Voltage Estimation of PH Meter Calibrator Using Integration of Kalman/FIR and R-T-S Smoothing Method

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Abstract. In order to further improve the accuracy of voltage estimation of PH meter calibrator, this paper introduce the scheme for the voltage estimation of PH meter calibrator using integration of Kalman filter (KF)/finite impulse response (FIR) filter and Rauch-Tung-Striebel (R-T-S) smoothing method. Firstly, the state-space for the voltage estimation is designed. Secondly, the integrated method of KF/FIR and R-T-S smoothing method is investigated. The proposed method employs the Kalman filter as the forward filter, as for the backward smoothing, the proposed R-T-S smoothing method is adopted. Then, the final estimation of the voltage is computed the average of the output of the KF/R-T-S smoothing-integrated method. By taking practical tests, the performance of the proposed integration of KF/FIR filter and R-T-S smoothing method is verified.

Keywords: KF/FIR · R-T-S smoothing · Voltage estimation

1 Introduction

The voltage estimation is essential for PH meter calibration, and its accuracy directly affects the accuracy of the PH meter calibrator. In terms of data fusion filters, Kalman filter is one of the most well-known examples [2], which has bee widely used in the field of the navigation [1]. However, it's supposed to point out that the Kalman filter (KF)'s performance depends on the accurate of the noise statistics [6], which is hard to obtain. Therefore, researchers proposed the finite impulse response (FIR) filter to overcome this shortcoming [5]. And this method has been used in the field of navigation [3,4].

In this work, the voltage estimation of PH meter calibrator using integration of Kalman filter (KF)/finite impulse response (FIR) filter and Rauch-Tung-Striebel (R-T-S) smoothing method will be proposed. In this work, contributions can be reflected in following aspects:

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(1) For the voltage estimation, the state-space has been designed.
(2) The voltage estimation of PH meter calibrator using integration of KF/FIR filter and R-T-S smoothing method has been proposed.
(3) The performance of the proposed integration of KF/FIR filter and R-T-S smoothing method is verified by taking a practical test.

2 State Space for the Voltage Estimation

In this section, the state-space for the voltage estimation will be designed. The Eq. (1) is the state equation of the KF/FIR filter used for the voltage estimation.

$$\mathbf{Vol}_{k|k-1} = \mathbf{A}\mathbf{Vol}_{k-1} + \mathbf{w}_k, \tag{1}$$

where k is the time index, $\mathbf{Vol}_k$ represents the state vector when the time index is k, in this work, the $\mathbf{Vol}_k$ only includes the voltage value at the time index k, and we set $\mathbf{A} = 1$, which represents the state transition matrix, and the $\mathbf{w}_k \sim (0, \mathbf{Q}_k)$ represents the system noise.

The Eq. (2) is the measurement equation of the KF/FIR filter used for the voltage estimation.

$$\mathbf{z}_k = \mathbf{H}\mathbf{Vol}_{k|k-1} + \mathbf{v}_k, \tag{2}$$

where $\mathbf{z}_k$ represents the observation vector when the time index is k, in this work, the $\mathbf{z}_k$ is the voltage value measured by the PH meter calibrator at the time index k, the measurement conversion matrix is represented by $\mathbf{H}$ and the $\mathbf{w}_k \sim (0, \mathbf{R}_k)$ represents the measurement noise.

3 KF/FIR Filter for the Voltage Estimation

From the model (1) and (2), we can see that the data fusion model is linear, therefore, the KF/FIR filter is used as data fusion filter, which will be introduced in this section. With the model (1) and (2) of the KF/FIR filter used for the voltage estimation, in the Algorithm 1, the pseudo code has been shown. The state prediction and state update of the Kalman filter are shown in the algorithm, at lines 3–4 and lines 5–7 respectively.

