



# Weak Vibration Signal Extraction Method of Mechatronics Equipment Based on Stochastic Resonance

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**Abstract.** Existing weak vibration signal extraction methods are limited by the small frequency parameters of the signal, and it is difficult to determine the critical amplitude of the signal, resulting in the delay of weak vibration signal extraction. Therefore, this paper designs a weak vibration signal extraction method for mechatronics equipment based on stochastic resonance. Based on the particle dynamics equation of Brownian motion, a bistable stochastic resonance model is constructed. By using the model, the parameters of potential well function are adjusted to achieve the optimal matching of signal, noise and nonlinear function to achieve stochastic resonance. According to the relationship between the output signal-to-noise ratio and the parameters of the potential well function, the output signal-to-noise ratio of the weak vibration is calculated, and the appropriate noise intensity is determined to enhance the weak periodic signal. The single oscillator is extended to an array equation group composed of four oscillators with different initial phase of driving signal, and the weak vibration signal with any initial phase is extracted. The experimental results show that: compared with the traditional method, the time of weak vibration signal extraction of this method is less, which shows that the weak vibration signal extraction of this method is more real-time.

**Keywords:** Stochastic resonance · Mechatronics · Weak vibration signal · Signal extraction

## 1 Introduction

With the rapid development of science and technology in our country, mechatronics equipment has been applied to all walks of life of the national economy, which strongly reflects the manufacturing level and engineering innovation ability of our country. Mechanical and electrical integration equipment components due to vibration fluctuations and other reasons, the failure will lead to the catastrophic collapse of the system, thus reducing the reliability and availability of the equipment, in turn increasing the downtime, resulting in a lot of financial losses, and each failure poses a threat to the

safety of workers [1]. Therefore, the health monitoring of equipment components is the key task to ensure the reliability of industrial process. However, many fault features are often difficult to extract in the background of strong noise, or in order to prolong the life cycle of the equipment, it is necessary to extract the early weak vibration signal, so as to diagnose the early fault of the equipment or the weak vibration signal in the background of strong noise, to minimize the impact of mechanical and electrical integration equipment fault.

Weak vibration signal extraction is an important branch of signal processing technology. It uses physics, electronics signal processing theory and other methods to study the characteristic frequency of the target signal, the variation law and characteristics of noise, so as to extract the characteristics of weak vibration signal submerged by strong noise. Due to the small amplitude of weak vibration signal and the strong noise produced by equipment operation. In addition, when extracting the characteristics of weak vibration signal, due to the influence of the noise of measuring instruments and sensors, the amplitude of the target signal to be extracted is often weaker, so that the weak vibration signal cannot be extracted [2]. The key point of weak vibration signal extraction is to extract weak characteristic frequency under strong noise background or early weak fault of mechanical equipment, so as to prevent mechanical fault.

Weak vibration signal extraction mainly depends on the quality of the signal. The signal-to-noise ratio of weak fault features in strong noise background is very low, and the signal quality is very poor. At present, the weak vibration signal extraction method is to extract the weak vibration signal from the angle of noise suppression, but at the same time, the characteristics of the weak signal are also suppressed, so that the signal-to-noise ratio is lower. When the frequency of the target signal is very close to that of the noise signal, the useful signal is damaged while processing the noise signal. At this time, it is difficult to extract the weak vibration signal. Due to the limitations of weak signal extraction technology, people continue to explore the weak vibration signal extraction and detection technology, so as to extract the weak vibration features in the strong noise background more efficiently and accurately. Most of the traditional weak vibration signal extraction methods suppress the strong noise signal, while stochastic resonance has the opposite advantages compared with the traditional noise reduction methods Using noise energy to transform weak vibration signal can enhance weak vibration characteristics and weaken part of the noise, so it can play a good role in the extraction of vibration signal in strong noise.

