Concept-Based Modeling as a Method Combining Digital and Analogue Means for Problem-Solving

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Abstract. In this paper we present Concept-Based Modeling (CBM), an innovative pedagogical method for problem-solving in engineering education, which combines analogue and digital tools. We outline the scientific rationale for CBM and discuss how it compares to traditional teaching with respect to optimizing the pedagogical value of both analogue and digital means. CBM is based on conceptual modeling of quantities derived directly from first principles and streamlined for the use of computer algebra systems (CAS). The method was evaluated in a pilot survey in a statics course for engineering students in their freshman year at Halmstad University. We conclude that CBM improves students' problem-solving skills by the reciprocal action between conceptual understanding and modeling of a problem. Student evaluations suggest that CBM enables students to handle more realistic problems and that CAS as a professional tool prepares them for their future working life. Future studies will address CBM for more advanced courses, as the students' knowledge develops over time.

Keywords: Concept-Based Modeling · Digital · Analogue · Problem-Solving · Computer Algebra System

1 Introduction

Problem-solving is a complex skill that requires both domain knowledge and knowing what strategies to apply in which problem situations. Teaching problem-solving skills carries the additional challenge of arranging teaching tools and materials that support the student's process of identifying relevant knowledge and rules, while allowing sufficient degrees of freedom for students' own creativity, exploration and independence. In this paper, we propose one pedagogical method that combines analogue and digital tools, where traditional, "pen-and-paper" methods precede the use of a computer algebra system for processing calculations, in the engineering and physics domains. Together, these two approaches have shown to increase the problem-solving capabilities of students.

Our aim is to highlight how conceptual thinking and creative exploration can work together with a computer algebra system (CAS) in a way that takes optimal advantage of both human and machine capacities. At the centre of this enterprise is a technique

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referred to as *Concept-Based Modeling* (CBM), which has been developed and assessed over five years of teaching in mechanical engineering at the university by the project leader and author (HL). In addition, we suggest how the CBM technique can be further empirically evaluated and developed as a pedagogical method, drawing from the input and course evaluations by students.

Hence, our guiding questions for the present paper can be formulated as follows:

- What is the scientific rationale for combining analogue and digital means in a common instructional design, in the form of CBM?
- How does teaching using CBM add pedagogical value to other, traditional methods of teaching for students in the engineering domains?
- What are the implications of CBM for educational interventions that optimize the combination of analogue and digital teaching tools?

The outline of this paper is as follows. First, we provide a background as to the relevance and rationale of our approach to teaching and learning problem-solving skills in the engineering domain. We specifically address the added value of combining digital and analogue means in a common pedagogical design, using CBM. Then, in the main part of this paper, we describe how CBM works in practice, including the results of a pilot survey among university students who used CBM in a course on statics. Finally, we make some concluding remarks and suggest some areas for follow-up work.

2 Background

2.1 Learning and Teaching How to Solve an Engineering Problem

Decades of research have failed to demonstrate any general problem-solving strategies independent on substantial subject knowledge; this is the case also for ostensibly less fact-based subjects like mathematics (Sweller, Clark & Kirshner, 2010). In order to become an expertise problem solver, such as an engineer who uses mathematics, one must learn how and when to apply one's general knowledge of mathematical laws and methods to quite specific contexts and situations – whether this concerns calculating an orbit to the moon, the construction of a bridge or figuring out when a stool remains at rest (an example which we will return to).

The vast range of possible, including hypothetical and yet unknown, problems to be solved leaves great room for creativity and motivational factors in the process. This concerns not only what is the most effective solution but also which is the more interesting, desirable and even aesthetically pleasing way to go about it. In short, if anything general can be said about actual problem-solving as carried out by human beings, we hold that it is driven by three main factors: will, creativity and knowledge.

Whilst education traditionally focuses on the latter (different forms of knowledge), teaching problem-solving must not become a task of simply having the student to recognize facts and mechanically perform calculations to reach a particular, by the teacher intended, solution. If good learning assumes willingness to solve a problem and creative insight, then good teaching should provide situations designed to offer such opportunities as part of the problem-solving process.

