Solar System Dynamics with Jet Propulsion Laboratory Ephemeris

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Abstract. The spectrum and orbits of celestial bodies are calculated via Fourier transform, which presents fascinating spectrum and behavior of this complicated solar system dynamics. This study investigates the multiple frequencies and Fourier series solution of celestial bodies' orbits based on data science approach. The Sun, Mercury, Venus, Earth, Moon, Mars and Jupiter's orbits are calculated and reported.

Keywords: n-body problem · Solar system dynamics · Fourier transform · Fourier series solution · Floquet theory

1 Introduction

The solar system, an n-body problem, is a complex and fascinating nonlinear dynamical system. Nonlinear problems have a tendency to multiply frequencies. The spectrum and behavior of nonlinear systems is appreciably richer than that of linear systems. Furthermore there generally does not exist an approach to solve the equations and the singularity structure of the solution of a nonlinear differential equation is affected by the initial conditions. [1]

Compared with the major drawback to numerical studies in Poincaré's time that lack of an efficient means of obtaining solutions to the equations of motion [2], advanced telescope and computers provide effective tools to study solar system dynamics.

The planetary and lunar ephemerides called DE440 and DE441 have been generated by fitting numerically integrated orbits to ground-based and space-based observations. Compared to the previous general-purpose ephemerides DE430, seven years of new data have been added to compute DE440 and DE441, with improved dynamical models and data calibration. [3] The NASA Jet Propulsion Laboratory(JPL) website provides web interface, command line interface, mail interface, and API. The ephemeris data in the its Horizons system can be accessed through these interfaces. Taking the Horizons network application as an example, a user can select the ephemeris type, target celestial body, reference system, time interval, and time step to generate the target celestial body ephemeris. The ephemeris not only contains the information such as the position and velocity of the target celestial body within the selected time interval but

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also provides its orbital parameters such as semi-major axis, period, eccentricity, inclination, etc.

The data from astronomical observations exhibits that a solar system is more than a collection of celestial bodies' motion in Kepler's fashion. It has an intricate dynamical structure, which can be largely understood by an inverse square law of force to its constituent bodies. To deeply understand the solar system dynamics require a better knowledge of the fine structure of constituent bodies' orbits. In the paper, the similar method in the remarkable figure-8 solution of three-body problem [4] is applied to calculate the orbit of celestial bodies. The results are good approximation of the orbit and reveal many key information and structure of solar system dynamics.

2 Fourier Analysis on Ephemeris Dataset

Directly solve the Newtonian equations of n-body problem [5] along with initial conditions is a mission impossible for now. This study is not trying to solve the dynamical system, but calculate the celestial orbits from ephemeris. The parametric equations of the celestial orbit $r(t)$ in a plane is a mapping from R to R^2, which are written as Fourier series expression:

$$r(t) = \begin{cases} x(t) = x_0 + a_0 sin(2\pi\omega_0 t + \phi_0) + a_1 sin(2\pi\omega_1 t + \phi_1) + ... \\ y(t) = y_0 + b_0 sin(2\pi\alpha_0 t + \psi_0) + b_1 sin(2\pi\alpha_1 t + \psi_1) + ... \end{cases} \tag{1}$$

Follow the similar routine in the article the remarkable figure-8 solution of three-body problem. The goal is to solve parameters set $\{x_0, \omega_k, \phi_k\, y_0, \alpha_k, \psi_k\}$ in the parametric equations based on ephemeris. The Fourier transform calculates the inner product between the signal and a series of trigonometric functions of frequencies [13]. When the corresponding frequency exists in the signal, the inner product reaches a maximum value, and a peak will appear in the scanning result. Parseval's theorem, a generalization of the Pythagorean theorem in Hilbert spaces, states the sum of the square of a function is equal to the sum of the square of its Fourier transform [14]. Both the amplitudes and frequencies of the corresponding trigonometric functions are obtained. Then construct the $x(t)$ and $y(t)$ functions with obtained amplitudes and frequencies and fit the constant x_0 and y_0 and phase parameters ϕ_k and ψ_k, all parameters of parametric curve equations are solved.

