A Stochastic Bandwidth Scanning Game

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Abstract. In this paper we consider a dilemma that arises in bandwidth scanning problems associated with the design of agents' scanning strategies based on the principle of rationality and the principle of insufficient reasons. On one hand, engaging tools that estimate a network's parameters allows an agent to act rationally to maximize its payoff. On the other hand, utilizing such engagement incurs extra costs associated with scanning. In particular, if the agent does not employ such tools, then the involved expenses can be reduced, although such a strategy might also cause a reduction in detection probability since in such cases the agent has to design strategy based on the principle of insufficient reasons (also called principle of indifference). In this paper we model this dilemma as a non-zero sum stochastic game between two players (Scanner and Invader). The equilibrium is found in closed form in stationary strategies via solving the corresponding Shapley-Bellman equations, and its dependence on network parameters is illustrated.

Keywords: Non-zero sum stochastic game · Detection · Equilibrium

1 Introduction

The open nature of the wireless medium, in spite of a lot of benefits given by the ability to access spectrum dynamically, also makes cognitive radios a powerful tool for conducting malicious activities or policy violations by secondary users. Therefore, detecting malicious users or unlicensed activities is a crucial problem facing dynamic spectrum access [15], and one of the challenges to enforcing the proper usage of spectrum is the development of an intrusion detection systems that can scan large amounts of spectrum and identify illegal activity [5]. Since, in such security problems, there are at least two agents with different goals (the adversary aims to sneak into bands undetected for their illegal usage, while the intrusion detection system intends to prevent illegal spectrum usage), *game theory* is an ideal tool to employ. As examples of applying game theory to detect an adversary to prevent malicious attack on networks, we mention [1–3, 6, 10–12, 16, 19–22, 24]. In all of these papers the players were *rational,* i.e., each of the players wants to maximize its payoff.

Such rational behavior is based on information the player can obtain associated with the network parameters, i.e. the communication environment and scenario. In this paper using a game-theoretical approach we consider a dilemma

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that arises in bandwidth scanning problems. On one hand, using tools that allow for the estimation of network parameters allows a player to act rationally to maximize its payoff. On the other hand, such engagement assumes an extra scanning cost. Meanwhile, not engaging such tools by a player allows to reduce involved expenses, although such non-action might also cause a reduction in detection probability since in such a case the player has to design strategy based on the principle of insufficient reasons (also called principle of indifference). In this paper we solve this dilemma via finding equilibrium strategies in the closed form of a non-zero sum stochastic game.

2 Non-Zero Sum Stochastic Game

In this section, we describe our model, which involves a scenario where a *primary user* owns a collection of frequency bands. An adversary, called the *Invader*, attempts to "sneak" usage from some of these bands. The primary user can scan bands to detect such malicious activity. Motivated by such functionality we call the primary user the *Scanner.* All actions (scanning by Scanner and sneaking by Invader) are performed in discrete time slots $1, 2, \cdots, \infty$, and the game continues until the Invader is detected. If the Invader is detected, it is eliminated and the game is over.

We assume that at each time slot Scanner and Invader can choose between rational and indifferent modes to scan and sneak, respectively. Denote these mode by $\mathbb{R}$ and $\mathbb{I}$, respectively.

- **(a)** In rational mode, the player acts based on information obtained by using special tools to estimate the bands' parameters so as to adjust its efforts for optimization of the corresponding output.
- **(b)** In indifferent mode, the player does not engage such tools due to the expenses involved. This leads to a lack of information about the network parameters, and makes the player implement a strategy based on other principles, such as the principle of insufficient reasons (principle of indifference) [13]. In this case, the player spreads its efforts equally across the bands.

Let C_S and C_{S0} (C_I and C_{I0}) be costs for employing rational and indifferent modes by Scanner (Invader). Of course,

$$C_S > C_{S0} \text{ and } C_I > C_{I0}. \tag{1}$$

Let α^1 and α^0 be probabilities to detect the Invader if both players implement rational modes or indifferent modes, respectively. Let α be the probability to detect the Invader if the players implement different modes. To avoid bulkiness in the formulas following the basic example of one-shot bandwidth scanning model given in Sect. 3 below we assume that

$$0 < \alpha \leq \alpha^1 < \alpha^0 < 1. \tag{2}$$

Other possible relationships between detection probabilities could be investigated following the suggested in this paper approach.

