Using Off-the-Shelf AR and VR Software for Teaching Immersive Perspectives to 9th Grade Students

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Abstract. Recent research has argued that an immersive reformulation of the concepts of anamorphosis and perspective can have beneficial effects on the teaching of these concepts to young students, and a didactic itinerary for Portuguese 9th grade students has been proposed and executed along these lines. A part of this reformulation is an integration with VR and AR visualizations which was implemented with off-the-shelf software. We report on both the advantages and limitations of these off-the-shelf platforms and propose adaptations and alternatives that might improve the didactic experience.

Keywords: Anamorphosis · Spherical Perspective · Curvilinear Perspective · Virtual Reality · Augmented Reality · Equirectangular · VR panorama

1 Introduction

It has been argued that an immersive reformulation of the concepts of anamorphosis and perspective can have beneficial effects on the teaching of these concepts, and that these benefits might be apparent even as a first approach for young students [1]. Following this speculation and earlier ad-hoc experiments, a didactic itinerary for Portuguese 9th grade students has been proposed and executed along these lines, involving a class of students in a Portuguese school [2]. An important part of this itinerary is an integration with Virtual Reality (VR) and Augmented Reality (AR) visualizations which was implemented with off-the-shelf software. In this paper we report on both the advantages and limitations of these off-the-shelf platforms and propose alternatives that may improve the didactic experience.

2 The Itinerary

The didactic itinerary has been described in some detail in [2], so here we will only provide the briefest outline of both the itinerary and its underlying conceptual basis.

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2.1 The Underlying Principles: Anamorphosis and Immersive Perspectives

The main notion underlying the approach is that anamorphosis can be redefined in an immersive way to make it the fundamental concept from which all central perspectives arise as derived concepts. This notion of immersive anamorphosis has been developed in several papers, of which a thorough review may be read in [1]. Briefly, anamorphosis is defined as an equivalence relation between 3D objects. Two 3D objects are called anamorphically equivalent with regards to a point O (or anamorphs of each other relative to O) if their points define the same cone of rays from O. This is a reformulation of Euclid's optics [1, 3, 4] taken to its logical conclusion. It is based on radial occlusion, i.e., the empirical observation (that under certain optical conditions) two points A and B look the same to a human observer at a point O if and only if they are on the same ray from O. Anamorphosis, as a equivalence relation, is a geometrical abstraction of this empirical observation. This principle leads directly to a notion of vanishing points that is more general and elegant than the usual one. Each object has a canonical anamorph which is its radial projection onto the unit sphere around O. A spatial line is anamorphic to a meridian (a meridian being defined as one half of a great circle) on this sphere, and this meridian ends at two points which are the vanishing points of the line, and meeting points for all the lines parallel to it.

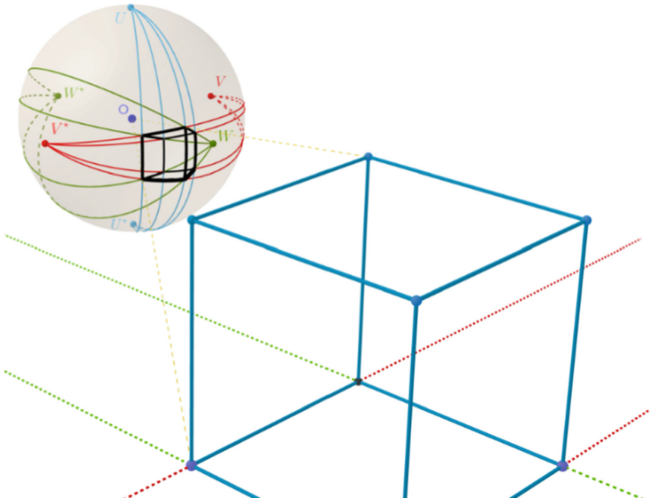

Fig. 1. Anamorphosis of a cube onto the visual sphere as seen in [1]. Each line maps to a meridian ending at two vanishing points. The cube defines a total of six vanishing points.

Under this definition, each line has exactly two vanishing points, diametrically opposite to each other on the unit sphere (or on the visual field). In Fig. 1 we can see a cube and its spherical anamorph, obtained by radial projection towards the center O. Each edge, extended to infinity, defines a line l which projects as a meridian, ending at two antipodal vanishing points. These points are located where a parallel to l through O meets the sphere [1]. Hence the images of parallels meet at common vanishing points. The cube, in particular, defines six different vanishing points.