3 Experimental Results on Celestial Body Orbits

With the above approach, empirical experimental results on celestial body orbits are following (Fig. 1):

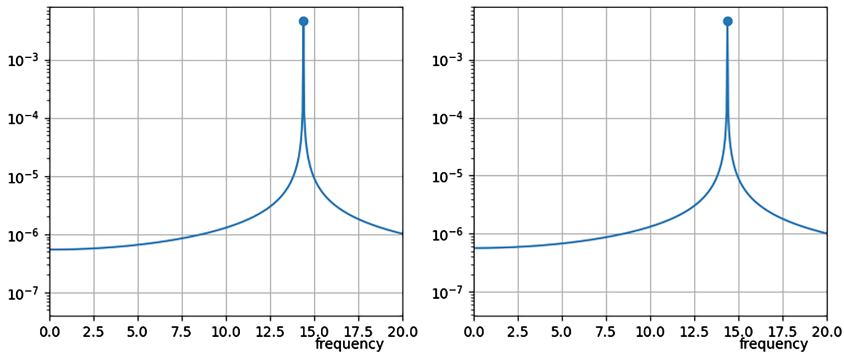

Fig. 1. The spectrum of the Sun orbit

3.1 Sun

From the JPL ephemeris, the Sun's orbiting around barycenter of solar system. And the orbit of the Sun is (Fig. 2 and Table 1)

$$r_{Sun} = \begin{cases} x(t) = 0.00462012 * sin(14.38140578 * 2\pi t - 0.565526749) \\ y(t) = 0.00465284 * sin(14.38140578 * 2\pi t - 2.1324948) \end{cases} \tag{2}$$

Table 1. The parameters of the Sun orbit.

$x(t)$	a_k	ω_k	ϕ_k	x_0
	0.00462012	14.38140578	−5.65526749e-01	-2.09289787e-10
$y(t)$	b_k	α_k	ψ_k	y_0
	0.00465284	14.38140578	−2.13249480	−1.08304895e-10

3.2 Mercury

Compared with other planets' orbits, Mercury's orbit probably is the most extraordinary and intriguing curve in the solar system. The smallest frequency is ω and 2ω till 5ω frequencies are observed in the spectrum. The complicated spectrum to some extent explains the precession of Mercury orbit (Figs. 3 and Table 2 and 3).

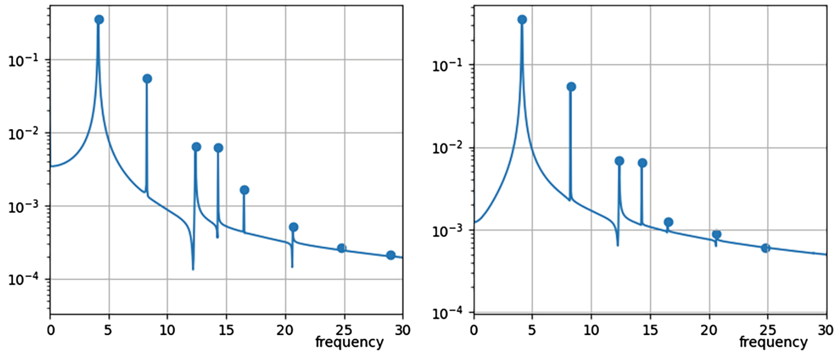

Fig. 2. The spectrum of the Mercury orbit

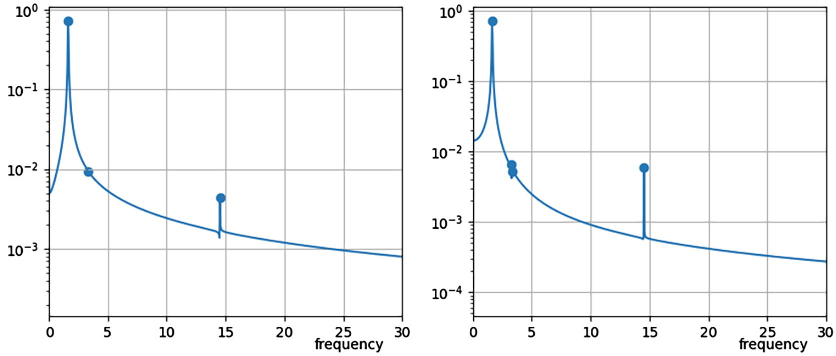

Fig. 3. The spectrum of the Venus orbit

Table 2. The parameters of the Mercury orbit.