At each time slot, the instantaneous payoff to Invader is the difference between the probability of its non-detection and cost for the employed sneaking mode. Similarly, at each time slot the instantaneous payoff to the Scanner is the difference between probability of Invader's detection and cost for employed scanning mode. If the Invader is not detected then the game moves to the next time slot and is played recursively with discount factor δ, which can be considered as a measure of urgency for sneaking usage of the radio spectrum. We describe this non-zero sum stochastic game via bimatrix form (Γ_S, Γ_I) (see, (3) below), where Γ_S and Γ_I are Scanner and Invader components of this game and each entry corresponds to an action pair of the Scanner and Invader. For example, if both the Scanner and Invader use their mode $\mathbb{R}$, then the Invader is not detected with probability $\overline{\alpha^1}$ where $\overline{\xi} \triangleq 1 - \xi$. In this case the instantaneous payoff to the Scanner is zero and to the Invader is one, and the game will be replayed with discount rate δ. Invader is detected with probability α^1. In this case, the instantaneous payoff to Scanner is one and to the Invader it is zero, and the game is over.

Thus, this non-zero-sum stochastic game can be presented in recursive form as follows:

$$(\Gamma_S, \Gamma_I) = (M_S(\Gamma_S), M_I(\Gamma_I)), \tag{3}$$

where

$$M_S(\Gamma_S) \triangleq \begin{array}{c c} & \begin{array}{cc} \mathbb{R} & \mathbb{I} \end{array} \\ \begin{array}{c} \mathbb{R} \\ \mathbb{I} \end{array} & \begin{pmatrix} \alpha^1 + \overline{\alpha^1}\delta\Gamma_S - C_S & \alpha + \overline{\alpha}\delta\Gamma_S - C_S \\ \alpha + \overline{\alpha}\delta\Gamma_S - C_{S0} & \alpha^0 + \overline{\alpha^0}\delta\Gamma_S - C_{S0} \end{pmatrix} \end{array}, \tag{4}$$

$$M_I(\Gamma_I) \triangleq \begin{array}{c c} & \begin{array}{cc} \mathbb{R} & \mathbb{I} \end{array} \\ \begin{array}{c} \mathbb{R} \\ \mathbb{I} \end{array} & \begin{pmatrix} \overline{\alpha^1} + \overline{\alpha^1}\delta\Gamma_I - C_I & \overline{\alpha} + \overline{\alpha}\delta\Gamma_I - C_{I0} \\ \overline{\alpha} + \overline{\alpha}\delta\Gamma_I - C_I & \overline{\alpha^0} + \overline{\alpha^0}\delta\Gamma_I - C_{I0} \end{pmatrix} \end{array}. \tag{5}$$

We solve this game in *stationary strategies*, i.e., strategies that are independent of history and the current time. Let $\boldsymbol{x}^T = (x, \overline{x})$ be the stationary (*mixed*) strategy for Scanner assigning the probabilities x and $\overline{x}$ to using actions $\mathbb{R}$ and $\mathbb{I}$.

Let $\boldsymbol{y}^T = (y, \overline{y})$ be the stationary (*mixed*) strategy for Invader assigning the probabilities y and $\overline{y}$ to using actions $\mathbb{R}$ and $\mathbb{I}$, respectively.

Thus, the Scanner and Invader have the same set of feasible stationary (mixed) strategies denoted by $\mathcal{S}$.

By (3)–(5), stationary equilibrium strategies $\boldsymbol{x}$ and $\boldsymbol{y}$ of Scanner and Invader, respectively, have to be functions of the corresponding accumulated payoffs, i.e., $(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{x}(\boldsymbol{v}), \boldsymbol{y}(\boldsymbol{v}))$ where $\boldsymbol{v} = (v_S, v_I)$ with v_S and v_I are the accumulated payoffs corresponding these equilibrium strategies. By (3)–(5), $(\boldsymbol{x}(\boldsymbol{v}), \boldsymbol{y}(\boldsymbol{v}))$ is a stationary equilibrium if and only if the following equations hold [18]:

$$v_S = (\boldsymbol{x}(\boldsymbol{v}))^T M_S(v_S)\boldsymbol{y}(\boldsymbol{v}), \tag{6}$$

$$v_I = (\boldsymbol{x}(\boldsymbol{v}))^T M_I(v_I)\boldsymbol{y}(\boldsymbol{v}), \tag{7}$$

where $\boldsymbol{v} = (v_S, v_I)$ and

$$\boldsymbol{x}(\boldsymbol{v}) = \underset{\boldsymbol{x}\in\mathcal{S}}{\operatorname{argmax}}\, \boldsymbol{x}^T M_S(v_S)\boldsymbol{y}(\boldsymbol{v}), \tag{8}$$

$$\boldsymbol{y}(\boldsymbol{v}) = \underset{\boldsymbol{y}\in\mathcal{S}}{\operatorname{argmax}} (\boldsymbol{x}(\boldsymbol{v}))^T M_I(v_I)\boldsymbol{y}. \tag{9}$$

