respectively; the carrier frequencies of the three signals are $f_G = 1.57$ GHz, $f_D = 12.38$ GHz and $f_i = 4.2$ GHz; the direct wave intensity of the three signals is -130.1 dBw, -111.83 dBw and -120.61 dBw; the power difference between the direct wave and its corresponding echo is 40 dB; the sampling points are 10^5 . According to the relationship between time delay, distance, target velocity, and Doppler frequency shift [6], it can be obtained that the time delay is about $0.4 \mu\text{s}$, $0.6 \mu\text{s}$, and $0.8 \mu\text{s}$, respectively. The simulation results are shown in Fig. 3.

Figure 3 shows that the detection performance of echo signals decreases with the increase of target velocity. This is because the Doppler frequency shift of each echo signal is different when the target velocity is different. The increase of target velocity will not only affect the receiver's reception, but also increase the corresponding Doppler frequency shift, which will affect the detection of echo signal. Therefore, this method is suitable for detecting low-speed flying targets.

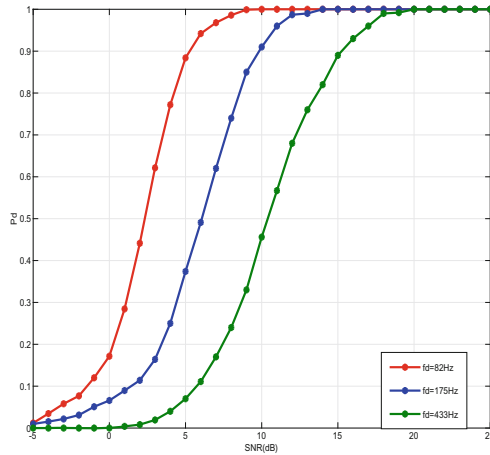


Fig. 3. Rao detection probability at different speeds

5 Conclusion

This paper theoretically deduces the detection method of multi-satellite weak target echo signal based on Rao detection. After the detection performance of the detector is analyzed theoretically, the adaptive detection threshold is set to complete the detection of the echo signal. Finally, the simulation of each aspect that may affect the detection algorithm's performance is carried out. The proposed method has low complexity and good detection performance, and it can detect target using multi-satellite illumination.

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