$x(t)$	a_k	ω_k	ϕ_k	x_0
	3.50467135e-01	4.14963279	−0.38550222	−0.02612611
	5.52317329e-02	8.25604025	−0.56562112	
	6.28872009e-03	12.40567304	1.42246356	
	6.18558570e-03	14.30758807	2.10120671	
	1.66348475e-03	16.51208049	3.15631628	
	5.16742457e-04	20.66171329	0.15308947	
	2.59717752e-04	24.76812074	2.37382634	
	2.11588049e-04	28.91775354	−0.12321694	
$y(t)$	b_k	α_k	ψ_k	y_0
	0.35567523	4.14963279	−1.94391569	−0.11630011
	0.0547745	8.25604025	−2.17981089	
	0.00688961	12.40567304	−0.00765279	
	0.00655814	14.30758807	0.63202588	
	0.00123301	16.51208049	2.24184979	
	0.00090034	20.66171329	−0.62221043	
	0.00060899	24.81134608	1.90326548	

Venus

$$r_{Venus} = \begin{cases} x(t) = 0.00484 + 0.722 * sin(1.6243 * 2\pi t - 1.5506) \\ +0.00929 * sin(3.3365 * 2\pi t - 0.7637)+ \\ +0.00439 * sin(14.531 * 2\pi t - 1.3818) \\ y(t) = -0.0055 + 0.7203 * sin(1.6243 * 2\pi t - 3.1206) \\ +0.00649 * sin(3.2487 * 2\pi t - 2.2564) \\ +0.00587 * sin(14.531 * 2\pi t - 2.9785) \\ +0.00518 * sin(3.3365 * 2\pi t + 1.2477) \end{cases} \quad (3)$$

Table 3. The parameters of the Venus orbit.

$x(t)$	a_k	ω_k	ϕ_k	x_0
	0.722046238285391	1.62438233582074	−1.550658589715679	0.004844528508381947
	0.009293114570175261	3.336569122226385	−0.7637043592846432	
	0.004392768924506664	14.531636571801755	−1.3818932852477146	
$y(t)$	b_k	α_k	ψ_k	y_0
	0.7203208150919663	1.624382197321208	−3.1206136326637246	-0.0055026645368427465
	0.006492773205732966	3.248764394642416	−2.2564182236835815	
	0.005878785580178873	14.53163533279243	−2.978595685659841	
	0.005186376485286768	3.33656883774086	1.2477130223295143	

3.3 Earth and Moon

Compared to the Earth's spectrum, the Moon's spectrum is almost identical but with an additional frequency. And this frequency is related to the Moon's orbiting around the Earth (Figs. 4 and 5, 6 and Tables 4, 5, 6, 7).

$$r_{Earth} = \begin{cases} x(t) = 0.00563 + 0.999 * sin(0.9992 * 2\pi t - 2.956) \\ +0.0081 * sin(1.998 * 2\pi t - 2.997) \\ +0.00008578 * sin(3.097 * 2\pi t + 3.0125) \\ +0.0000445 * sin(13.39 * 2\pi t - 6.2829) \\ y(t) = -0.02441 + 0.999 * sin(0.9992 * 2\pi t + 1.7562) \\ +0.00827 * sin(1.998 * 2\pi t + 1.715) \\ +0.00007418 * sin(2.9978 * 2\pi t + 1.6755) \\ +0.0000281 * sin(13.39 * 2\pi t - 1.5753) \end{cases} \quad (4)$$