4 The R-T-S Smoothing Method

The R-T-S smoothing method will be investigated in this section, which is based on the (1) and (2). The R-T-S smoothing method is one of the most famous smoothing method, it requires the outputs of the forward filter, and we adopt the KF as the forward filter in this work, which is listed in Algorithm 1, and

Algorithm 1: The KF/FIR filter based on the model (1) and (2)

```
Data: $\mathbf{z}_k$, $\mathbf{Vol}_0$, $\mathbf{P}_0$, $\mathbf{Q}$, $\mathbf{R}$,$q^E$
Result: $\hat{\mathbf{Vol}}_k$,$\hat{\mathbf{P}}_k$
begin
    for $k = 1 : N$ do
        if $k < q^E$ then
            $\hat{\mathbf{Vol}}_{k|k-1} = \mathbf{A}\hat{\mathbf{Vol}}_{k-1}$;
            $\hat{\mathbf{P}}_{k|k-1} = \mathbf{A}\hat{\mathbf{P}}_{k-1}\mathbf{A}^T + \mathbf{Q}$;
            $\mathbf{K}_k = \hat{\mathbf{P}}_{k|k-1}\mathbf{H}_k^T\left(\mathbf{H}_k\hat{\mathbf{P}}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}\right)^{-1}$;
            $\hat{\mathbf{Vol}}_k = \hat{\mathbf{Vol}}_{k|k-1} + \mathbf{K}_k\left(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{Vol}}_{k|k-1}\right)$;
            $\hat{\mathbf{P}}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\hat{\mathbf{P}}_{k|k-1}$;
        else
            $m = k - q^E + 1,\ \ t = m + M^E - 1$;
            $\tilde{\mathbf{Vol}}_t = \hat{\mathbf{Vol}}_t$;
            $\tilde{\mathbf{P}}_t = \hat{\mathbf{P}}_t$;
            for $b = m + M^E : k$ do
                $\tilde{\mathbf{Vol}}_{b|b-1} = \mathbf{A}\tilde{\mathbf{Vol}}_{b-1}$;
                $\tilde{\mathbf{P}}_{b|b-1} = \mathbf{A}\tilde{\mathbf{P}}_{b-1}\mathbf{A}^T + \mathbf{Q}$;
                $\mathbf{G}_b = \left[\mathbf{H}^T\mathbf{H} + \left(\mathbf{A}\mathbf{G}_{b-1}\mathbf{A}^T\right)^{-1}\right]^{-1}$;
                $\mathbf{K}_b = \mathbf{G}_b\mathbf{H}^T$;
                $\tilde{\mathbf{Vol}}_b = \tilde{\mathbf{Vol}}_{b|b-1} + \mathbf{K}_b\left(\mathbf{z}_b - \mathbf{H}\tilde{\mathbf{Vol}}_{b|b-1})\right)$;
                $\tilde{\mathbf{P}}_b = (\mathbf{I} - \mathbf{K}_b\mathbf{H})\,\tilde{\mathbf{P}}_b\,(\mathbf{I} - \mathbf{K}_b\mathbf{H})^T + \mathbf{K}_b\mathbf{R}\mathbf{K}_b{}^T$;
            end for
            $\hat{\mathbf{Vol}}_k = \tilde{\mathbf{Vol}}_b$;
            $\hat{\mathbf{P}}_k = \tilde{\mathbf{P}}_b$;
        end if
    end for
end
† $N$ is the last time index
† $q^E$ is the dimension of $\hat{\mathbf{Vol}}_k$
```

the R-T-S smoothing method uses the KF's outputs $\hat{\mathbf{Vol}}_k$ and $\mathbf{P}_k, k \in [1, N]$. With the model (1) and (2)of the R-T-S smoothing method used for the voltage estimation, in Algorithm 2, the pseudo code has been shown. It can be seen from the algorithm that the R-T-S smoothing is a reverse smoothing.

Algorithm 2: R-T-S smoothing algorithm for the model (1) and (2)