Based on the above research background, this paper designs a method of weak vibration signal extraction for mechatronics equipment based on stochastic resonance, which has important practical value for fault diagnosis under strong noise background. The overall work of the new method is as follows:

- (1) Based on the particle dynamics equation of Brownian motion, a bistable stochastic resonance model is built, and the parameters of the potential well function are adjusted by the model, so that the optimal matching of signal, noise and nonlinear function is achieved, so as to realize the stochastic resonance.
- (2) According to the relationship between the output signal-to-noise ratio and the potential well function parameters, the output signal-to-noise ratio of weak vibration is

calculated, so as to determine the appropriate noise intensity and realize the noise enhancement of weak periodic signal.

- (3) A single oscillator is extended into an array system of equations composed of four oscillators with different initial phases of the driving signals, from which weak vibration signals with arbitrary initial phases are extracted.

## 2 Method Design

### 2.1 Construction of Bistable Stochastic Resonance Model

The extraction of weak vibration signal of mechatronics equipment relies on the matching of noise, periodic signal and nonlinear function to achieve resonance. In the nonlinear function, when the periodic driving and noise signal match, the weak periodic driving and random noise interference will produce a synergistic effect. This effect not only does not make the increased noise make the system output signal disordered, but also greatly improves the signal-to-noise ratio of the output response, so as to achieve the purpose of detecting weak vibration signal [3]. In order to match the relationship among the three, the bistable stochastic resonance model is needed firstly, which lays the foundation for the research of weak vibration signal feature extraction in this paper.

The bistable stochastic resonance model is based on the particle dynamics equation of Brownian motion. When a particle moves irregularly in a medium, it will collide with the medium molecules. At the microscopic level, Brownian particles collide with medium molecules randomly. The bistable stochastic resonance model can be expressed as:

$$\frac{ds}{dt} = -J(s) + A \cos(\omega t + \phi) + \sigma(t) \quad (1)$$

In Eq. (1),  $s$  is the trajectory of the particle;  $t$  is the running time;  $A$ ,  $\omega$ ,  $\phi$  is the amplitude, angular frequency and phase of the periodic signal;  $J(s)$  is the symmetric bistable potential well;  $\sigma(t)$  is the zero mean additive white Gaussian noise, which meets the following conditions:

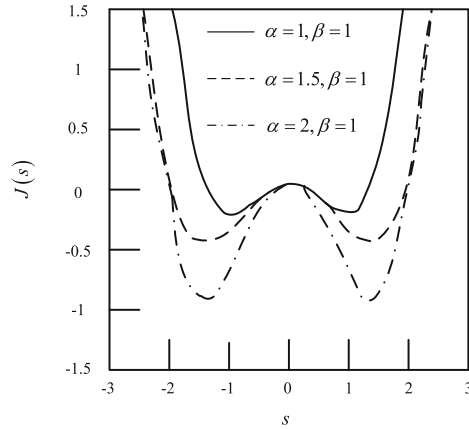
$$\langle \sigma(t)\sigma(0) \rangle = 2Q\varepsilon(t) \quad (2)$$

In Eq. (2),  $\sigma(0)$  is the white noise at the beginning of the motion;  $Q$  is the noise intensity;  $\varepsilon(t)$  is the standard deviation of the distribution. The calculation formula of  $J(s)$  is as follows:

$$J(s) = -\frac{1}{2}\alpha s^2 + \frac{1}{4}\beta s^4 \quad (3)$$

In Eq. (3),  $\alpha$ ,  $\beta$  is the bistable well parameter. By adjusting the parameters, the bistable well with different shapes can be obtained, as shown in Fig. 1.

In the case of the default periodic force, the trajectory function will jump between the local steady state ( $-1$  or  $1$ ). The jump caused by noise satisfies Kramers rate. In the case of periodic signal, the probability of particle jumping is determined by the equilibrium position [4]. The response amplitude of bistable potential well has a function