As a case in point, consider traditional engineering education, which leans heavily on mathematics. A main ingredient in traditional mathematics teaching consists of the teacher demonstrating procedures on blackboards/whiteboards, a form of pedagogical practice analysed in detail by Greiffenhagen (2014). Interestingly, traditional teaching of computer programming consists of similar practices (Tenenberg et al., 2018). Code is being written on whiteboards by the teacher who comments on each step.

In these contexts, examples and illustrations are often math heavy. To make sure the mathematical aspects are covered in all steps of the solution to the engineering problems, they are presented from the outset. This imposes great demands on well-developed mathematical abilities of the students – requirements that are becoming more difficult to defend given today's powerful computer algebra systems and the growing complexity of the problems that engineers must be able to deal with.

Hence, we see several issues that call for innovative teaching practices in engineering. First, creative and conceptual thinking are vulnerable mental processes that are easily disrupted. As soon as algorithms, programming or mathematical methods become in focus, the big picture is often backgrounded or lost. Second, students' problem-solving process is often hidden, in that the steps taken to reach a solution is not made visible and open to scrutiny for both the teacher and the students themselves. Learning implies making errors, revising and understanding where the errors come from, such that conceptual errors are not confused with syntax errors in a line of code. Third, the availability of digital educational resources today offers complements to traditional teaching materials which may effectively address these previous limitations. This calls for a closer examination into the role of traditional, "analogue", and innovative, "digital", means for problem-solving and, most importantly, how they should be combined for improving problem-solving practices in engineering education and related domains.

2.2 Adding Pedagogical Value by Combining Digital and Analogue Means

Thinking requires hard effort and although people seem naturally inclined to solve problems, we are not naturally good at it. As Willingham (2009, p. 3) puts it, "People are naturally curious, but we are not naturally good thinkers; unless the cognitive conditions are right, we will avoid thinking". The argument goes, that hard thinking is so demanding, that our brain works to save us from the effort whenever possible, resulting in what Kahneman (2011) referred to as the fast, automatic "System 1" thinking (in contrast to the slow, effortful "System 2" thinking). At the same time (and cogently), successfully solving a problem is rewarding and pleasurable, to make the effort worth it. The implication for education is that teachers carefully need to consider how they make students engage in purposeful cognitive processes. Any external support or tools should serve to encourage students' own, and *the right kind of,* efforts.

What then are "the right kind"? First, we must acknowledge that students need tools to help their thinking; some tools target specific cognitive processes (e.g. a calculator) whereas others serve to generally off-load information onto the environment (e.g. a keyboard, or a pen). Second, as noted above, students need knowledge. In education, we cannot assume that students (or anyone else) are cognitively well-equipped enough to learn new concepts or skills simply from exposure to a problem, which then triggers thinking in the direction of an intended learning outcome. However, provided with the right tools, and the right knowledge, which trigger efforts in the right direction for solving a problem, human beings are exceptionally good learners. In learning, we form ideas and hypotheses, using different tools to off-load our "thinking" onto the environment, and repeatedly compare outcomes of internal states to events in the outside world. In doing so, people can develop professional skills and produce outstanding results, as exemplified by the engineering ingenuity of modern society.

Computers, on the other hand, are not equipped to generate ideas, to formulate hypotheses or conceive of models in line with human thought and creativity. Computers are good at executing problem-solving steps, as programmed by humans, following algorithms and processing vast amounts of information in terms of predefined data – that is, computation. Although computers have been shown to surpass human performance in delimited domains such as playing chess, Go and trivia quizzes (Silver et al., 2017), present discourse emphasizes that the *collaboration* between human beings and computers is even more efficient in achieving specific outcomes, than either part alone (Tegmark, 2017; Polson & Scott, 2018).

For example, a study by Wang et al. (2019), which involved 20 professional data scientists, revealed that computers using artificial intelligence (AI) are not necessarily perceived as a threat or competition to human performance, even when it concerns the development of techniques which serve to automate the work practices of data scientists themselves. The authors concluded that the interviewees maintained a general view that "…the future of data science work will be a collaboration between humans and AI systems, in which both automation and human expertise are indispensable". If this is true from the extensive experience of professionals in the field, who work with actual problems, it would make sense to foster a similar view in education for students who anticipate this future. In other words, students would benefit from learning to assess and use their own expertise in relation to the functions of computer systems.

