A Signal Cyclic Shifting Method Based on Frank Code for Range Ambiguity Suppression in SAR System

Qichen Zheng, Zhitan Zhou, and Qiang Yang(✉)

Harbin Institute of Technology, Harbin, China
yq@hit.edu.cn

Abstract. High-resolution and wide-band (HRWS) are the performance pursued by SAR systems, which cannot be improved at the same time due to the limitation of the minimum antenna area. How to suppress the range ambiguity is the key factor that achieve HRWS imaging. There are mainly three ways to achieve it, such as using multi-channel technology, transmitting signals with specific nature and build different physical models. But there are some problems with these methods, like increasing the system complexity or being unable to completely suppress the range ambiguity of each order. In this letter, a signal cyclic shifting method based on Frank code is proposed to suppress the range ambiguity problem of SAR system. By applying this method, a single antenna is used to sequentially transmit and receive Frank code subsequences in the flight direction of the platform without increasing the system complexity. Then process the echo signal with a matching filter constructed by the cyclic shift method. The post-processing is completed by traditional SAR imaging algorithms. Finally, the range unambiguity and low sidelobes images will be obtained, which may cause ambiguity in the azimuth spectrum. Through the simulation of wide-swath scenes and comparing imaging results of different transmitted signals, it has verified the effectiveness of the method based on Frank code.

Keywords: Frank code · Signal cyclic shifting · Range ambiguity suppression · Synthetic aperture radar (SAR)

1 Introduction

By actively emitting electromagnetic waves to achieve all-weather detection on the ground, SAR plays a unique role in remote sensing and other fields. It is difficult to improve both high-resolution and wide-band imaging in SAR, which requires a higher pulse repetition frequency (PRF) to achieve high resolution and a lower PRF to get wide swath. In the spaceborne SAR scenario, a higher PRF should be selected for high resolution in the azimuth direction, which leads to the reduction of the width of the unambiguity mapping band. When the antenna transmission beam covers the non-fuzzy mapping band and adjacent areas, the target point echoes with specific distance relationships in different mapping bands in the irradiation scene are received at the same time, resulting in range ambiguity.

H.-H. Chen and W. Meng (Eds.): WiSATS 2024, LNICST 606, pp. 3–13, 2025.
https://doi.org/10.1007/978-3-031-86203-8_1

Some methods have been proposed to suppress the range ambiguity problem in SAR systems. By using multi-channel technology, setting up multiple receiving antennas in the direction of the track, it is possible to increase the azimuth spatial sampling rate of the SAR system, thereby allowing the reduction of PRF and suppressing range ambiguity. A sampling method based on the phase center of non-uniform displacement is proposed to complete the reconstruction of the azimuth signal [1]. On the basis of multi-channel technology, Krieger et al. [2] used digital beamforming (DBF) technology to construct a narrow beam with high gain to complete the imaging of the target area and suppress the distance ambiguity. Cerutti-Maori et al. [3] discussed the signal reconstruction of Multiple-Input Multiple-Output Synthetic Aperture Radar (MIMO-SAR) equipped with multiple channels, and reconstructed the SAR signal from the aliasing signal through a signal reconstruction algorithm to obtain HRWS imaging. All of the above methods effectively suppress range ambiguity, but increase the complexity of the system. Targeting the diversity of signals emitted by SAR systems, Mittermayer and Martinez [4] use the signal with up and down chirp modulation. The approach disperses the ambiguous energy to the distance dimension instead of eliminating it. Wang et al. [5] first used particle swarm optimization to obtain a set of orthogonal LFM signals, then alternately emitted orthogonal LFM signals encoded by azimuth phase coding (APC) technology. The experimental results showed that the odd-order range ambiguity could be completely suppressed and the even-order range ambiguity could be partially suppressed. Jun Yang et al. [6] proposed an APC coding technique based on a multi-channel SAR system. This approach can distinguish the target signal spectrum from the ambiguous signal spectrum by using a space-time filter. In addition to this, inspired by the cocktail effect, Chang et al. [7] constructed the suppression range ambiguity problem as a blind source separation problem and used the Second Order Blind Identification Algorithm (SOBI) to effectively suppress the range ambiguity.