Thus, by (8), for fixed $\boldsymbol{v}$ and $\boldsymbol{y}(\boldsymbol{v})$, $\boldsymbol{x}(\boldsymbol{v})$ is the best response to $\boldsymbol{y}(\boldsymbol{v})$. By (9), for fixed $\boldsymbol{v}$ and $\boldsymbol{x}(\boldsymbol{v})$, $\boldsymbol{y}(\boldsymbol{v})$ is the best response to $\boldsymbol{x}(\boldsymbol{v})$. Also, v_A and v_I are such that (6) and (7) also have to hold. Thus, (6)–(9) is the non-zero-sum, and for the Shapley-Bellman equation for the zero-sum game (see, [18]). In Sect. 4 below we solve in closed form, these Shapley(-Bellman) equations.

Finally, we note that in the literature the stochastic game approach has been employed to model different network security problems [4,8,9,17,23].

3 A Basic Example of Rational and Indifferent Behaviour

In this section, we give a basic model as an example of rational and indifferent behaviour in bandwidth scanning/sneaking. The model involves a scenario where a *primary user* (Scanner) owns n frequency bands $1, 2, \dots, n$. Invader will attempt to "sneak" usage on *only* one of these bands. By assumption, the Scanner can only scan a single band at a time to detect such malicious activity. We assume that the Invader will be detected with probability γ_i, $\gamma_i \in (0, 1)$, if it sneaks in band i and Scanner scans that band. If Scanner does not scan the band that Invader is using, then the Invader sneaks safely, i.e., its detection probability is zero. We assume that the payoff to the Scanner is one if the Invader is detected, and it is zero otherwise. Note that in optimization framework such kind of problems relates to search theory founded by B. Koopman [14].

Let a (mixed) strategy for the Scanner be $\boldsymbol{p} = (p_1, \dots, p_n)$, where p_i is the probability that it scans band i. So, $\sum_{i=1}^n p_i = 1$ and $p_i \geq 0$, $i = 1, \dots, n$.

Let a (mixed) strategy for the Invader be $\boldsymbol{q} = (q_1, \dots, q_n)$, where q_i is the probability that it sneaks in band i. Thus, $\sum_{i=1}^n q_i = 1$ and $q_i \geq 0$, $i = 1, \dots, n$. Then, the expected payoff to the Scanner, which reflects the detection probability of the Invader, for the players employing strategies $\boldsymbol{p}$ and $\boldsymbol{q}$, is given as follows:

$$V(\boldsymbol{p}, \boldsymbol{q}) = \sum_{i=1}^{n} \gamma_i p_i q_i. \tag{10}$$

3.1 Rational Players

Next we consider when both players are rational, i.e., they know the network parameters. Based on this information the Scanner wants to maximize detection probability of the Invader, i.e., to maximize $V(\boldsymbol{p}, \boldsymbol{q})$ on $\boldsymbol{p}$ for each fixed $\boldsymbol{q}$.

Meanwhile, the Invader wants to minimize such probability. Thus, this is a zero sum game [18]. We look for (Nash) equilibrium strategies. Recall that $(\boldsymbol{p}, \boldsymbol{q})$ is an equilibrium in the zero-sum game if and only if the following inequalities hold for all feasible $(\tilde{\boldsymbol{p}}, \tilde{\boldsymbol{q}})$:

$$V(\tilde{\boldsymbol{p}}, \boldsymbol{q}) \leq V(\boldsymbol{p}, \boldsymbol{q}) \leq V(\boldsymbol{p}, \tilde{\boldsymbol{q}}) \tag{11}$$