Once spherical anamorphosis is defined, spherical *perspectives* are then defined as planifications (or *flattenings*) of the spherical anamorphosis. Each chart of the sphere (with certain technical restrictions) defines a different perspective. The flattening ruins the anamorphic *trompe l'oeil* but preserves the visual information. In particular, vanishing points are preserved. Other central perspectives (such as linear perspective or cylindrical perspective) may be viewed as partial subsets (crops) of spherical perspectives, linear perspective being the only one that preserves anamorphosis (i.e., the *trompe l'oeil* effect) [5], at the expense of losing at least one vanishing point per line. These immersive redefinitions of anamorphosis and perspective(s) resolve – or rather dismiss as ill-defined - many so-called "paradoxes" of perspective, such as Leonardo's paradox, "perspective deformations", or the supposed opposition between Euclid's optics and perspective [1, 4].

Beyond these philosophical aspects, this notion of anamorphosis has proved useful as a guiding concept in solving (that is, finding systematic methods to draw in) spherical perspectives, starting with the azimuthal equidistant case [5], then the equirectangular case [6], and recently the cubical case [7]. These methods are gaining importance as drawing in immersive perspectives, and in particular in equirectangular perspective, is gaining attention in several fields, from illustration [8], to architecture [9], to video games [10]. As immersive photography and video becomes mainstream, so does the implicit language of their underlying perspectives become more relevant to the arts. In particular, the equirectangular spherical perspective corresponds to the projection most widely used for storing 360-degree photography, and its visual language – for instance its specific deformations – are now familiar to the casual owner of a 360-degree consumer camera, and increasingly carefully studied by those involved in serious immersive photographic work, such as that used in the documentation of cultural heritage [11, 12].

2.2 The Itinerary

Immersive anamorphosis turns out to be not only philosophically but also didactically elegant. It allows for the rapid and integrated teaching of both classical anamorphosis and of many different perspectives (both linear and curvilinear). Several authors [13–15] have remarked upon the link between new visualization technologies such as VR, AR, full dome projections and video mapping, with the classical immersive technologies such as illusionary church ceilings and 19th-century panoramas. Immersive anamorphosis makes this connection technical and formal in a useful way. For these reasons this concept has been at the center of an intensive module at Aberta University (UAb) in Portugal for many years, taught by A. B. Araújo, first to classes of illustrators and artists, then to school teachers, and in later years as a module in the Ph.D. of Digital Media Arts at UAb [1, 15]. This module teaches, with practical exercises, a large plethora of concepts, from handmade anamorphoses, to curvilinear perspectives, to VR panoramas, at a pace only possible due to the way immersive anamorphosis unites the subjects and naturally segues one into the next. Some informal experiences with younger students, as well as the experience of working with their teachers, led this researcher to believe that the module could be adapted to younger age groups, using the concept of the anamorphic Dürer machine as a guiding thread, with AR/VR to aid visualization. This idea has been developed and implemented as the Ph.D. project of Manuel Flores, who is a teacher at

a school in the North of Portugal. The study was conducted in this school, involving a total of 37 students from two classes of the 9th year of elementary education, aged 14 to 16 years old. The experiment is part or a wider formative itinerary [16, 17], aimed at rethinking the study of perspective and related subjects (e.g. descriptive geometry) in elementary school Visual Education, and to foster the capacity for visualization and abstract thinking.

This itinerary implied a careful adaptation and selection of topics previously thought out for adult students [1, 15], requiring a toning down of abstract concepts and a greater reliance on VR and AR mobile apps to help the visual imagination and abstract reasoning of these young students. We will focus on these latter aspects in this paper. The reader should refer to [2] for a more general view of the itinerary. The fitting of these topics into the official program of Visual Education was possible because the program allows a certain freedom of approach to the classical notions of perspective and anamorphosis, through practical work. The itinerary had to secure the teaching of these ordinary topics, innovating without contradicting the standard expectations. This was possible through a novel use of old examples such as Dürer's perspective machine, which was subverted into a more general device, Pozzo's anamorphoses in church ceilings [18], which embody immersive concepts, and the panoramas of Richard Barker [19], which lead to the study of curvilinear perspectives, using a perspective machine in line with Baldassare Lanci's original creation [20].