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Data: $\mathbf{z}_k$, $\mathbf{Q}$, $\mathbf{R}$, $\hat{\mathbf{Vol}}_k$, $\mathbf{P}_k, k \in [1, N]$
Result: $\hat{\mathbf{Vol}}_k^S$, $\hat{\mathbf{P}}_k^S$
begin
    $\hat{\mathbf{Vol}}_N^S = \hat{\mathbf{Vol}}_N$;
    $\hat{\mathbf{P}}_N^S = \hat{\mathbf{P}}_N$;
    for $k = N : 2$ do
        $\hat{\mathbf{P}}_{k|k-1} = \mathbf{A}\hat{\mathbf{P}}_{k-1}\mathbf{A}^T + \mathbf{Q}$;
        $\mathbf{K}_{k-1}^S = \hat{\mathbf{P}}_{k-1}\mathbf{A}^T\hat{\mathbf{P}}_{k|k-1}^{-1}$;
        $\hat{\mathbf{Vol}}_{k-1}^S = \hat{\mathbf{Vol}}_{k-1} + \mathbf{K}_{k-1}^S\left(\hat{\mathbf{Vol}}_k^S - \mathbf{A}\hat{\mathbf{Vol}}_{k-1}\right)$;
        $\hat{\mathbf{P}}_{k-1}^S = \hat{\mathbf{P}}_{k-1} + \mathbf{K}_{k-1}^S\left(\hat{\mathbf{P}}_k^S - \hat{\mathbf{P}}_{k|k-1}\right)\left(\mathbf{K}_{k-1}^S\right)^T$;
    end for
end
† Superior $S$ denotes smoothness
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5 The Integration of KF/FIR Filter and R-T-S Smoothing Method

In this section, the integration of Kalman and R-T-S smoothing algorithm for the voltage estimation will be investigated. With the Algorithm 1 and Algorithm 2, and in Algorithm 3, the integration of Kalman and R-T-S smoothing algorithm with the model (1) and (2) can be shown. The structure of the integration of Kalman and R-T-S smoothing method for the voltage estimation is shown in Fig. 1. From the algorithm and the figure, one can infer easily that the proposed integration of Kalman and R-T-S smoothing method includes three steps:

(1) The forward filter (lines 2–8).
(2) The backward smoothing (lines 9–16).
(3) The $\hat{\mathbf{Vol}}$ is the mean of the $\hat{\mathbf{Vol}}_k, k \in [1, N]$.

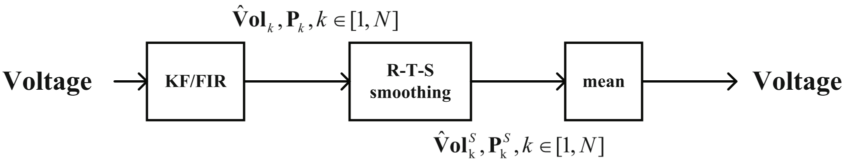

Fig. 1. The structure of the integration of Kalman and R-T-S smoothing method for the voltage estimation.