This is a zero sum game with a diagonal payoff matrix and its equilibrium strategies are given in closed form as follows [7]:

$$p_i^1 = q_i^1 = \frac{1/\gamma_i}{\sum_{j=1}^n (1/\gamma_j)}, i = 1, \dots, n \tag{12}$$

with the equilibrium payoff to the Scanner, which reflects the detection probability, being equal to

$$\alpha^1 \triangleq V(\boldsymbol{p}^1, \boldsymbol{q}^1) = \frac{1}{\sum_{j=1}^n (1/\gamma_j)}. \tag{13}$$

3.2 Strategies Based on Principle of Indifference

If a player does not have tools (for example, to estimate band's parameters reflected by detection probabilities) to adjust its efforts, or it does not engage such tools due to the expenses involved, the player must design strategy based on other principles, such as the principle of insufficient reasons (principle of indifference) [13]. This principle might be applied in absence of information regarding the network's bands. In this case, the player would spread its effort equally among the bands.

Thus, based on the principle of insufficient reasons, the Invader implements strategy $\boldsymbol{q}^0$ which uniformly distributes sneaking efforts over bands, i.e.,

$$\boldsymbol{q}^0 \triangleq (1/n, \dots, 1/n). \tag{14}$$

Similarly, Scanner, based on the principle of insufficient reasons, implements strategy $\boldsymbol{p}^0$, which uniformly distributes scanning efforts over bands, i.e.,

$$\boldsymbol{p}^0 \triangleq (1/n, \dots, 1/n). \tag{15}$$

3.3 Relations Between Detection Probabilities

In the following lemma we establish relationships between the detection probabilities for different combinations of player's behavior.

Lemma 1. **(a)** *If both players implement strategies based on principle of indifference then the detection probability is equal to*

$$\alpha^0 \triangleq V(\boldsymbol{p}^0, \boldsymbol{q}^0) = \sum_{i=1}^n \gamma_i / n^2. \tag{16}$$

(b) *If only one player implements strategy based on principle of indifference then the detection probability is equal to*

$$\alpha = V(\boldsymbol{p}^1, \boldsymbol{q}^0) = V(\boldsymbol{p}^0, \boldsymbol{q}^1) = \frac{1}{\sum_{i=1}^{n}(1/\gamma_i)}. \tag{17}$$

(c) *Between detection probabilities the following relations hold:*

$$0 < \alpha = \alpha^1 < \alpha^0 < 1. \tag{18}$$

Note that (18) is a particular case of assumption (2).

The proof of Lemma 1 can be found in Appendix A.1.

4 Solution of Non-Zero Sum Stochastic Game

In this section we design equilibrium strategies in closed form. First we derive conditions when equilibrium is in pure strategies. In the remaining cases, we derive equilibrium in mixed strategies and prove its uniqueness.

4.1 Equilibrium in Pure Strategies

In this section we establish conditions for the equilibrium to be in pure strategies.

A Pair of Actions $(\mathbb{R}, \mathbb{R})$ is Equilibrium: Let us consider the situation where a pair of actions $(\mathbb{R}, \mathbb{R})$ is equilibrium. In other words, when $((1, 0), (1, 0))$ is equilibrium.

Theorem 1. *In the stochastic game Γ, a pair of actions $(\mathbb{R}, \mathbb{R})$ is an equilibrium with corresponding accumulated payoffs (v_S, v_I) if and only if*

$$v_S = \frac{\alpha^1 - C_S}{1 - \overline{\alpha^1}\delta} \text{ and } v_I = \frac{\overline{\alpha^1} - C_I}{1 - \overline{\alpha^1}\delta} \tag{19}$$

and

$$\frac{1 - \overline{\alpha}\delta}{1 - \overline{\alpha^1}\delta}(\alpha_1 - C_S) > \alpha - C_{S0} \text{ and } \frac{1 - \overline{\alpha}\delta}{1 - \overline{\alpha^1}\delta}(\overline{\alpha_1} - C_I) > \overline{\alpha} - C_{I0}. \tag{20}$$

The proof can be found in Appendix A.2.

A Pair of Actions $(\mathbb{I}, \mathbb{I})$ is Equilibrium: Let us examine a situation where a pair of actions $(\mathbb{I}, \mathbb{I})$ is an equilibrium. In other words, when $((0, 1), (0, 1))$ is equilibrium.