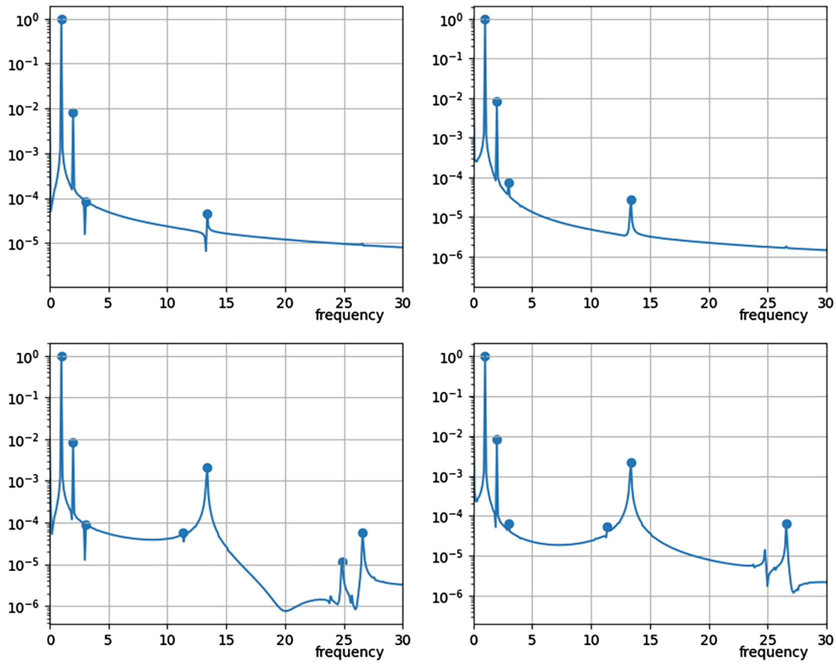

Fig. 4. The spectrum of the Earth (above) and the Moon (below) orbits

$$r_{Moon} = \begin{cases} x(t) = 0.00563 + 0.999 * sin(0.9992 * 2\pi t - 2.956) \\ +0.00819 * sin(1.9985 * 2\pi t - 2.9972) \\ +0.0000857 * sin(3.0978 * 2\pi t + 3.0125) \\ +0.0000445 * sin(13.39 * 2\pi t - 6.2829) \\ y(t) = 0.00562 + 0.9997 * sin(0.9992 * 2\pi t - 2.956) \\ +0.00819 * sin(1.9985 * 2\pi t - 2.996) \\ +0.00216 * sin(13.39 * 2\pi t + 3.1433) \\ +0.0000892 * sin(3.09781 * 2\pi t - 0.1109) \\ +0.000058628 * sin(26.5812 * 2\pi t - 9.5023) \\ +0.00005715 * sin(11.292 * 2\pi t - 0.0903) \\ +0.00001162 * sin(24.882 * 2\pi t + 17.664) \end{cases} \quad (5)$$

Table 4. The parameters of the Earth orbit.

$x(t)$	a_k	ω_k	ϕ_k	x_0
	0.9997748660222578	0.9992980392952218	−2.956069503600092	0.005631151618628536
	0.00819465834	1.9985960785904435	−2.9972887382364446	
	8.578019811562512e-05	3.0978239218151877	3.0125661488611186	
	4.450130727494664e-05	13.390593726555972	−6.282992564638444	
$y(t)$	b_k	α_k	ψ_k	y_0
	0.9999477284733309	0.9992980392952218	1.756293670621902	−0.024416071137540876
	0.008270111469806747	1.9985960785904435	1.7152149916162314	
	7.418618310416173e-05	2.9978941178856653	1.6755640652416326	
	2.810243449994342e-05	13.390593726555972	−1.5753480530011705	

Table 5. The parameters of the Moon orbit.

$x(t)$	a_k	ω_k	ϕ_k	x_0
	0.9997748660222578	0.9992980392952218	−2.956069503600092	0.005631151618628536
	0.00819465834	1.9985960785904435	−2.9972887382364446	
	8.578019811562512e-05	3.0978239218151877	3.0125661488611186	
	4.450130727494664e-05	13.390593726555972	−6.282992564638444	
$y(t)$	b_k	α_k	ψ_k	y_0
	0.9997675581302258	0.9992962074199679	−2.9560100551847914	0.005628223108971562
	0.008195272766757626	1.9985924148399359	−2.9969485331690473	
	0.0021638451544734893	13.39056917942757	3.1433211327678907	
	8.922910322914388e-05	3.0978182430019006	−0.11096492776134088	
	5.862871713916423e-05	26.581279117371146	−9.502320791218168	
	5.7155917433008134e-05	11.292047143845638	−0.09033365985711901	
	1.1623288774874141e-05	24.8824755647572	17.664534249111323	