Importantly, from our perspective, digital tools such as computers and computer software, make people better equipped to solve problems by the innovative means they offer for complementing human functioning. Considering technological development over time, the distinction between digital and analogue technologies has become increasingly blurry. For the objectives of this paper, we intend technologies that allow capturing human bodily movements in a way that is analogous to the movement and traces on a medium. Pens or brushes used on paper is a prime example. However, todays' digital technologies, like tablets, can emulate the use of pens or brushes on paper in ways which allow for very similar or identical actions by the drawing hand. When we refer to use of "analogue means" we therefore include drawing tablets, when used to emulate analogue technologies, because they allow for more or less identical bodily actions to creative drawing using a hand-held pen or brush.

Our suggestion here is, that analogue and digital means, in this sense, respectively might serve different pedagogical functions. More precisely, the analogue means of drawing and writing on paper seem more apt at aiding our slow, conceptual thinking and generation of ideas, whereas the digital means of computation and automation seem more apt at performing the actual calculations which, once conceived of, produce the outcome that informs us of the next step in the problem-solving process.

CBM takes into account both students' need for analogue "thinking" (conceptualization) and the computer's capacity of digital "automation" (through algorithms and calculation). In learning with CBM, the student typically has to reconceive and revise her model from the results it generates from a Computer Algebra System (CAS). A central aspect of CBM is that the CAS is used to free the analogue means (the modeling using pen and paper) from the demand to represent all mathematical aspects. The presence of digital means thus opens new possibilities for using analogue means.

In CBM, analogue means are used to model the concepts of "first principles" (see below), letting the computer algebra system take care of all the algebra, arithmetic or more advanced mathematics. The reduction of the mathematical content is intended to help the students to get an aerial view of the engineering problem, which allows them to use cognitive resources to address the fundamental levels of the problem. More specifically, the analogue means are introduced to help the students clarify how the relevant first principle applies to the problem.

The primary concern of CBM is not whether the outcome at first constitutes a "correct" solution (although this would be the ultimate goal of any problem-solving) but that the student makes the critical parts of her own problem-solving process more precise, identifiable and visible, in order to guide her learning further. In the following paragraphs we detail how CBM meets these criteria and is carried out in practice, starting with its theoretical foundation and then exemplifying its use with a sample problem.

3 How Concept-Based Modeling Works

3.1 The Foundation of CBM in Scientific Theory and Knowledge

Scientific knowledge can be described in terms of theory, principles, and concepts. At the core of our scientific knowledge, we find what we call theories. These are separate and non-overlapping descriptions of the world. For example, the theory of Newtonian mechanics and the theory of thermodynamics deals with two different aspects of the world. While Newtonian mechanics efficiently describes the detailed motion of particles, thermodynamics tells us how energy may be transported and what forms it can take.

Each theory is furthermore founded on a complete set of laws or first principles. In Newtonian mechanics we find Newton's three laws of motions and Newton's universal law of gravity, and in thermodynamics we have the zeroth to the third law of thermodynamics. Every law contains fact-based universal pieces of information about nature that we call concepts. These concepts are fundamental to our understanding of the world. Some concepts are non-divisible "atomic facts" (Wittgenstein, 1962) like the mass, or the speed of a body. Others are combinations of atomic facts like the concept of momentum. To simplify this hierarchy of facts, we will call every piece of fact a concept.

3.2 The Three-Step Process of CBM

CBM divides the problem-solving process into two distinct tasks: modeling, and computation. Following the computation, the final result is assessed, a "reality check". This three-step process is depicted in Fig. 1.

The modeling task is to identify the first principles of the problem, for example Newton's second law of motion or the first law of thermodynamics, and to model each of the involved physical concepts, part by part, all the way down to the most basic of mathematical concepts. In the computation task, the analyst merely transfers the conceptually modelled first principles into a computer algebra system of choice for the succeeding mathematical treatment. These two tasks require completely different mindsets and are advantageously separated. In the CBM process, it is crucial that the initial modeling activity precedes the coding and execution of digital programs in the CAS. Modeling and computation are thus clearly separated. This serves to create a space in which pen and paper are used to create visual images to support the conceptual exploration of the engineering problem, before the mathematical processing commences.