Theorem 2. *In the stochastic game Γ, a pair of actions $(\mathbb{I}, \mathbb{I})$ is an equilibrium with corresponding accumulated payoffs (v_S, v_I) if and only if*

$$v_S = \frac{\alpha^0 - C_{S0}}{1 - \overline{\alpha^0}\delta} \text{ and } v_I = \frac{\overline{\alpha^0} - C_{I0}}{1 - \overline{\alpha^0}\delta} \tag{21}$$

and

$$\frac{1 - \overline{\alpha}\delta}{1 - \overline{\alpha^0}\delta}(\alpha_0 - C_{S0}) > \alpha - C_S \text{ and } \frac{1 - \overline{\alpha}\delta}{1 - \overline{\alpha^0}\delta}(\overline{\alpha_0} - C_{I0}) \quad > \overline{\alpha} - C_I. \tag{22}$$

The proof can be found in Appendix A.3.

A Pair of Actions $(\mathbb{R}, \mathbb{I})$ is Equilibrium: Let us consider situation where a pair of actions $(\mathbb{R}, \mathbb{I})$ is an equilibrium. In other words, when $((1,0),(0,1))$ is an equilibrium.

Theorem 3. *In the stochastic game Γ, a pair of actions $(\mathbb{R}, \mathbb{I})$ is an equilibrium with corresponding accumulated payoffs (v_S, v_I) if and only if*

$$v_S = \frac{\alpha - C_S}{1-\overline{\alpha}\delta} \text{ and } v_I = \frac{\overline{\alpha} - C_{I0}}{1-\overline{\alpha}\delta} \tag{23}$$

and

$$\frac{1-\overline{\alpha^0}\delta}{1-\overline{\alpha}\delta}(\alpha - C_S) > \alpha^0 - C_{S0} \text{ and } \frac{1-\overline{\alpha^1}\delta}{1-\overline{\alpha}\delta}(\overline{\alpha_0} - C_{I0}) \quad > \overline{\alpha^1} - C_I. \tag{24}$$

The proof can be found in Appendix A.4.

A Pair of Actions $(\mathbb{I}, \mathbb{R})$ is an Equilibrium: Let us consider a situation where a pair of actions $(\mathbb{I}, \mathbb{R})$ is an equilibrium. In other words, when $((0,1),(1,0))$ is equilibrium.

Theorem 4. *In the stochastic game Γ, a pair of actions $(\mathbb{I}, \mathbb{R})$ is an equilibrium with corresponding accumulated payoffs (v_S, v_I) if and only if*

$$v_S = \frac{\alpha - C_{S0}}{1-\overline{\alpha}\delta} \text{ and } v_I = \frac{\overline{\alpha} - C_I}{1-\overline{\alpha}\delta} \tag{25}$$

and

$$\frac{1-\overline{\alpha^1}\delta}{1-\overline{\alpha}\delta}(\alpha - C_{S0}) > \alpha^1 - C_S \text{ and } \frac{1-\overline{\alpha^0}\delta}{1-\overline{\alpha}\delta}(\overline{\alpha} - C_I) \quad > \overline{\alpha^0} - C_{I0}. \tag{26}$$

The proof can be found in Appendix A.5.

Uniqueness of Equilibrium in Pure Strategies: Let us prove that only one equilibrium in pure strategies could exist.

Theorem 5. (a) *None of a pair of conditions (20), (22), (24) and (26) can hold simultaneously.*
(b) *Only one equilibrium in pure strategies could exist.*

The proof can be found in Appendix A.6.

4.2 Equilibrium in Mixed Strategies

In this section we consider the situation where the equilibrium $(\boldsymbol{x}, \boldsymbol{y})$ is in mixed strategies, i.e., $x \in (0,1)$ and $y \in (0,1)$.

Theorem 6. *In the stochastic game Γ, the equilibrium is unique. Moreover,*

(a) *if one of four conditions (20), (22), (24) and (26) holds, then equilibrium is in pure strategies, and it is given by Theorem 1–Theorem 4, respectively;*
(b) *if none of the four conditions (20), (22), (24) and (26) holds then equilibrium is in mixed strategies. Moreover, it is equal to $((x, \overline{x}), (y, \overline{y}))$ with corresponding accumulated payoffs (v_S, v_I), where*