Algorithm 3: The integration of KF/FIR filter and R-T-S smoothing method for the model (1) and (2)

```
Data: $\mathbf{z}_k$, $\mathbf{Vol}_0$, $\mathbf{P}_0$, $\mathbf{Q}$, $\mathbf{R}$,$q^E$
Result: $\hat{\mathbf{V}}\mathbf{ol}$
begin
    for $k = 1 : N$ do
        if $k < q^E$ then
            $\hat{\mathbf{V}}\mathbf{ol}_{k|k-1} = \mathbf{A}\hat{\mathbf{V}}\mathbf{ol}_{k-1}$;
            $\hat{\mathbf{P}}_{k|k-1} = \mathbf{A}\hat{\mathbf{P}}_{k-1}\mathbf{A}^T + \mathbf{Q}$;
            $\mathbf{K}_k = \hat{\mathbf{P}}_{k|k-1}\mathbf{H}_k^T\left(\mathbf{H}_k\hat{\mathbf{P}}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}\right)^{-1}$;
            $\hat{\mathbf{V}}\mathbf{ol}_k = \hat{\mathbf{V}}\mathbf{ol}_{k|k-1} + \mathbf{K}_k\left(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{V}}\mathbf{ol}_{k|k-1}\right)$;
            $\hat{\mathbf{P}}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\hat{\mathbf{P}}_{k|k-1}$;
        else
            $m = k - q^E + 1, \quad t = m + M^E - 1$;
            $\tilde{\mathbf{V}}\mathbf{ol}_t = \hat{\mathbf{V}}\mathbf{ol}_t$;
            $\tilde{\mathbf{P}}_t = \hat{\mathbf{P}}_t$;
            for $b = m + M^E : k$ do
                $\tilde{\mathbf{V}}\mathbf{ol}_{b|b-1} = \mathbf{A}\tilde{\mathbf{V}}\mathbf{ol}_{b-1}$;
                $\tilde{\mathbf{P}}_{b|b-1} = \mathbf{A}\tilde{\mathbf{P}}_{b-1}\mathbf{A}^T + \mathbf{Q}$;
                $\mathbf{G}_b = \left[\mathbf{H}^T\mathbf{H} + \left(\mathbf{A}\mathbf{G}_{b-1}\mathbf{A}^T\right)^{-1}\right]^{-1}$;
                $\mathbf{K}_b = \mathbf{G}_b\mathbf{H}^T$;
                $\tilde{\mathbf{V}}\mathbf{ol}_b = \tilde{\mathbf{V}}\mathbf{ol}_{b|b-1} + \mathbf{K}_b\left(\mathbf{z}_b - \mathbf{H}\tilde{\mathbf{V}}\mathbf{ol}_{b|b-1}\right)$;
                $\tilde{\mathbf{P}}_b = (\mathbf{I} - \mathbf{K}_b\mathbf{H})\tilde{\mathbf{P}}_b(\mathbf{I} - \mathbf{K}_b\mathbf{H})^T + \mathbf{K}_b\mathbf{R}\mathbf{K}_b{}^T$;
            end for
            $\hat{\mathbf{V}}\mathbf{ol}_k = \tilde{\mathbf{V}}\mathbf{ol}_b$;
            $\hat{\mathbf{P}}_k = \tilde{\mathbf{P}}_b$;
        end if
    end for
    $\hat{\mathbf{V}}\mathbf{ol}_N^S = \hat{\mathbf{V}}\mathbf{ol}_N$;
    $\hat{\mathbf{P}}_N^S = \hat{\mathbf{P}}_N$;
    for $k = N : 2$ do
        $\hat{\mathbf{P}}_{k|k-1} = \mathbf{A}\hat{\mathbf{P}}_{k-1}\mathbf{A}^T + \mathbf{Q}$;
        $\mathbf{K}_{k-1}^S = \hat{\mathbf{P}}_{k-1}\mathbf{A}^T\hat{\mathbf{P}}_{k|k-1}^{-1}$;
        $\hat{\mathbf{V}}\mathbf{ol}_{k-1}^S = \hat{\mathbf{V}}\mathbf{ol}_{k-1} + \mathbf{K}_{k-1}^S\left(\hat{\mathbf{V}}\mathbf{ol}_k^S - \mathbf{A}\hat{\mathbf{V}}\mathbf{ol}_{k-1}\right)$;
        $\hat{\mathbf{P}}_{k-1}^S = \hat{\mathbf{P}}_{k-1} + \mathbf{K}_{k-1}^S\left(\hat{\mathbf{P}}_k^S - \hat{\mathbf{P}}_{k|k-1}\right)\left(\mathbf{K}_{k-1}^S\right)^T$;
    end for
    $\hat{\mathbf{V}}\mathbf{ol} = \frac{1}{N}\sum_{k=1}^{N}\hat{\mathbf{V}}\mathbf{ol}_k$;
end
```

6 Experimental Results

In this test, we will conduct a real experiment to proving the effectiveness of the KF/FIR filter and R-T-S smoothing integrated method with the model (1) and (2), which is proposed in this work. Firstly, the details of the experiment will be introduced, secondly, the differences of test results between the KF/FIR filter and the integration of KF/FIR filter and R-T-S smoothing method will be investigated. In the test, the sample times are set as 20, and the reference voltage value is 100.05 mV. Figure 2 displays the errors estimated by the measurement, KF/FIR filter, and the integration of KF/FIR + R-T-S smoothing. Here, the error estimated by the measurement means the errors of the measurement. From the figure, one can infer that both the KF/FIR filter and KF/FIR + R-T-S smoothing are able to estimate the voltage value, and to further illustrate the effectiveness of the proposed algorithm, the cumulative distribution functions (CDFs) of the measurement, KF/FIR filter, and KF/FIR + R-T-S smoothing are shown in Fig. 3. The root mean square errors (RMSEs) of the measurement, KF/FIR filter, and KF/FIR + R-T-S smoothing are listed in Table 1. When the reference value is 100.0096, Table 2 shows the final outputs of KF/FIR filter and KF/FIR + R-T-S smoothing. It has been clearly shown in the figure that the KF/FIR + R-T-S smoothing proposed has the smallest error compared with the KF/FIR, which verify the effectiveness of the KF/FIR + R-T-S smoothing integrated method.