**Fig. 1.** Bistable well with different shapes.

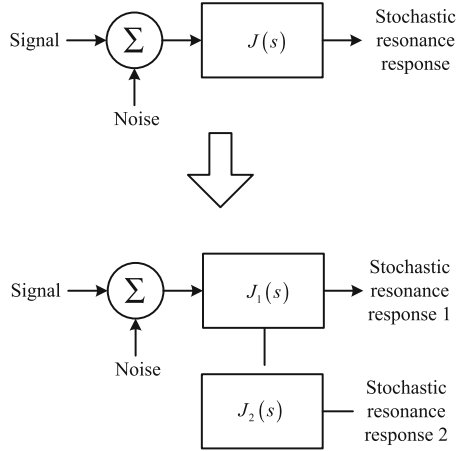
relation with noise intensity. Under the periodic excitation of different amplitude, the output amplitude of stochastic resonance changes with the increase of noise intensity, not monotone, and decreases with the increase of noise intensity when the maximum value is reached. According to the derivation process, the bistable stochastic resonance model is constructed.

## 2.2 Adjusting Well Function Parameters Based on Stochastic Resonance

According to the analysis in the previous section, stochastic resonance is actually the optimal matching relationship among signal, noise and nonlinear function. However, in practical engineering applications, the situation of signal and noise is often unknown, and the optimal matching relationship between them and nonlinear function is not always. At this time, if we want to realize weak signal detection through the stochastic resonance of the oscillator, we need to change one, two or all of the characteristics of signal, noise or nonlinear function [5]. For the measured signal, the feasible method is to adjust the parameters of the function so that the signal, noise and nonlinear function can achieve the optimal matching and achieve stochastic resonance. The occurrence of stochastic resonance has an optimal limit on the amount of noise input into the nonlinear function. When different amounts of noise are input, the output response of the system varies greatly. However, when the noise is classified and different types of noise, such as white noise, color noise and additive noise, are combined with the periodic driving input function, the output response of the function is different [6].

In the process of signal response, not only the input noise of function affects the occurrence of stochastic resonance, but also the selection and adjustment of well parameters can not be ignored. When the noise intensity is 0, the whole bistable well will no longer keep balance, and the well will tilt back and forth at a certain frequency driven by periodic signal. As long as the signal amplitude is kept less than the critical value of the function, Brownian particles can only move in a certain well at the same frequency. However, after noise is input to the function, even if the signal amplitude is less than the

critical value of the function, particles can easily transition from one potential well to another [7]. At this time, the output response is switched between the two wells according to the modulation frequency of the signal. When the signal amplitude exceeds 0, the signal introduces a periodic change to the switching of the potential well, which effectively synchronizes the switching caused by noise, so that the small periodic component of the output result is enhanced. The cascade bistable system is formed by connecting several continuous bistable wells in series, and its structure is shown in Fig. 2.



**Fig. 2.** Cascaded piecewise linear stochastic resonance potential well

The cascading method of continuous bistable well is used to transfer the high frequency energy to the low frequency region. The high frequency component is filtered out gradually, and the low frequency component is highlighted to achieve good noise reduction effect. When stochastic resonance occurs, the output response signal ratio of the continuous bistable well reaches the maximum. At this time, the output power spectrum consists of two parts, which can be expressed as:

$$P(f) = P_1(f) + P_2(f) \quad (4)$$

In Eq. (4),  $P(f)$  represents the output power spectrum;  $P_1(f)$  represents the power spectrum caused by the input periodic signal, which is equal to the frequency of the input signal;  $P_2(f)$  represents the power spectrum caused by noise, which is in the form of Lorentz distribution. In a certain range, the larger the noise intensity is, the smoother the characteristic curve of Lorentz distribution is; the smaller the noise intensity is, the steeper the curve is. The output power of noise has the characteristics of Lorentz distribution [8], and most of the energy is concentrated in the low frequency band. Therefore, only with the help of the noise energy can Brownian particles cross the barrier and make a reciprocating transition between two potential wells in the continuous bistable system at the signal frequency, thus stochastic resonance occurs in the low frequency band. With the increase of the parameters of the potential well function, the value of the optimal signal-to-noise ratio decreases gradually, and the noise intensity

corresponding to the optimal signal-to-noise ratio also increases. That is to say, the smaller the parameters of the potential well function, the better the output response of the bistable potential well can be achieved with less noise energy. Based on the above adjustment process, stochastic resonance is used to determine the well function parameters to obtain the best signal-to-noise ratio.