$$v_S = \frac{\dfrac{C_S - \alpha}{\alpha^1 - \alpha} + \dfrac{C_{S0} - \alpha^0}{\alpha^0 - \alpha}}{\dfrac{\overline{\alpha}\delta - 1}{\alpha^1 - \alpha} + \dfrac{\overline{\alpha^0}\delta - 1}{\alpha^0 - \alpha}} \text{ and } v_I = \frac{\dfrac{C_I - \overline{\alpha}}{\alpha^1 - \alpha} + \dfrac{C_{I0} - \overline{\alpha^0}}{\alpha^0 - \alpha}}{\dfrac{\overline{\alpha}\delta - 1}{\alpha^1 - \alpha} + \dfrac{\overline{\alpha^0}\delta - 1}{\alpha^0 - \alpha}} \tag{27}$$

and

$$x = \frac{C_I + \alpha + (1 - \overline{\alpha}\delta)v_I}{(\alpha - \alpha^1)(1 + \delta v_I)} \text{ and } y = \frac{C_S - \alpha + (1 - \overline{\alpha}\delta)v_S}{(\alpha^1 - \alpha)(1 - \delta v_S)}. \tag{28}$$

The proof can be found in Appendix A.7.

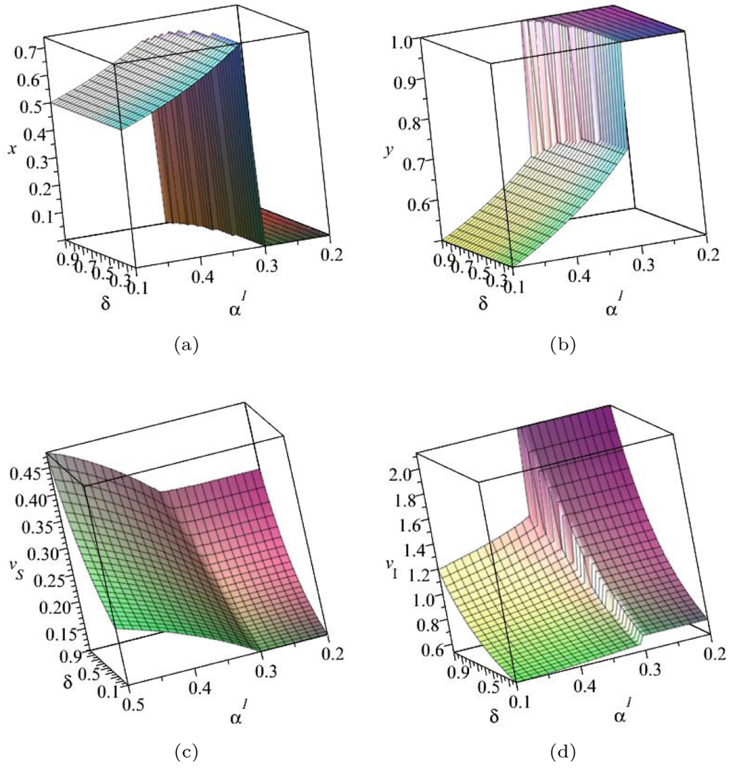

Fig. 1. (a) Probability x, (b) probability y (c) payoff to Scanner and (d) payoff to Invader as functions on detection probability α^1 and discount factor δ.

5 Discussion of the Results

Let us illustrate the obtained results on a network with scanning costs $C_{S0} = 0.1$ and $C_S = 0.2$, sneaking cots $C_{I0} = 0.1$ and $C_I = 0.2$, detection probability $\alpha^0 = 0.5$ when both players implement indifferent mode and detection probability $\alpha = 0.2$ when only one of the players implements indifferent mode. Figure 1 illustrates that an increase of discount factor leads to an increase in payoffs of both players. An increase in detection probability α^1 when both players implement a rational mode leads to an increase in payoff to Scanner and a decrease in payoff to Invader. For small probability α^1, the players follow pure strategies implementing opposite modes. For large probability α^1 the players implement mixed strategies.

6 Conclusions

A dilemma, arising in bandwidth scanning problems, that occurs because the agents implement strategies designed based on either a rational principle or the principle of insufficient reasons, has been modeled as a non-zero sum stochastic game between two players (Scanner and Invader). The equilibrium for this stochastic game has been found in closed form in stationary strategies via solving the corresponding Shapley(-Bellman) equations, and its uniqueness has been proven. The proven uniqueness of equilibrium reflects stability of the designed trade-off selection between scanning strategies designed based on a rational principle and the principle of insufficient reasons.

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A Appendix

A.1 Proof of Lemma 1

First, note that (12), (14) and (15) imply (a) and (b).