2.3 The Durer Machine as an Anamorphosis Machine

The itinerary starts with a very practical piece of work: students are invited to consider the concept of radial occlusion in a practical way, through a modification of the Dürer perspective machine that turns it into a machine for drawing anamorphoses [15]. Students fix a long thread to a fixed point O (say, the top of a tripod) and make the thread pass through the vertices of a box until it hits the ground. Thus they obtain a large scale anamorphosis of the box, with respect to point O, on the floor of the school's main hall (the construction is later connected to descriptive geometry concepts - see [2, 15]) (Fig. 2).

This work fits well with the Visual Education program. Linear occlusion is the concept at play in both linear perspective and so-called "oblique" anamorphosis, which are the same projection done closer or further away from the foot of the perpendicular from O. The exercise stresses the importance of the observation point, a main program topic. This importance, often vaguely acknowledged in the study of perspective, is clearer in this kind of anamorphosis, since even a slight shift leads to clear optical deformations.

Once the physical anamorphosis is obtained, the students are prodded with questions: they are asked to imagine the lines that extend the boxes's edges to infinity, both on the real box and on the projection. Do these parallel's converge? In what sense, and if so, where? In this, we use an old device of perspective: we lead the students to understand that in following a line from point O by pointing at it with their index finger, the finger becomes parallel to the line at infinity. The real lines will seem to converge in the direction where the finger points. In classical perspective one concludes that the images of the lines will converge at the point of the canvas where the index finger points when the arm is parallel to the line [21]. In our case, we ask the student to consider that our

Fig. 2. Left: Implementing a Dürer "anamorphosis machine" with a thread fixed on a tripod. Right: The plane anamorph of a cardboard box on the floor of the hall.

"canvas" is immersive; it is the whole room, say, with its floors, and ceilings, and walls. Wherever the machine points, it will hit a surface. So, in following the line with the pointing finger, we become aware that both arms can point parallel to the line, but in opposite directions, and both will end up pointing at "the canvas", at a point where lines will seem to converge. "Which of these two points is the vanishing point, then?". With no reason to pick either, students naturally reach the conclusion: "Both."

Although it is relatively easy to reach this fundamental point of the analysis, it is a very abstract point, and it is not clear to us that the students felt secure in their conclusions or that they could grasp them in full. When prodded to imagine the projections of the lines travelling through the surrounding environment to reach the vanishing points they had clear difficulty. For instance, it is relatively easy to understand that the verticals of the box must converge at a point above one's head and another below one's feet, just by pointing fingers following the spatial lines. And it is even quite apparent to the eye that the verticals of the projected box on the school hall do seem to converge to a point under the tripod. But it is quite the feat of visual imagination to follow the projection of the verticals towards the point at the zenith, as these must travel first along the hall and corridor, then up the wall, and then finally back towards one's position in the ceiling above. This is confusing an leads even adult students to say things like "the vanishing point is at O" though this contradicts both the finger pointing exercise and their own previously achieved analytic conclusions.

If it is hard to make the students fully grasp the projections on the irregular room as a canvas, It is also hard to lead the students to imagine the projection onto the canonical environment, i.e., the visual sphere. They are prodded to "imagine a large transparent sphere" around their eye, and say what the line projections will be like and where they will project. In this case the canvas is the simplest possible one, but the effort is one of abstraction, rather than of intricate visualization.

It is to solve these difficulties that AR comes into place.

2.4 Using AR to Visualize Anamorphoses

At this point the students were invited to follow a QR code through their mobile phones to activate an Augmented Reality app. This app summoned a new anamorph of the box, this time a digital one in the form of a 3D model of a transparent box, with the edges extended by lines going to infinity. The 3D box was placed on top of the real one (and of its planar anamorph) on the video feed of the phone's camera. The students could then move the viewing direction at will, looking around point O to see where the lines converged, for all six vanishing points of the cube (Fig. 3).