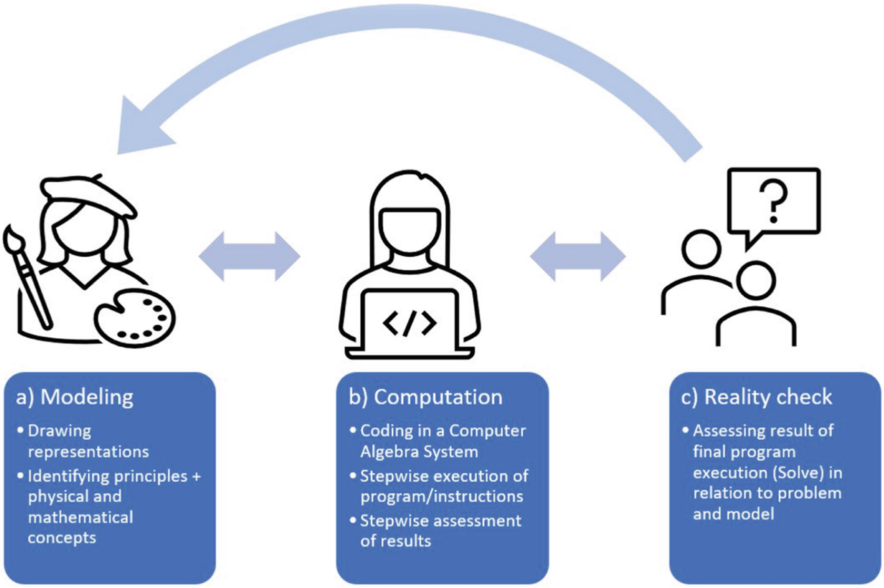

Fig. 1. The three-step process of CBM.

In the modeling step (a), the students creatively draw and write models, equipped with colored pens and paper or drawing tablets. The computation step (b) can be described as a stepwise process involving several iterations of coding, execution, and assessment. The final step in the process, the reality check step (c), begins when the entire code is run and a final result in the system is presented. It differs from the stepwise assessments in step (b) in that the students need to assess the plausibility of the final result (for example, that it is not a negative number if it needs to be positive). Errors that cannot be detected in step (b) can potentially show up in step (c).

If a step does not function as intended, something is wrong in the code and/or the mathematical conceptualization, and that needs to be addressed. The computer is the indispensable dialogue partner in this process, but it is the students who build the structure in which the computation flows. This way of structuring solutions in a transparent and logical way is a crucial part of the students' learning process.

When the engineering problem has been presented by the teacher/textbook (or defined by the students), but before the proper modeling (step a) has begun, initial cognitive activity which has been referred to as *problem conceptualization* (Delahunty et al., 2020) takes place[1]. This relates to our points above (Sects. 2.1–2.2) and is important for the subsequent activities in CBM, since the students' previous knowledge will be of consequence for how the problem-solving task is approached.

When the students move on to modeling, their internal conceptualizations of the problem begin to be externalized through the drawing of visual representations in a way which resonates with the students´ understanding. They are encouraged to be artistic and creatively draw nice colourful pictures, so that their minds really get a feel for the problem. The identification and modeling of the first principles valid to the problem are also conceptual in nature. Questions like "are forces and motion, or heat and energy important?" are central to the conceptual big picture so as to not distract the creative and mindful flow of this solution step. Whether the end result is plausible and reasonable, becomes a subsequent concern in what we call the "reality check" in step (c).

3.3 Targeting the Limitations of Traditional Teaching by Combining Analogue and Digital Means in CBM

At this point, we may summarize how the CBM method incorporates both analogue and digital means to meet the challenges to learning and teaching problem-solving, following the steps described above.

First, the modelling step aims to off-load the students' need of representing too much information at once (i.e., what is commonly referred to as "information overload" or, in the academic literature, "cognitive load"). It has been argued that external representations are particularly crucial when people are facing ill-structured problems, or at least problems that appear ill-structured in relation to the level of their previous knowledge. Kiverstein and Rietveld (2018) uses the term *representation hunger* to denote this property of cognition. Delahunty et al. (2020, p. 399) concludes that novice problem solvers can be described as "more'representation-hungry' necessitating a more active engagement with discrete cognitive schema and mental models as well as active combination and modification mechanisms".

Based on this literature, we conclude that the initial problem conceptualization and the following creative drawing in CBM are often highly cognitively demanding. The students need to form a representation of the engineering problem, using the relevant first principles with corresponding physical and mathematical concepts. Drawing can be extremely helpful in this phase and since the students are generally more or less novice in the domain and can creatively interact with the external representations that is being produced during the drawing.