### 2.3 Calculate the Signal-to-Noise Ratio of Weak Vibration Output

According to the relationship between the output signal-to-noise ratio and the parameters of the potential well function, the weak vibration output signal-to-noise ratio is further calculated. Stochastic resonance is a nonlinear phenomenon. When a certain amount of noise and weak periodic vibration signal are added to the input of nonlinear function, the output response end is not affected by the added noise, but can improve the signal-to-noise ratio of the output end. When the amount of noise in the input periodic signal is appropriate, there is an optimal signal-to-noise ratio at the output response end of the nonlinear function. The essence of the above process is that the energy of the noise is transferred to the energy of the weak vibration signal through the nonlinear function, so that the energy of the weak vibration signal is enhanced, which fully reflects the cooperative relationship among noise, periodic signal and nonlinear potential well function [9]. According to the above analysis, the amplitude of weak vibration periodic signal can be enhanced under noise excitation. But in practical engineering applications, more attention is paid to the proportion of signal and noise. The formula for calculating the power spectral density of stochastic resonance output is as follows:

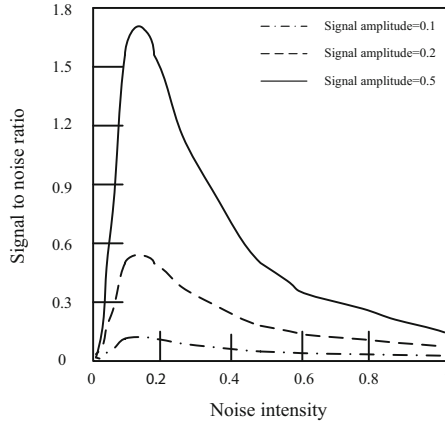
$$R(\omega) = \int_{-\infty}^{+\infty} e^{-t} \langle \langle s(t + \Delta t)s(t) \rangle \rangle dt \quad (5)$$

In formula (5),  $R(\omega)$  represents the power spectral density of the random resonance output;  $s(t + \Delta t)s(t)$  represents the total average of the noise realization; and  $\int_{-\infty}^{+\infty} e^{-t} \langle \langle \rangle \rangle dt$  represents the average of the initial phase of the input. When other frequency components are taken into account, the formula of SNR is obtained, namely:

$$snr = \pi \left( \frac{As}{Q} \right)^2 r_k \quad (6)$$

In Eq. (6),  $r_k$  is the relaxation rate of the transition. The SNR curve is very similar to the output amplitude curve, as shown in Fig. 3.

According to Fig. 3, the signal-to-noise ratio also increases with the increase of noise, reaches the maximum and then decreases. When the noise intensity is small, the oscillator can only vibrate in one potential well; when the noise intensity gradually increases, the oscillator can occasionally jump between potential wells; when the noise intensity is appropriate, the weak periodic signal can be enhanced with the help of noise, and the period of the output signal is more obvious; when the noise intensity is too large, the periodic component is polluted by noise. Therefore, only when the intensity of noise is appropriate, can the weak periodic signal be enhanced by noise.



**Fig. 3.** Relationship between output SNR and noise intensity of bistable stochastic resonance

## 2.4 Extracting Weak Vibration Signal of Mechatronics Equipment

In addition to the characteristics of strong noise background, the signals collected in practical engineering usually contain other frequency components. For weak vibration fault, the acquisition signal is not only fault frequency, but also the components of frequency conversion, mode frequency and DC component exist simultaneously. When using the method of stochastic resonance to extract the characteristic frequency of signal, the above interference frequency includes frequency conversion, mode frequency, other characteristic frequency and DC component, which have adverse effects on the signal we want to extract. The practicability of stochastic resonance can not be extended. The energy of high frequency signal output at all levels can be transferred to the low frequency part in turn by using the cascade of bistable potential wells. At the same time, the DC component in the original signal has a great influence on the occurrence of stochastic resonance. The probability of system transition will be reduced when the DC component increases to a certain extent.