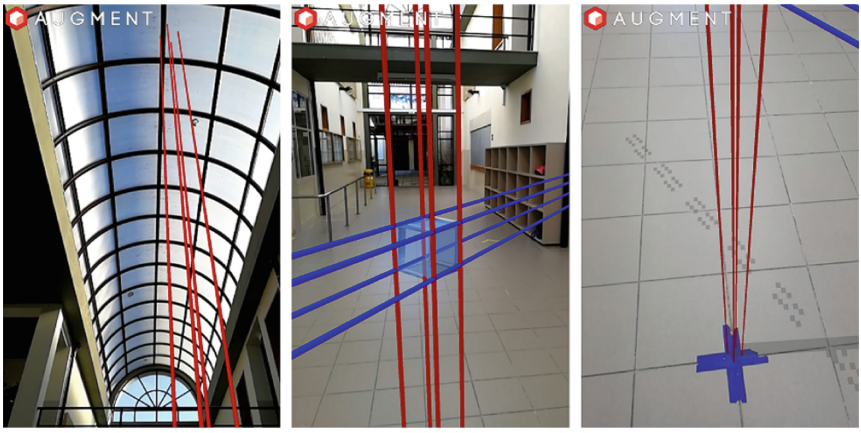

Fig. 3. Vanishing points of an anamorphosis with augmented reality

More than confirming the spotting of the vanishing points, these lines allow students to see the specific path that both the real lines and the projected lines on immersive canvas would have to follow. For instance take the red verticals of Fig. 3. They can be interpreted as just spatial lines going up and down from the real box. But they can also be interpreted as the projection of those spatial lines onto the surfaces of the room, and now we can see their path; when moving down, they travel along the corridor, towards us, reaching their vanishing point on the floor right below the tripod, where an X was marked (Fig. 3 - right). Going up, the situation is more complex: the lines must travel first down the corridor, away from the viewer and diverging from each other, then up the wall, parallel to each other, then jumping closer to us to project on the overpass, then back up the wall and over to the ceiling, where they start moving towards the viewer, to converge directly above his head. No matter how circuitous this path, with many breaks and jumps, the student can just follow it by eye, and see where it hits the "canvas" at each moment. It becomes easier to see that this anamorphosis could be physically painted on the walls, and that in this immersive "canvas" the painted lines would look straight and continuous from O, while breaking into confusing paths when seen from anywhere else. It is clear to the students that the AR is just showing cheaply and easily an effect that could be achieved (slowly and expensively) with nothing more than paint and the principle of radial occlusion. AR enables visual demonstration that otherwise would be

impractical with the means and time allotted to a teacher, and this in turns makes a wider gamut of complex subjects approachable in the classroom.

We then ask students to imagine that these AR lines are, instead of travelling straight in space or projected along the walls, moving on the visual sphere. This still pushes the students to the edge of their abstraction and imagination abilities, but the AR helps. In either case it becomes clear that, in some sense, vanishing points are properties of lines and there are always two of them; that they "live" in the visual sphere but may or may not manifest visually in the drawing, depending on whether the canvas happens to cover where they hit, and that linear perspective can only display one at a time through a limitation not of lines but of the plane as a display surface.

It must be stated that even with AR, due to the natural limitation of abstraction on the part of the young students of 15 years of age, it was not then certain to us that all of them had clearly understood the notion of vanishing points applied to anamorphosis. Nevertheless, they performed these exercises enthusiastically and seem genuinely interested, some of them repeated several times the exercise, combining the AR app and the finger pointing procedure. One student in particular stated: "I already knew where the vanishing points were, but now it makes more sense to me".

2.5 Using Anamorphoses as AR Markers

In alternative to the QR code, we also experimented with using anamorphoses as AR markers to trigger the 3D box. Students could initiate the process by pointing the camera at the anamorphic drawing of the box, but the camera had to be at the observation spot O in order for the anamorphosis to be recognized as a marker. This effectively "gamifies" the process of finding the observation point, and stresses how much the anamorphosis changes in appearance once even a little bit away from it. This gives a didactical value to the triggering process itself, even when the marker is used not to show the box but mere unrelated animations that serve as confirmation and "prize" for locating the correct point of view (Fig. 4).

This struggle to find O is of course the classical game that gives anamorphosis its meaning ("to form again"), but with a mechanical umpire to pass judgement on the success of the attempt. This gamification makes the observation point more memorable than theoretical admonitions could. Psychologically, one speculates that maybe this also stresses the equivalence of the anamorphic drawing with the real box, when the machine recognizes the drawing "as a box". This in a way seems to confirm with the viewer that the anamorphs do indeed in some way identify with each other – it is perhaps the same childish elation that any portrait painter has felt when first realizing that the face recognition software of a mobile phone camera identified his drawing as a person's face. The mechanical identification is perhaps satisfying because seems impartial and free of self-delusion (if the machine says it is so, it is so, even if what the "so" is may be unclear). This use of anamorphic images as markers seems to us be ripe for further exploration. For more uses of anamorphoses as markers see for instance [22], or, in a different but related context, the puzzle game Superliminal where placing the viewpoint correctly turns anamorphs of a 3D models into the model itself [23].