[1] Delahunty et al. (2020, p. 399) define problem conceptualization as "the period of cognitive processing occurring before the externalization of any representation (e.g. a sketch, mathematical expression, verbal communication etc.)".

Furthermore, premature involvement of mathematical operations and calculations will activate different knowledge networks, potentially competing for resources in the working memory that is needed to identify and model the relevant first principles and concepts. Less formally expressed from the practical teaching viewpoint informing CBM, conceptual thinking and creative problem conceptualization and representation are vulnerable processes that can easily be disrupted by the details of analysis. Finally, drawing also makes aspects of the student´s reasoning available to both teachers and peers for comments and feedback which are important aspects of the learning setting.

Second, in contrast to modeling in the CBM process, the computation step is supported by digital means. The student turns to the functions of the digital tools. The creative mindset is temporarily left to focus on the details and get every number, variable and equation correctly declared. Here, the big picture disappears in favour of the details. The student starts to create a simple program, which is solved in small steps by trial and error until the final result or answer appears on the screen.

Step (b) can be described as a dialogue with the computer, in which inputs are made and temporary results are reviewed. The dialogue with the computer also affords a trial-and-error approach where different coding options can be tried out to assess which one seems to work. However, in order to successfully complete the computation step, the student needs to build a structure for the code that relates the appropriate concepts to the relevant first principles (see the sample problem in the Appendix, in which Positions, Forces, Moments are stepwise modeled and finally put into the equation for equilibrium - the relevant first principles). While part of this process can consist of trial-and-error solutions, the students also need to organize the code in an appropriate way (see Appendix).

The digital tools (here, the CAS) ideally contain as many pre-defined high-level functions and algorithms allowing automated operation as possible. This means that as much as possible of the mathematical and computational processing will be black boxed and not directly accessed by the students. This is to allow them to focus on the most conceptual part of the work with mathematics and to avoid introducing cognitively demanding coding.

The CAS is utilized to off-load the highly knowledge intense mathematical processing to the computational system, thereby minimizing the need for attention and previous knowledge. This does not mean that the CAS approach is simply made easier than traditional methods. Rather, we suggest that the combination of low-stakes trial and error approach in combination with the need to structure the whole problem-solving process, from step (a) to step (c), are very important for deep learning, or what Ohlsson (2011) refers to as non-monotonic change. There are many potential instances of positive or negative feedback (from the computer as well as from teachers and peers) which can gradually and qualitatively alter the cognitive networks activated in the students. Next, we show how these steps manifest in practice by illustrating how a sample problem is solved by using CBM.

3.4 Sample Problem

For a concretisation of the CBM problem-solving process, we refer to the Appendix. There we work through a typical statics problem, which combines text and a picture, detailing the modeling and computation steps from Fig. 1. Importantly, all the problem-solver does is to model the concepts of the first principles that governs the problem. All arithmetic, algebra and more advanced mathematics are handed over to the CAS.

3.5 Evaluation with Students

To study the student experience of CBM as a problem-solving method, we conducted a pilot survey at the end of the CBM course in statics for engineering students in their freshman year, 2019 and 2021. During the pandemic year 2021 all teaching was online, but there were no indications that this affected the overall student experience and learning with CBM, probably because CBM made use of digital drawing tablets and computers that connected well with web seminars. In total, 33 students responded.

The survey was divided into two parts: CBM in teaching and CAS as a tool. The full survey comprised twelve items, including items relating to the student experience which are beyond the scope of the present paper (e.g., if CAS had increased one's curiosity for mathematics and physics). Here, we focus on three stand-alone, free-response items of the first part (CBM in teaching) and three rating items of the second part (CAS as a tool), which all targeted engineering-relevant problem-solving skills. In the first part, the students were asked to describe their overall experience of CBM in relation traditional teaching (where problem-solving is made purely by hand) as well as with respect to special skills required by the teacher, and for meeting the needs of the labour market. In the second part, the students rated their agreement with different statements about using a CAS (e.g. "*CAS facilitates the use of mathematics*") on a 5-point likert scale, where 1 meant "Disagree" and 5 meant "Totally agree". The results of the second part are summarized in Table 1. The averaged total can be seen as a measure of students' general attitude to the utility of using CAS in the context of CBM.