With the rapid development of the Very Large Scale Integration Circuit (VLSI) technology, the problems of high data rate, large data volume and complex design circuit encountered in the generation of broadband radar signals have been effectively solved. The generation of broadband radar digital signals has become possible [8]. In this paper, a range ambiguity suppression method for SAR system based on Frank code phase modulation of wideband radar signals is proposed. The Frank code can be expressed in matrix form, and each row of the matrix is regarded as a Frank code subsequence. The subsequence has good autocorrelation characteristics, which can be used to obtain a ranged compression waveform with low sidelobes. Signal with low side lobes is conducive to alleviating the problem of strong scattering point sidelobes covering weak target points in SAR imaging. The Frank code sequence also has a good cross-correlation characteristic, according to which the range ambiguity can be effectively suppressed. This paper is organized as follows. Section 2 explains the cause of range ambiguity in SAR systems. Section 3 proves the autocorrelation and cross-correlation characteristics of Frank code by mathematical formula, and gives the process of using Frank code to suppress distance ambiguity. Section 4 simulates the fuzzy scene of the SAR system and gave the peak sidelobe ratio. By comparing the imaging results of the LFM signal and the Frank code, it is verified the effectiveness of the method of using the wideband radar signal based on

Frank code phase modulation to suppress the range ambiguity. Section 5 gives a brief summary.

2 Range Ambiguity

Spaceborne SAR usually has an antenna with a large beamwidth to obtain high-resolution imaging in azimuth. In this way, when the observation ground is wide, the target points in different mapping zones on the ground will reach the receiving antenna at the same time. The traditional imaging algorithms cannot effectively distinguish the target points in different areas, resulting in distance ambiguity.

In Fig. 1, if the distance from the target point in different mapping bands to the spaceborne SAR satisfies the following equation, the time to reach the receiving antenna will be the same and cannot be distinguished, resulting in distance ambiguity:

$$R_a = R_t + \frac{u}{2 \cdot PRF} \cdot c \tag{1}$$

where $u = \pm 1, \pm 2, \ldots$, R_t is the distance of the target point in the non-fuzzy mapping zone, R_a is the distance of the point in the u-order ambiguity strips.

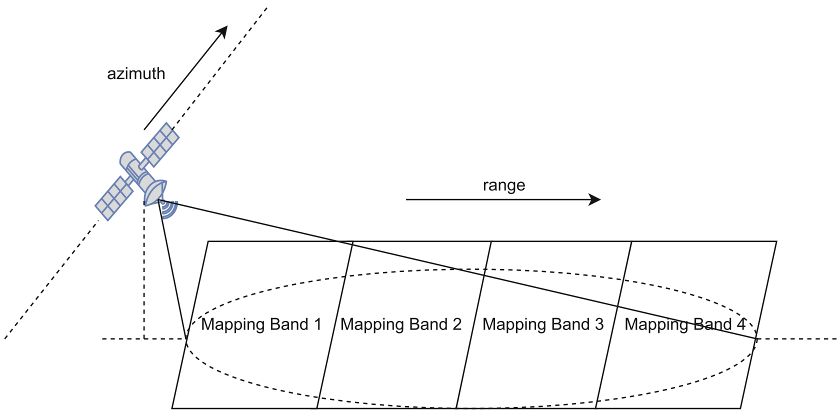

Fig. 1. Distance fuzzy schematic

3 Range Ambiguity Suppression Based on Frank Code

3.1 Frank Code Construction

R.L. Frank designed the Frank code in 1963. There are several ways to describe a Frank code. For a Frank code length of L^2, its phase sequence can be expressed as:

$$\varphi_{i,j} = \left(\frac{2\pi}{L}\right)(i-1)(j-1), i = 1, \ldots, L, j = 1, \ldots, L \tag{2}$$

The phase sequence of Frank codes can be more visually represented in the form of a matrix:

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 2 & \cdots & L-1 \\ 0 & 2 & 4 & \cdots & 2(L-1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & L-1 & 2(L-1) & \cdots & (L-1)^2 \end{bmatrix} * \left(\frac{2\pi}{L}\right) \tag{3}$$

In order to facilitate the derivation of the autocorrelation and cross-correlation characteristics of Frank codes, in Fig. 2, The Frank code in the form of the matrix above is represented as F_0 shown in the figure. F_{n_i} is the element in line i of F_0. F_i is the matrix obtained by shifting row i along the direction of the column.