By symmetry, function $\sum_{i=1}^{n} \gamma_i \sum_{i=1}^{n}(1/\gamma_i)$ achieves its minimum at such $\gamma_i \in [0,1]$, $i = 1,\dots,n$ that $\gamma_1 = \cdots = \gamma_n$. Meanwhile,

$$\sum_{i=1}^{n} \gamma_i \sum_{i=1}^{n}(1/\gamma_i) = n^2 \text{ for } \gamma_1 = \cdots = \gamma_n. \tag{29}$$

This jointly with (13) and (16), imply that

$$\alpha^1 < \alpha^0. \tag{30}$$

Finally, (b) and (30) imply (18), and the result follows. ∎

A.2 Proof of Theorem 1

By (6)–(9), we have that $(\mathbb{R},\mathbb{R})$ is equilibrium with v_S and v_I as the corresponding accumulated payoffs if and only if the following relations hold:

$$\alpha^1 + \overline{\alpha^1}\delta v_S - C_S = v_S, \tag{31}$$

$$\overline{\alpha^1} + \alpha^1\delta v_I - C_I = v_I \tag{32}$$

with

$$\alpha^1 + \overline{\alpha^1}\delta v_S - C_S > \alpha + \overline{\alpha}\delta v_S - C_{S0}, \tag{33}$$

$$\overline{\alpha^1} + \overline{\alpha^1}\delta v_I - C_I > \overline{\alpha} + \overline{\alpha}\delta v_I - C_{I0}. \tag{34}$$

Solving (31) and (32) by v_S and v_I, respectively, imply (19). Substituting v_S and v_I given by (19) into (33) and (34) imply (20). ■

A.3 Proof of Theorem 2

By (6)–(9), we have that $(\mathbb{I},\mathbb{I})$ with v_S and v_I as the corresponding accumulated payoffs if and only if the following relations hold:

$$\alpha^0 + \overline{\alpha^0}\delta v_S - C_{S0} = v_S, \tag{35}$$

$$\overline{\alpha^0} + \overline{\alpha^0}\delta v_I - C_{I0} = v_I \tag{36}$$

with

$$\alpha^0 + \overline{\alpha^0}\delta v_S - C_{S0} > \alpha + \overline{\alpha}\delta v_S - C_S, \tag{37}$$

$$\overline{\alpha^0} + \overline{\alpha^0}\delta v_I - C_{I0} > \overline{\alpha} + \overline{\alpha}\delta v_I - C_I. \tag{38}$$

Solving (35) and (36) by v_S and v_I, respectively, imply (21). Substituting v_S and v_I given by (21) into (37) and (38) imply (22). ■

A.4 Proof of Theorem 3

By (6)–(9), we have that $(\mathbb{R},\mathbb{I})$ is an equilibrium with v_S and v_I as the corresponding accumulated payoffs if and only if the following relations hold:

$$\alpha + \overline{\alpha}\delta v_S - C_S = v_S, \tag{39}$$

$$\overline{\alpha} + \overline{\alpha}\delta v_I - C_{I0} = v_I \tag{40}$$

with

$$\alpha + \overline{\alpha}\delta v_S - C_S > \alpha^0 + \overline{\alpha^0}\delta v_S - C_{S0}, \tag{41}$$

$$\overline{\alpha} + \overline{\alpha}\delta v_I - C_{I0} > \overline{\alpha^1} + \overline{\alpha^1}\delta v_I - C_I. \tag{42}$$

Solving (39) and (40) by v_S and v_I, respectively, imply (23). Substituting v_S and v_I given by (23) into (41) and (42) imply (24). ■

A.5 Proof of Theorem 4

By (6)–(9), we have that $(\mathbb{I}, \mathbb{R})$ is an equilibrium with v_S and v_I as the corresponding accumulated payoffs if and only if the following relations hold:

$$\alpha + \overline{\alpha}\delta v_S - C_{S0} = v_S, \tag{43}$$

$$\overline{\alpha} + \overline{\alpha}\delta v_I - C_I = v_I \tag{44}$$

with

$$\alpha + \overline{\alpha}\delta v_S - C_{S0} > \alpha^1 + \overline{\alpha^1}\delta v_S - C_S, \tag{45}$$

$$\overline{\alpha} + \overline{\alpha}\delta v_I - C_I > \overline{\alpha^0} + \overline{\alpha^0}\delta v_I - C_{I0} \tag{46}$$

Straightforward calculation based on (43)–(46) implies the result. ■

A.6 Proof of Theorem 5

(b) follows from (a) and Theorem 1–Theorem 4.