In order to minimize this effect, the input signal is preprocessed before the random resonance occurs. The main purpose is to eliminate the adverse effects of the peak spectrum and DC component on the stochastic resonance. When the potential well response reaches random resonance, increasing the noise intensity of input signal again will make the potential well unable to recognize the periodic component after the output response. The over resonance phenomenon occurs in the potential well output. For the input signal under the strong noise background, according to the relationship between the signal-to-noise ratio of the output signal and the amplitude and the noise intensity of the input signal, the bistable potential well is prone to over resonance. By adjusting the noise in the input signal, the output signal can reach the optimal random resonance [10].

Because of the existence of the detection window, only a single vibrator can detect the weak vibration signal of the initial phase within the detection window. It is impossible to detect the weak vibration signal whose initial phase is beyond the scope of the detection window. But the range of detection window in the range of  $[-180, 180]$  varies with the change of the initial phase of the driving signal. Therefore, the single vibrator is extended to an array equation system composed of four different initial phase drivers to detect the weak vibration signals in any initial phase. In conclusion, as long as the amplitude of the characteristic signal to be measured is large enough, at least one potential well can be transformed from chaos state to large-scale periodic state in the array, so as to detect the weak vibration signal to be measured. In other words, as long as the amplitude of weak vibration signal is large enough, the array vibrator can extract any initial phase characteristic signal with the same frequency as the driving signal.

### 3 Experimental Study

In order to verify the effectiveness of the weak vibration signal extraction method of mechatronics equipment based on stochastic resonance, the following experiments are designed.

#### 3.1 Experimental Preparation

The weak vibration test data of mechatronics equipment is provided by the Electrical Engineering Laboratory of West storage University of the United States. An acceleration sensor is placed on the bearing pedestal of the motor drive end and fan end. A 16 channel data recorder is used to collect the vibration signal of the rolling bearing of the motor. The sampling frequency of the signal includes 12 kHz and 48 kHz.

#### 3.2 Analysis of Weak Vibration Signal Extraction Test Results

In order to test the application effect of the method of weak vibration signal extraction of mechanical and electrical integration equipment based on stochastic resonance, the signal extraction time of different methods is tested by comparing with the existing signal extraction methods. Three test points are arranged on the platform rotating axis in order to collect vibration signals of different components of the equipment, and the sampling frequency of the signals is 12 kHz. The test results are shown in Fig. 4.

According to the comparison results in Fig. 4, the weak vibration extraction time of different components of the equipment is obtained. The extraction time of weak vibration of each component obtained by this method changes in the range of 3–5 s, and the fluctuation amplitude is small. However, the time of the existing extraction methods fluctuates in the range of 6–8 s, which indicates that the extraction time is delayed, which is not conducive to the real-time extraction of weak vibration signals. The weak vibration signal extraction time of mechatronics equipment at different sampling frequencies is further counted, as shown in Table 1 and Table 2.

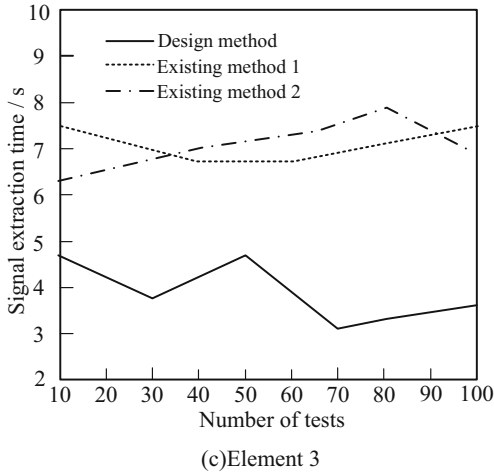
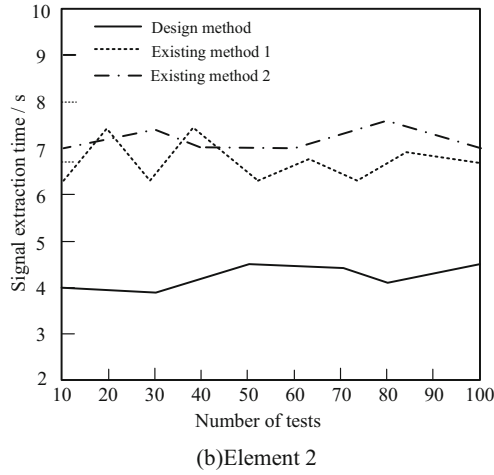
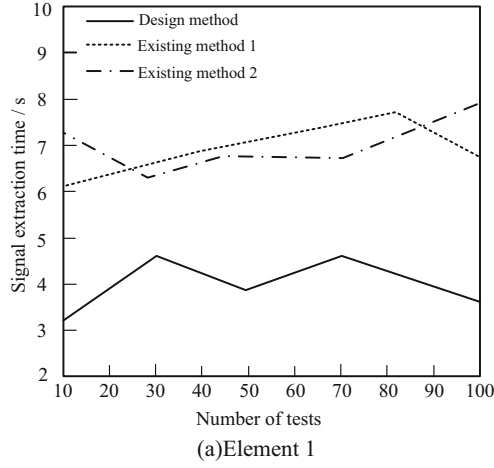