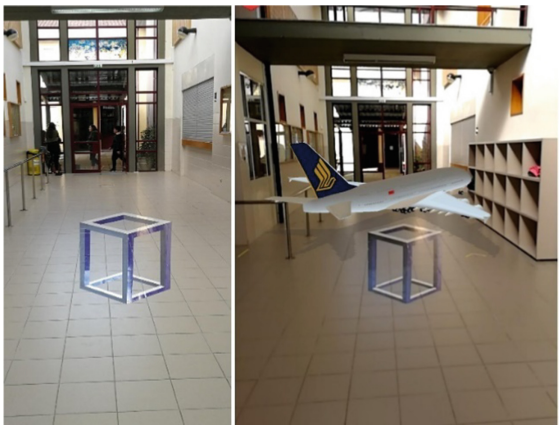

Fig. 4. An AR model triggered by using the anamorphic cube (as seen from O) as a marker.

2.6 Limitations and Possible Improvements

In the previous sections we described the use of AR in a somewhat idealized form. The actual experience, although essentially as described, suffers from some glitches and insufficiencies that somewhat disturb the flow of the exercise. We used off-the-shelf software for triggering and displaying the AR box. This was good enough for prototyping, but not completely satisfying.

The 3D box with the extended edges was modelled with SketchUp software, and exported in COLLADA (.dae) format to be uploaded on the Augment platform [24]. The Augment mobile app was then used for the AR display. The Augment platform allows experiences in AR, displaying 3D models in a real environment and in real time through smartphones and/or tablets. Its activation may be done through markers or QR codes. With this experience we have tried both possibilities, as described above. However, we found some difficulties in activating the object on the right position, i.e., overlapping the digital cube with the anamorphic projection (Fig. 5).

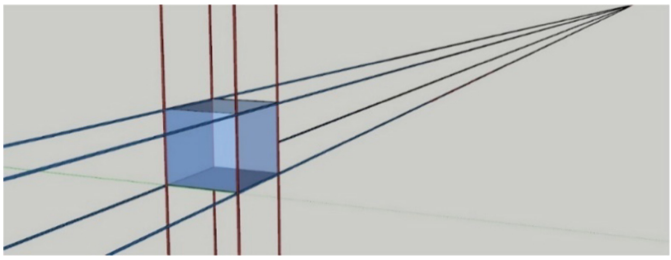

Fig. 5. 3D model of the box and its extended edges (thin cylinders) modelled in SketchUp.

The activation was done from the point of observation as intended. However, there was a need for some adjustments on the screen of the smartphone to improve its positioning. The app ignored distance to target in the initial display, being configured to

simply display the full model centred onscreen. Since in the 3D model the length of the extended edges was much larger than the dimensions of the box, this implied that the model was almost invisible at start-up and had to be pinch-zoomed into view, and then finessed until it clearly overlapped with the physical anamorphosis on the hall's floor. Once the positioning of the digital object was done correctly, it was possible to move smartphones in various directions of the vanishing points as the object would stay on the correct position. But the initial fiddling was regrettable.

In order to reduce this difficulty, the length of the extended edged was reduced, also making the 3D model lighter for processing, but then (as can be seen in Fig. 3) the meeting of the lines at the vanishing points are no longer so much seen as suggested, as the lines do not extend far enough to create a credible illusion.

Although these problems do not negate the learning experience, they do take up time in useless fiddling and break the flow of the exercises and the explanations. Solutions for this may be as simple as finding another piece of off-the-shelf software whose configurations are more adequate to our needs. There is no technical impediment to getting the 3D model perfectly aligned with the anamorphosis at startup – it requires no great precision of absolute positioning through geolocation, or even great precision in measuring absolute viewing direction, since the anamorphic drawing, when used as marker, can itself calibrate the positioning of the 3D model. So this is perfectly feasible for apps such as Augment – they just need to be adequately configurable.

In time we would like to make our own app to ensure that configurability; a prototype using Unity 3D and Vuforia was already tested, but was not ready for classroom use within the timeframe of the study.