Let us now prove (a). First note that by (2) we have that

$$\overline{\alpha} \geq \overline{\alpha^1} > \overline{\alpha^0}. \tag{47}$$

Assume that (20) and (22) hold simultaneously. Then, (47) and the second of inequalities (20) imply

$$\overline{\alpha} - C_I > \frac{1 - \overline{\alpha^1}\delta}{1 - \overline{\alpha}\delta}(\overline{\alpha} - C_{I0}). \tag{48}$$

Combining this inequality with the second of inequalities (20) imply

$$\frac{1 - \overline{\alpha}\delta}{1 - \overline{\alpha^0}\delta}(\overline{\alpha_0} - C_{I0}) > \overline{\alpha} - C_I > \frac{1 - \overline{\alpha^1}\delta}{1 - \overline{\alpha}\delta}(\overline{\alpha} - C_{I0}). \tag{49}$$

Meanwhile, by (47),

$$(1 - \overline{\alpha}\delta)^2 < (1 - \overline{\alpha^0}\delta)1 - \overline{\alpha^1}\delta \tag{50}$$

and

$$\overline{\alpha^0} - C_{I0} < \overline{\alpha} - C_{I0} \tag{51}$$

By (50) and (51), we have that

$$\frac{1 - \overline{\alpha}\delta}{1 - \overline{\alpha^0}\delta}(\overline{\alpha_0} - C_{I0}) < \frac{1 - \overline{\alpha^1}\delta}{1 - \overline{\alpha}\delta}(\overline{\alpha} - C_{I0}). \tag{52}$$

This contradicts to (49). Thus, conditions (20) and (22) cannot hold simultaneously.

The remaining pairs of conditions can be checked similarly. ■

A.7 Proof of Theorem 6

First note that there exists at least one equilibrium in stochastic game Γ, since Γ is a recursive game with discount factor $\delta \in (0, 1)$ [18].

By proofs of Theorem 1–Theorem 4, it follows that if equilibrium is in pure strategies, then it is given by Theorem 1–Theorem 4, and one of four conditions (20), (22), (24) and (26) have to hold. Moreover, by Theorem 5, only one of these four conditions can hold, and (a) follows.

Thus, if none of the four conditions (20), (22), (24) and (26) holds then the equilibrium is in mixed strategies. By (6)–(9), we have that $((x, \overline{x}), (y, \overline{y}))$ with $x \in (0, 1)$ and $y \in (0, 1)$ is an equilibrium with v_S and v_I as the corresponding accumulated payoffs if and only if the following relations hold:

$$(\alpha^1 + \overline{\alpha^1}\delta v_S - C_S)y + (\alpha + \overline{\alpha}\delta v_S - C_S)\overline{y} = v_S, \tag{53}$$

$$(\alpha + \overline{\alpha}\delta v_S - C_{S0})y + (\alpha^0 + \overline{\alpha^0}\delta v_S - C_{S0})\overline{y} = v_S \tag{54}$$

and

$$(\overline{\alpha^1} + \overline{\alpha^1}\delta v_I - C_I)x + (\overline{\alpha} + \overline{\alpha}\delta v_I - C_I)\overline{x} = v_I, \tag{55}$$

$$(\overline{\alpha} + \overline{\alpha}\delta \Gamma_I - C_{I0})x + (\overline{\alpha^0} + \overline{\alpha^0}\delta \Gamma_I - C_{I0})\overline{x} = v_I. \tag{56}$$

Solving (53) by y implies the second of two Eqs. (28). Solving (54) by y, we have that

$$y = \frac{C_{S0} - \alpha^0 + (1 - \overline{\alpha^0}\delta)v_S}{(\alpha - \alpha^0)(1 - \delta v_S)}. \tag{57}$$

Substituting this y into the second of two Eqs. (28) and solving by v_S implies the first of two Eqs. (27).

Solving (55) by x implies the first of (28). Solving (56) by x implies

$$x = \frac{C_{I0} - \overline{\alpha^0} + (1 - \overline{\alpha^0}\delta)v_I}{(\alpha^0 - \alpha)(1 + \delta v_I)}. \tag{58}$$

Substituting this x into the first of two Eqs. (28) and solving by v_I implies the second of two Eqs. (27). ■

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