Table 1. The RMSEs of KF/FIR filter and KF/FIR + R-T-S smoothing.

Methods	RMSE (mV)
KF/FIR	0.20233
KF/FIR + R-T-S smoothing	0.18532

Table 2. The final outputs of KF/FIR filter and KF/FIR + R-T-S smoothing when the reference value is 100.0096.

Methods	Value (mV)
KF/FIR	100.0356
KF/FIR + R-T-S smoothing	100.0302

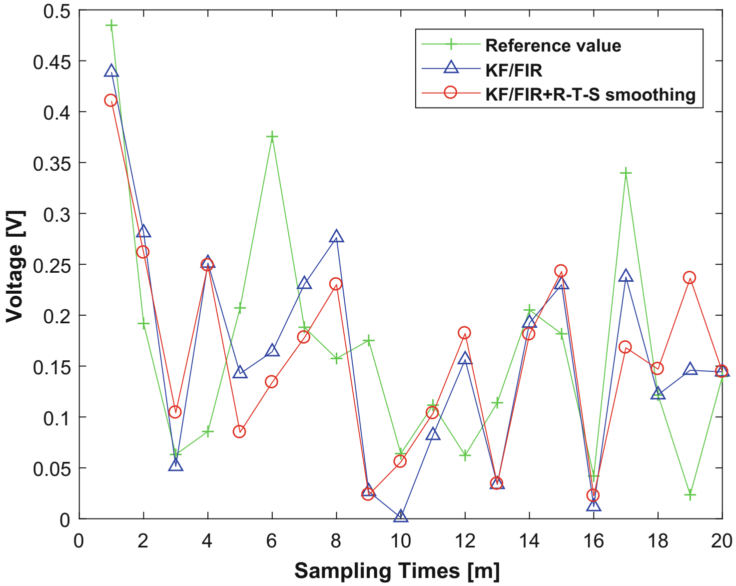

Fig. 2. The errors estimated by the measurement, KF/FIR filter, and the integration of KF/FIR filter and R-T-S smoothing method (KF + R-T-S smoothing).

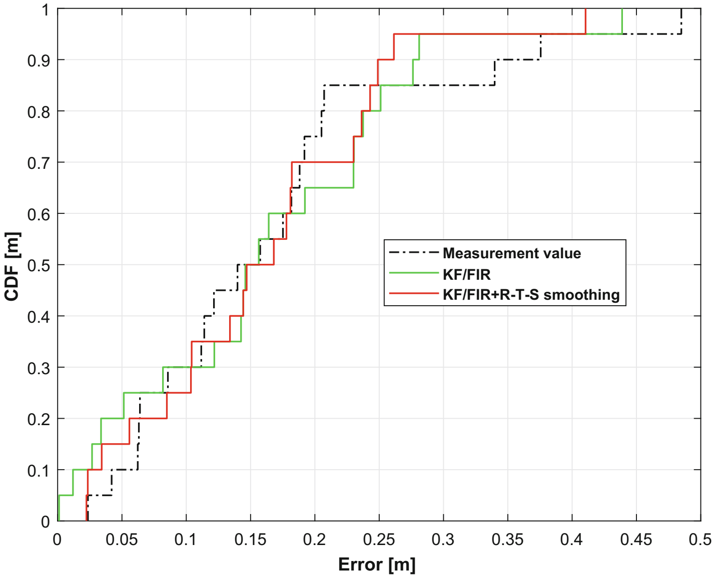

Fig. 3. The CDFs of the measurement, KF/FIR filter, and KF/FIR + R-T-S smoothing.

7 Conclusion

In this work, the voltage estimation of PH meter calibrator using integration of KF/FIR filter and R-T-S smoothing method has been proposed. The contribution of this work is reflected in the following aspects:

(1) The state-space for the voltage estimation has be designed.
(2) The voltage estimation of PH meter calibrator using integration of KF/FIR filter and R-T-S smoothing method has been proposed.
(3) The performance of the proposed integration of KF/FIR filter and R-T-S smoothing method is verified by taking a practical test.

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