However, from a theoretical viewpoint, another method may be more desirable. After all, any 3D model will always be but an approximation of what we want. The whole point of immersive/spherical anamorphosis is that it is a *compactification* of the visual information [1, 15], that is, it puts all the visual information of an unbounded, infinite environment into a compact (bounded and closed) surface of representation, such as a sphere. On the sphere, the drawing obtained contains all the points of the extended edges up to and including the vanishing points; a 3D model, by contrast, can never fully represent an infinite object, and much less its vanishing points.

Hence, a better approach might be to program an AR panorama viewer with some special properties: it should display a transparent sphere covered with a VR panorama, with the camera at the center of the sphere; It should take as input for the panorama transparent background images such as *png*s, in equirectangular format (up ahead we will see why this format is desirable); In the areas that are left transparent in the source, it should display the image of the mobile camera feed thus mixing features of AR with those of a VR panorama viewer; It should use the anamorphic image in the live feed to configure the initial position of the sphere so as to match the equirectangular model with the physical model; it should allow the camera to leave O so as to see the anamorphosis break, and watch the apparent lines turning to meridians on the visual sphere.

There is no technological hurdle in any of these conditions; it is just that no off-the-shelf app verifies them all. All we need is to program or reuse an equirectangular renderer and create a superposition with a live camera feed. We plan to develop such an app for the next iteration of our study. With it, the 3D model of the box would be replaced

by a hand-drawn model of the cube in correct position and with lines actually meeting at vanishing points. Producing this model is itself an interesting exercise in spherical perspective drawing. It requires no special software but only a knowledge of formal equirectangular perspective following [6]; this is a finishing exercise for the students of the present version of the UAb course mentioned above [15], that neatly closes the circle between anamorphosis and perspective. It is too complex an exercise for 15 year olds, but it might be useful to have it done by the instructor and shown to the students in the latter part of the itinerary when we address equirectangular perspective.

3 Curvilinear Anamorphoses and Spherical Perspectives

The next step in abstraction in our itinerary is to make the projection surfaces curved. In fact, the VR display of our extended cube already suggested one such anamorphosis since the ceiling of the hall was a cylindrical surface (Fig. 3-left). On flat walls two points suffice to project a line but onto a curved surface many points may be required to approximate it. This example fits well with the official program as it evidences a main feature of linear perspective by breaking it.

We study the cylindrical case because of its concreteness and of how smoothly it segues from anamorphosis to perspective. The anamorphosis of a line onto a cylinder (with the observer on a point in the cylinder's axis) is an arc of ellipse, which becomes a sinusoid when we unroll cylinder to get the cylindrical *perspective* [25].

Students approached this perspective very concretely, working on a cylindrical perspective machine built by their instructor (Fig. 6 - left). The machine consists of a target for the eye to look through, placed at the axis of a half-cylinder made of transparent acrylic held fixed by a frame. A second layer of flexible, transparent acrylic serves as drawing surface. Students can use this machine to draw a cylindrical anamorphosis of the room they are in, simply by copying onto the acrylic panel what they see while looking across the target. Then the flexible panel is taken out of the support and allowed to unbend. Once straightened, the drawing on its surface is no longer a cylindrical anamorphosis but a cylindrical *perspective.* This transition from anamorphosis to perspective involves no change in the actual drawing; only the surface's shape has to change, and only by an isometry, so it is clear, in this case, that the perspective contains all the information of the anamorphosis. What was gained was the ease of drawing on a plane; what was lost was the *trompe l'oeil*: when seen in the cylinder, from the point O, the ellipses looked like straight lines, but they look curved (sinusoidal) when seen in the flat perspective (Fig. 6 - right).

After working in this perspective and getting familiarized with its distortions, students finally meet a proper spherical perspective: the equirectangular spherical perspective.