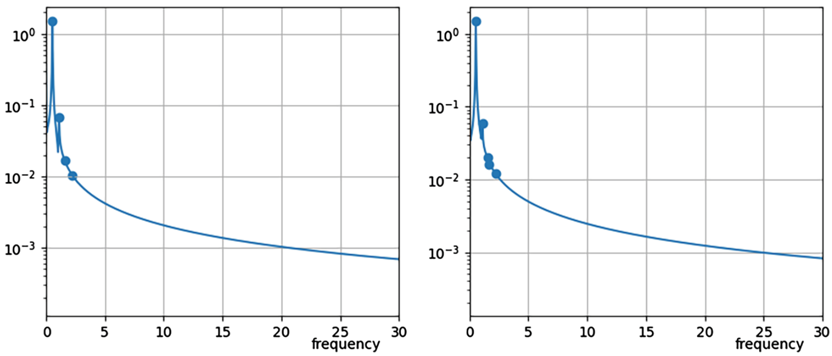

Fig. 5. The spectrum of Mars orbit

3.4 Mars

$$r_{Mars} = \begin{cases} x(t) = -0.196 + 1.513 * sin(0.531 * 2\pi t + 1.483) \\ +0.0674 * sin(1.106 * 2\pi t + 1.712) \\ +0.0166 * sin(1.638 * 2\pi t + 1.849) \\ +0.0104 * sin(2.213 * 2\pi t - 2.365) \\ y(t) = 0.0877 + 1.503 * sin(0.531 * 2\pi t - 0.088) \\ +0.0598 * sin(1.106 * 2\pi t - 0.07) \\ +0.0198 * sin(1.594 * 2\pi t + 0.553) \\ +0.0162 * sin(1.682 * 2\pi t - 0.8467) \\ +0.0119 * sin(2.2139 * 2\pi t + 1.73) \end{cases} \quad (6)$$

Table 6. The parameters of the Mars orbit.

$x(t)$	a_k	ω_k	ϕ_k	x_0
	1.51349432	0.53135985	1.48379458	−0.19604086
	0.06749398	1.10699968	1.71293731	
	0.01669802	1.63835953	1.84921259	
	0.01045105	2.21399936	-2.36533913	
$y(t)$	b_k	α_k	ψ_k	y_0
	1.50387014	0.53135985	-0.08802946	0.08773089
	0.05981574	1.10699968	-0.07085421	
	0.01986805	1.59407954	0.55306972	
	0.01621179	1.68263952	-0.84676325	
	0.01195149	2.21399936	1.73042012	

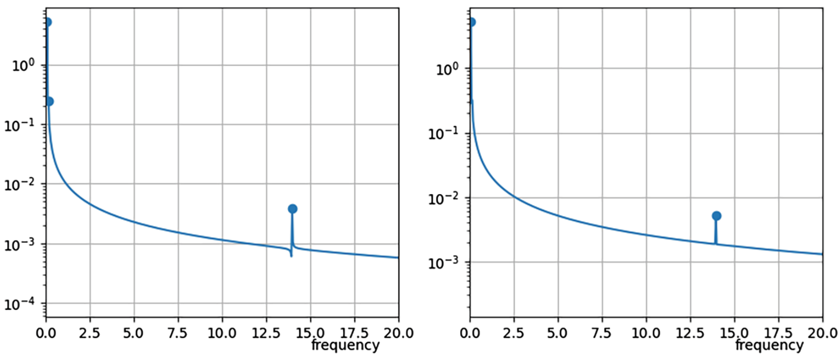

Fig. 6. The spectrum of the Jupiter orbit

3.5 Jupiter

$$r_{Jupiter} = \begin{cases} x(t) = -0.3735 + 5.193 * sin(0.08439 * 2\pi t + 2.1593) \\ +0.248 * sin(0.1687 * 2\pi t + 2.4809) \\ +0.0038 * sin(13.966 * 2\pi t + 2.5442) \\ y(t) = -0.0924 + 5.1888 sin(0.084 * 2\pi t + 0.614) \\ +0.005 * sin(13.908 * 2\pi t + 1.518) \end{cases} \quad (7)$$

Table 7. The parameters of the Jupiter orbit.

$x(t)$	a_k	ω_k	ϕ_k	x_0
	5.193316419776355	0.08439151728486118	2.1593208335992045	−0.3735655669789274
	0.24802121876459826	0.16878303456972235	2.4809038523072195	
	0.003868201103213779	13.966796110644525	2.5442808113133606	
$y(t)$	b_k	α_k	ψ_k	y_0
	5.18884012721591	0.08404144572604776	0.6140722455535058	−0.09240309219156834
	0.005049850165783217	13.908859267660903	1.518306472278778	

4 Conclusion

The Fourier transform of planets' orbit exhibit interesting spectrum. To fully understand and digest the observation, it requires more advanced mathematical method such as Floquet theory, spectrum theorem and even some unknown technique. Although the spectrum of planets' orbits especially the frequencies and amplitudes set still remain mysterious, they reveal many significant features of planets' motion. Due to the fact that solar system is a real n-body problem, the continuous differentiable orbits obtained better pictured solar system dynamics which is way more sophisticated than what described by Kepler's laws.

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