Table 1. Student ratings of agreement with statements about CAS (1 to 5; 1 = Disagree, 5 = Totally agree).

Statement	Student average rating ($N = 33$) M (SD)
CAS facilitates the use of mathematics	4,0 (1,2)
CAS works well as a tool for problem-solving	4,1 (1,0)
CAS has helped to develop my programming skills	3,4 (1,1)
Total	**3,8 (1,1)**

The main result from Table 1 is that students experienced CAS as an effective tool, particularly with respect to facilitating the use of mathematics and problem-solving, and to somewhat less degree for developing programming skills.

The students' free responses were reviewed by two of the authors (HL and BS) for common themes. As a pilot survey, we were primarily interested in finding out how the items in the survey would capture relevant student experiences of CBM, as a basis for constructing a more precise questionnaire for future studies. We took special interest in student free-response reflections on the relevance and effectiveness of the method, relating to the items in Table 1. In addition, we were interested in what special qualities or skills that students considered the teacher to need when using CBM (e.g., whether they would point to the teacher's digital competence as to skills in combining analogue and digital means, or more general pedagogical qualities such as communication skills).

Overall, students did not express any major differences between CBM and traditional teaching (e.g. "It is basically the same, it all starts on paper, the difference is that you type in everything on a computer.") but several pointed out that the use of computers seemed to make CBM more relevant, and that problem-solving became "easier" and "more efficient". Students also stressed the importance of combining approaches, e.g. "A combination is important. The computer helps a lot, but knowledge also decreases. You learn more by hand, but the computer makes the work much more efficient.". Only one response expressed a preference for traditional teaching, but did not specify why.

As to special skills needed by the teacher of CBM, students pointed particularly to the importance of good conceptual thinking and programming skills. Both analogue and digital skills were mentioned in this context (e.g. "both be able to explain on paper and be able to program", "Knowledge of the software and to be able to explain how the code is interpreted"). As to the role of CAS for courses that meet the needs of the labour market, the main themes related to introducing professional tools (e.g. "My experience is that CAS is used more and more in the labor market in general and in an increasingly digitalized world it feels like this will be the future.") and solving more realistic problems (e.g. "CAS allows us to solve more advanced problems that reflect reality and are not as dumb down as the typical school problems.").

4 Discussion

In this paper, we aimed at presenting how and why analogue and digital means can be combined in a common instructional design, in the form of CBM. We specifically targeted how teaching using CBM adds pedagogical value to other, traditional methods of teaching in engineering. In this section, we further address the implications of CBM for educational interventions that optimize the combination of analogue and digital teaching tools.

In short, our two-fold, analogue and digital, approach was motivated by considering basic qualities of human cognition in relation to the functionality of computerized systems. CBM exemplifies how learning can be made more appealing and efficient, through directing students' efforts towards practices that better meet the demands of complex problem-solving, such as that reflected in engineering education. The educational aspect is important, since learning involves iterative cycles of modeling and computation in relation to the outcome (Fig. 1) – in other words, practice and feedback – before students' have developed the professional skills needed in their actual work. Considering how human-computer interactions are likely to develop along these lines, the experience of using a multi-faceted approach like CBM makes an important preparation for students' future work practices.

As to how CBM is carried out in practice and what it means for optimizing the combination of analogue and digital teaching tools, we find a few points particularly worth highlighting. For example, the students associated CBM with solving more realistic problems and that problem-solving became more time efficient. Many students stated that the skills and methods trained in the classroom were the same as the ones they will use in their future working life. This makes CBM stimulating for the student, as well as for the teacher. Interestingly, students made less pronounced associations between programming skills and using CAS. One reason might be the high level of automation built into the systems in today's mathematical software. This often makes the creation of the solution so simple and straightforward that the student becomes unaware of the programming process.