We opted for this one for two main reasons. First, it is the standard for VR panorama files and 360-degree photography, so there are many free apps and online services to convert the drawings to interactive VR panoramas. On the desktop one can use free programs such as FSPViewer or Quicktime, and online it is enough to upload an equirectangular file to platforms such as Flickr or Facebook with appropriate tags or metadata. Second, equirectangular perspective is (nearly) an extension of cylindrical perspective

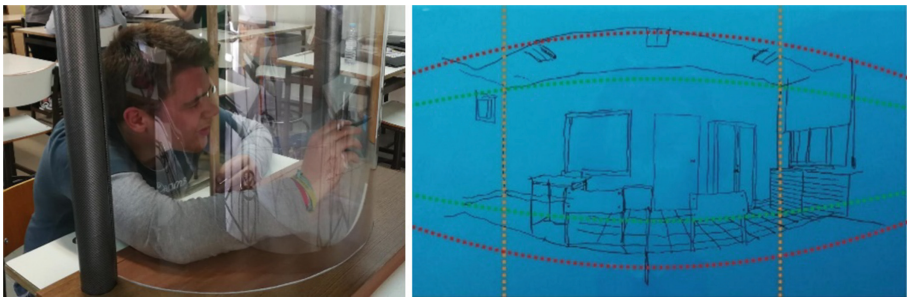

Fig. 6. A Dürer machine for drawing cylindrical anamorphoses/perspectives.

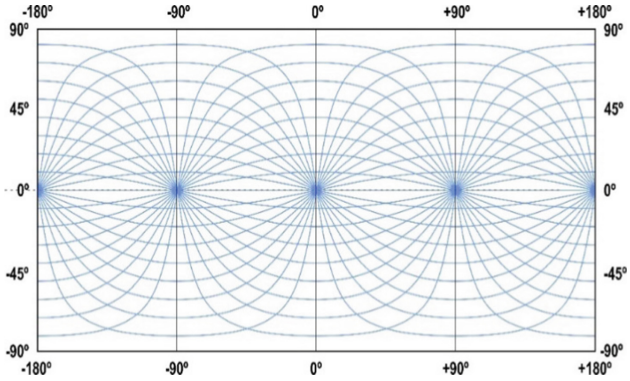

Fig. 7. An equirectangular grid of geodesics. Lines up to 60° elevation look acceptably similar to cylindrical perspective lines. After 60° the lines become strongly sigmoidal.

when cropped to low angular elevations. The equirectangular projections can be viewed as a series of two steps [1]: we first straighten the up-down meridians while keeping the horizon fixed. The meridians become vertical lines, and the sphere becomes a cylinder, which we then cut and unroll, just as with cylindrical perspective. If we plot the geodesics (images of planes through O) of this perspective we get curves that, for low values of angular elevation, are very similar to those of cylindrical perspective (See Fig. 7), differing from sinusoids by an error of about 1° for planes with angular elevations of up to 30°, and from arcs of circles with around a 2° error for up to 60° elevation [6]. So for a span of about 60°, we can draw more or less like in cylindrical perspective. This is enough to draw most panoramas (say, of a room) for the most part avoiding the high deformations zones near the zenith and nadir, except when those deformations shed light on the nature of the perspective itself. Hence we get an intuitive introduction to a full spherical perspective that preserves all vanishing points, and with great ease of VR visualization of the results by subverting 360 photography apps to display our drawings.

As an exercise, it was suggested that the students might represent the room of Vincent van Gogh from the painting "Bedroom in Arles", using an equirectangular grid (Fig. 8). The grid, measuring 14 × 28 cm, was printed on a A3, 120 g/m^2 sheet of paper, on

which the students overlap a 90 g/m^2 tracing paper sheet that they used as a support for their drawings. See Fig. 8 for an example of a student's work.

This work, executed with much collaboration among the students, was crucially helped by the use of off-the-shelf VR panorama mobile apps, namely the Android free app PhotoSphere and the 360 Sphere Viewer Lite by Google, which is compatible with the Google carboard VR glasses. Students where awed when seeing that their handmade drawings created an immersive environment they could observe from inside, interactively and immersively using their phones and cardboard glasses.

"I have never experienced anything like this!… It really works!" (Alexandre)

"It seems 3D!… Awesome… I have to paint it better… it is a bit bright. (Ana)

"It looks like I am inside a room… now I can really see the room very well!" (Tiago)

At various instances students took photos of the ongoing work, converted to a VR panorama and viewed it interactively (see Fig. 9) to find and correct problems with the drawing. This strategy stimulated students to be autonomous and persistent in the search of the solutions for the problems they found, as apparent in the following quotes:

"There are some missing lines here… if I improve the shadows … this is going to be perfect!" (Tiago).

"This is as if I was there inside… a cube!… Only it seems that I am very close to the wall" (Joana).