Fig. 4. Signal extraction time test of Mechatronics components.

**Table 1.** Test results of signal sampling frequency 12 kHz

Serial number	Extraction time of weak vibration signal/s		
	Method of this paper	Existing methods 1	Existing methods 2
1	8	18	19
2	9	16	18
3	8	18	19
4	10	17	22
5	11	18	21
6	8	21	23
7	9	18	22
8	9	19	23
9	8	21	22
10	7	19	19
11	9	18	20
12	9	18	21
13	11	19	21
14	9	18	20
15	10	19	19
16	11	21	18
17	9	16	19
18	8	17	20
19	9	16	21
20	11	16	22
Average value	9.15	18.15	20.45

**Table 2.** Test results of signal sampling frequency 48 kHz

Serial number	Extraction time of weak vibration signal/s		
	Method of this paper	Existing methods 1	Existing methods 2
1	6	15	16
2	5	14	16
3	6	15	17
4	5	13	15
5	4	15	16
6	6	13	16
7	5	12	15
8	6	13	14
9	4	14	13
10	7	15	15
11	6	13	15
12	5	14	16
13	6	12	16
14	4	13	14
15	6	14	13
16	5	15	13
17	6	14	14
18	7	12	15
19	6	13	16
20	5	13	15
Average value	5.5	13.6	15

According to the test results in Table 1 and Table 2 when the signal sampling frequency is 12 kHz, the weak vibration signal extraction time of this method is 9.15 s, which is 9 s and 11.3 s less than the existing methods; when the signal sampling frequency is 48 kHz, the weak vibration signal extraction time of this method is 5.5 s, which is 8.1 s and 9.5 s less than the existing methods. To sum up, the method in this paper effectively reduces the extraction time and makes the extraction process of weak vibration signals more real-time.

## 4 Concluding Remarks

In this study, a method of weak vibration signal extraction for mechatronics equipment was designed based on stochastic resonance. Based on the construction of bistable stochastic resonance model, the method used the model to adjust the potential well function parameters, so that the optimal matching of signal, noise and nonlinear function could be achieved, and the stochastic resonance could be achieved. Then, according to the relationship between the output signal-to-noise ratio and the parameters of the potential well function, the output signal-to-noise ratio of the weak vibration is calculated, and the appropriate noise intensity is determined to realize the noise enhancement of the weak periodic signal. Finally, the single oscillator is extended into a set of array equations composed of four oscillators with different initial phases of the driving signals, and the weak vibration signals with arbitrary initial phases are extracted. In this study, the experimental results proved that the method of weak vibration signal extraction is more real-time, shorten the extraction time.

Due to the complexity of actual working conditions and conditions, there is still room for further optimization of weak vibration signal extraction. When constructing the signal extraction model, membership function can be introduced to select the optimal parameter range to further improve the extraction effect of each classified signal.

**Fund Projects.** Key topics of Beijing Polytechnic.

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