Traditional teaching in the engineering sciences is mainly knowledge-based and emphasizes on breadth rather than understanding and depth. Facts can only be memorized and known, not (normally) understood. On the other hand, principles and concepts are the very foundation of our understanding. Hence, a concept-based teaching approach is focused on understanding and opens for deeper knowledge, also stated by Stern et al. (2017) and Alhumaid (2020). It is in this context we conclude that CBM is beneficial to teaching engineering science and problem-solving. Both are strengthened by the power of conceptual thinking (and CAS). The combination of the analogue and the digital further helps the student to structure his or her own thinking, and to separate the creative from the analytic processes of problem-solving. The analytic processes connected with mathematics are significantly simplified using CAS. Given the complexity of todays' engineering problems, it seems no longer defendable to do most of the calculations by hand. In fact, CBM proposes the contrary; the computer performs all computations and lets students focus on the problem formulation, modeling and reflection.

The main technical difference between what we call traditional teaching and teaching by means of CBM lies in the all-out use of a CAS. Ultimately, without the need of teaching mathematical tricks and clever simplifications a lot of time is freed to deepening the students conceptual understanding. However, trading the mathematical solution techniques of traditional teaching with the manipulation of a CAS is not only time saving, but it also makes a massive impact on how theory is being presented. In CBM, every concept and physical law are derived and applied in its most general form. This makes a contrast to the traditional teaching that typically derives partial facts and approximations and through lengthy explanations finally come up to (but seldomly reaching) the general form. Simply adding a CAS to an otherwise traditional teaching philosophy does not make it CBM. CBM is above all a problem-solving process which makes full use of the CAS.

Notably, some students remarked that the use of computers makes the learning more difficult due to the programming part. If you are new to a math-software, there is a learning curve that initially takes focus away from the course and the problem-solving process. Some students also expressed fears that knowledge was being lost when using computers, and there is a belief that you learn more by hand. This fear is widespread, also among teachers, but is historically unjustified (Schramm, 1998). This fear or anxiety of using CAS can, however, be overcome if it is blended into their everyday class work and systematically introduced (D'Souza et al., 2005). A good way of letting the students familiarize themselves with the software is to combine programming and CAS in the creation of new functions (Galán García et al., 2005). Such projects may help to transform the feared Black Box experience into a White Box experience.

5 Conclusions and Future Studies

Our work with CBM has demonstrated how analogue and digital means can be fruitfully combined for the teaching and learning of problem-solving in engineering, on a scientific basis. By using CAS, the students' mathematical abilities no longer set a definitive limit to what kind of problems that can be studied. Instead, the limit is set based on the conceptual understanding and the modeling capability of the student, in reciprocal action. An important educational implication of CBM, substantiated by evaluation with students, is that more realistic problems can be studied and that CAS as a professional tool prepares the student for their future working life. Finally, we conclude that CBM is important to the curriculum in engineering education due to its potential to match the future demands of a growingly complex technological development. Future work will extend our work with CBM to other and later topics, like Dynamics and Thermodynamics. This will show how conceptual modeling and computation using CAS can be further effectively combined for more advanced courses, as students' knowledge develops over time.

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Appendix

Sample Statics Problem Resolved Using CBM

Here, we work through a typical statics problem using CBM (see Fig. A1).

a) **Modeling**

Given the question, we embark on the first task of the solution process, the conceptual modeling of the problem. We need to decide the first principles that governs the problem. This is a static problem for which it is know that the stool needs to be in equilibrium with its surroundings, that is, the resulting forces and force-couples are zero ($\sum \mathbb{F} = 0$ and $\sum \mathbb{M}_C = 0$). This means that a free-body diagram must be drawn, and all the forces and force-couples must be modeled.

A free-body diagram is a careful drawing of the specific part (the body) that is to be studied (see Fig. A2). Here, we introduce the known lengths and angles along with an appropriate coordinate system (*x, y, z*). This is the part of the solution process where we really see the problem. Remember that we cannot draw what we have not seen.

Once the stool is drawn, we start to model the forces conceptually by adding vectors (arrows) in the direction of their application onto the stool. Generally, assuming