The VR visualizations allowed for an iterative process of trial-and-error. Although there is a systematic process to draw equirectangular perspectives [6], there was no way within the constraints of our program to take the study of the perspective that far. The frequent use of the VR panorama view permitted instead a quick, collaborative, and intuitive construction of the intended result by trial and error.

Fig. 8. Student work (André): Equirectangular perspective of van Gogh's Bedroom in Arles.

The use of VR panorama visualization apps enabled an immersive learning experience in which the students could feel inside their own drawings. Their response was joyous and enthusiastic (Fig. 10) as patent in their comments:

"Spectacular!… Ah! I wasn't expecting anything like this!… Awesome" (Catarina).

"It's fantastic…there is a vanishing point…. I see *six* vanishing points." (Beatriz).

"Classes were extremely rich because we had an experience with this type of drawing that, even if it is more rigorous, it is also a lot of fun!" (Rui).

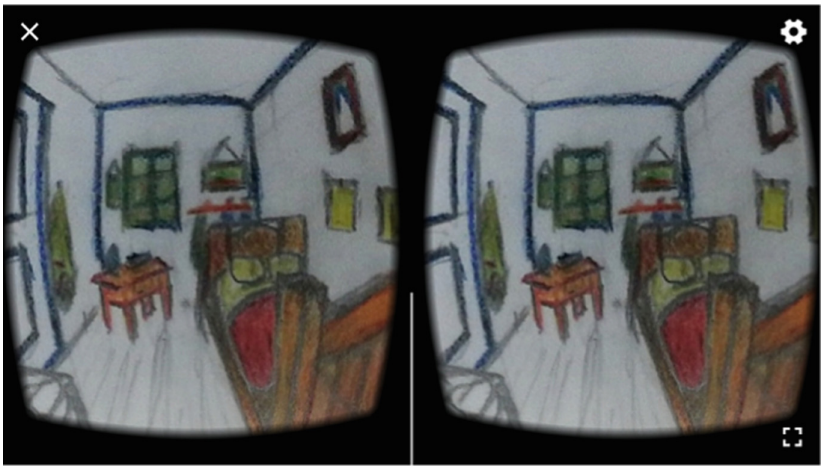

Fig. 9. Google cardboard VR visualization of André's equirectangular perspective drawing.

3.1 Problems and Proposals

The main problem with the VR panorama apps was not the visualization itself but the preprocessing required. The drawing must be photographed and carefully cropped to exactly 2×1 format before being viewed. This becomes tiresome if done repeatedly in an iterative process. It is fiddly, as the photo must be transformed to correct perspective errors, then cropped carefully and resized. This limits the frequency of checks and corrections that one is willing to undertake. It would be good to have an app that automatically detected the boundaries of the drawing, corrected for perspective, and cropped and resized the picture. It is certainly within the bounds of current technology to do so, and we plan to design such an app to improve the flow of the classwork.

The prototype presented in [26] shows a way forward in this regard. It is a live VR panorama viewer which allows for a camera to constantly feed both the flat drawing and its VR view to screens or VR glasses, as the drawing occurs. Then a constant check of the drawing can occur, which allows for new didactic possibilities. A collaboration is underway to adapt this prototype to the classroom. The prototype consists of a series of Python scripts running on a commercial visual programming software called TouchDesigner [27]. In the future it would be desirable to reimplement the whole prototype as a free software app. The authors of the prototype are aligned with the present authors in this purpose, which seems feasible in the near future.

Fig. 10. Students visualizing their works with virtual reality glasses type cardboard

4 Conclusions

Although a more systematic study is required, present experience seems to indicate the usefulness of the concept of immersive anamorphosis in elementary school Visual Education, when materialized by the modified Dürer machine and assisted by AR and VR. This is so even when immersive anamorphosis is only used as a novel way to approach and clarify the subjects of present curricula, but it is our view that a change in the curricula itself would be advantageous. This stems from both the enthusiastic response from students, the clarity with which they faced very abstract notions, and the quality of the works obtained, which is more extensively described in [2]. In the present paper we stressed the crucial role that mobile AR/VR apps played in this process, that would probably had been unfeasible without them, at least within present constraints of time and curriculum. Although these apps proved invaluable they also present inadequacies that we would like to address in future work. It is likely that off-the-shelf mobile immersive visualization apps should be replaced by free software made specifically for the present purposes, and the feasibility of this will be tested in future work.

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