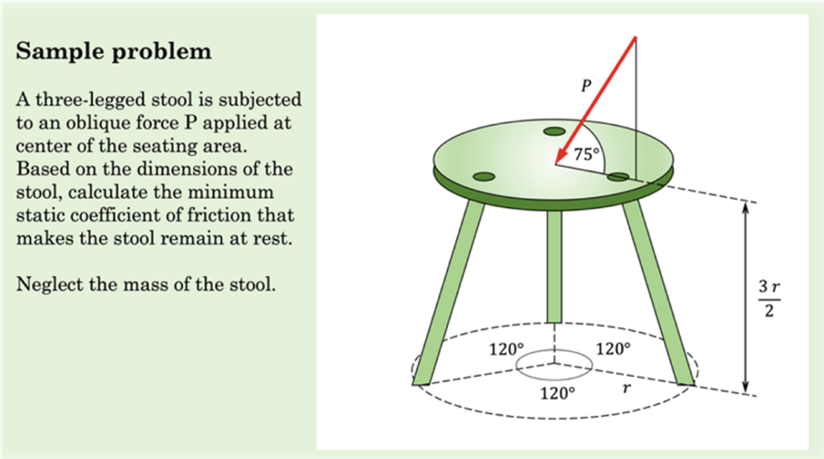

Fig. A1. A typical problem in a university level course in statics.

that each of the three legs are in contact with the floor, their reaction forces must be a combination of normal and friction forces. In finding the minimum static coefficient of friction we must assume that the friction forces are fully developed and equal to the coefficient of friction times the normal force. The load P is already given in the problem.

b) **Computation**
The next step of the solution process is to transform the conceptual models into a code that can be processed by a computer. In this example we make use of the CAS *Mathematica* ®. The coding always starts with defining the most basic of all concepts, the base vectors. Once we have these base vectors, we can define our rotation matrix and so on.

Base vectors

```
𝕚 = {1, 0, 0};
𝕛 = {0, 1, 0};
𝕜 = {0, 0, 1};
```

Rotation matrix

```
ℜ = RotationMatrix[(i - 1) 120 °, 𝕜];
```

After the mathematical concepts are defined, we are ready to go on and model all our physical concepts. Note that the concepts always appear on the left-hand side of the equal signs and their definition and models on the right:

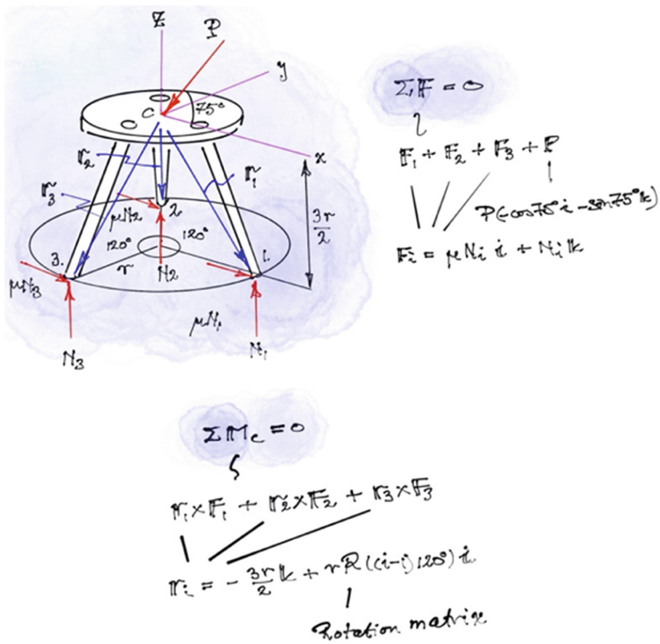

Fig. A2. The conceptual modeling of the loaded stool. The shaded equations represent the first principles valid for equilibrium. Each of the physical concepts are here modelled in steps all the way down to the most basic, the base vectors of the coordinate system.

Positions

```
r_i_ = -(3/2) r k + r R.i;
```

Forces

```
F_i_ = μ N_i i + N_i k;
P = P (-Cos[75 °] i - Sin[75 °] k);
```

Resultant

```
ΣF = P + Sum[F_i, {i, 1, 3}];
```

Moments about C

```
ΣMC = Sum[r_i × F_i, {i, 1, 3}];
```

Now we are ready to state the 1st principles, the equations for equilibrium:

Equilibrium

```
equilibrium = ΣF == 0 && ΣMC == 0;
```

And finally, we state the unknowns and apply the solver:

Solution

```
unknowns = {μ, N1, N2, N3};
```

```
Solve[equilibrium, unknowns] // N
{{μ → 0.267949, N1 → 0.0631562 P, N2 → 0.451385 P, N3 → 0.451385 P}}
```

From the solver, we get the answer that the minimum static friction coefficient, needed for the stool to remain at rest, is $\mu = 0.27$. This concludes the CBM study.

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