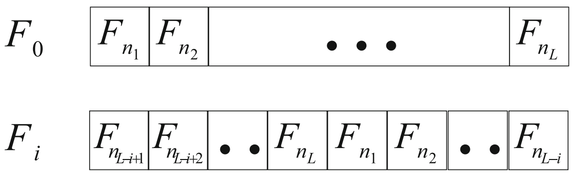

Fig. 2. Frank shift matrix

3.2 Autocorrelation Characteristics of Frank Code Subsequences

The Frank code sequence has good autocorrelation characteristics, and the mathematical formula is derived below. Firstly, find the sum of the autocorrelation functions of each row for F_0. The expression is as follows:

$$\sum_{i=1}^{L} \Re\left(F_{n_i}\right) = \sum_{n=1}^{L} \sum_{k=1}^{L-m} \exp\left\{j\frac{2\pi}{L}(n-1)(k-1)\right\} \exp\left\{-j\frac{2\pi}{L}(n-1)(k+m-1)\right\} \tag{4}$$

where $\Re(\cdot)$ is an autocorrelation function. m is the symbol offset when solving for the autocorrelation function. (4) Can be further simplified to:

$$\sum_{i=1}^{L} \Re(F_{n_i}) = \exp(j\frac{2\pi}{L}m) \sum_{n=1}^{L} \exp(-j\frac{2\pi}{L}nm) \tag{5}$$

Using the closed-form expressions for the sum of a geometric progression, the sum over n can be expressed by:

$$\sum_{n=1}^{L} \exp(-j2\pi nm/L) = \exp(-j2\pi m/L)\frac{1-\exp(-j2\pi m)}{1-\exp(-j2\pi m/L)} \tag{6}$$

The sum equals zero for $0 < m < L$ and equals L for $m = 0$. This characteristic can be summed up in the formula:

$$\tilde{\Re}(F_i) = \sum_{i=1}^{L} \Re(F_{n_i}) = \begin{cases} L^2, m = 0 \\ 0, m \neq 0 \end{cases} \tag{7}$$

where $\tilde{\Re}(\cdot)$ is a generalized correlation function defined in this letter. The function is to calculate the sum of the correlation functions of the elements in the corresponding rows of the matrix. This feature of the Frank code can be applied to SAR systems to obtain images with low sidelobes.

3.3 Cross-Correlation Characteristics of Frank Code Subsequences

The Frank code sequence has good cross-correlation characteristics, and the mathematical formula is derived below. Firstly, calculate the sum of the cross-correlation functions of the corresponding row elements of different Frank code shift matrices. The expression is as follows:

$$\tilde{\Re}\left(F_p, F_q\right) = \sum_{n=1}^{L}\sum_{k=1}^{L-m} \exp\left\{j\frac{2\pi}{L}(n+p-1)(k-1)\right\}\exp\left\{-j\frac{2\pi}{L}(n+q-1)(k+m-1)\right\} \tag{8}$$

where p is not equal to q. can be further simplified to:

$$\begin{aligned} &\tilde{\Re}\left(F_p, F_q\right) = \\ &\exp\left\{j\tfrac{2\pi}{L}(-qm+q+m-p)\right\} \times \sum_{n=1}^{L} \exp\left\{-j\tfrac{2\pi}{L}nm\right\} \times \sum_{k=1}^{L-m} \exp\left\{-j\tfrac{2\pi}{L}(q-p)k\right\} \end{aligned} \tag{9}$$

where the result of simplification is the same as that of. Therefore, It is only necessary to consider the case when $m = 0$. Using the closed-form expressions for the sum of a geometric progression, the sum over k can be expressed by:

$$\sum_{k=1}^{L} \exp\{-j2\pi(q-p)k/L\} = \exp(-j2\pi(q-p)/L)\frac{1-\exp(-j2\pi(q-p))}{1-\exp(-j2\pi(q-p)/L)} \tag{10}$$

The sum equals zero for $0 < |q-p| < L$. This characteristic can be summed up in the formula:

$$\tilde{\Re}\left(F_p, F_q\right) = 0, 0 \leq |m| < L \tag{11}$$

This feature of the Frank code can be applied to SAR systems to obtain images with free-ambiguity.

3.4 Proposed Scheme

The construction of different forms of Frank code has been given, and the two very important properties of Frank code sequence have been proved. Based on these two properties, a signal cyclic shifting method for suppressing range ambiguity in SAR system based on Frank code is proposed.

Taking the Frank code of $L = 4$ as an example, compared with the traditional SAR system that transmits LFM pulse signals sequentially in the azimuth direction, the method proposed in this paper can be summarized as transmitting signals of intra-pulse and inter-pulse joint modulation. The signals encoded by F_{n_1}, F_{n_2}, F_{n_3}, F_{n_4} are emitted alternately in the azimuth direction. In order to ensure that the range resolution is consistent with the LFM signal used by the traditional SAR system, the code length (instant width) of the transmission is set as $1/B$. In addition, in order to ensure the transmit power, the signal that uses the element of F_{n_i} for phase modulation is repeatedly transmitted.

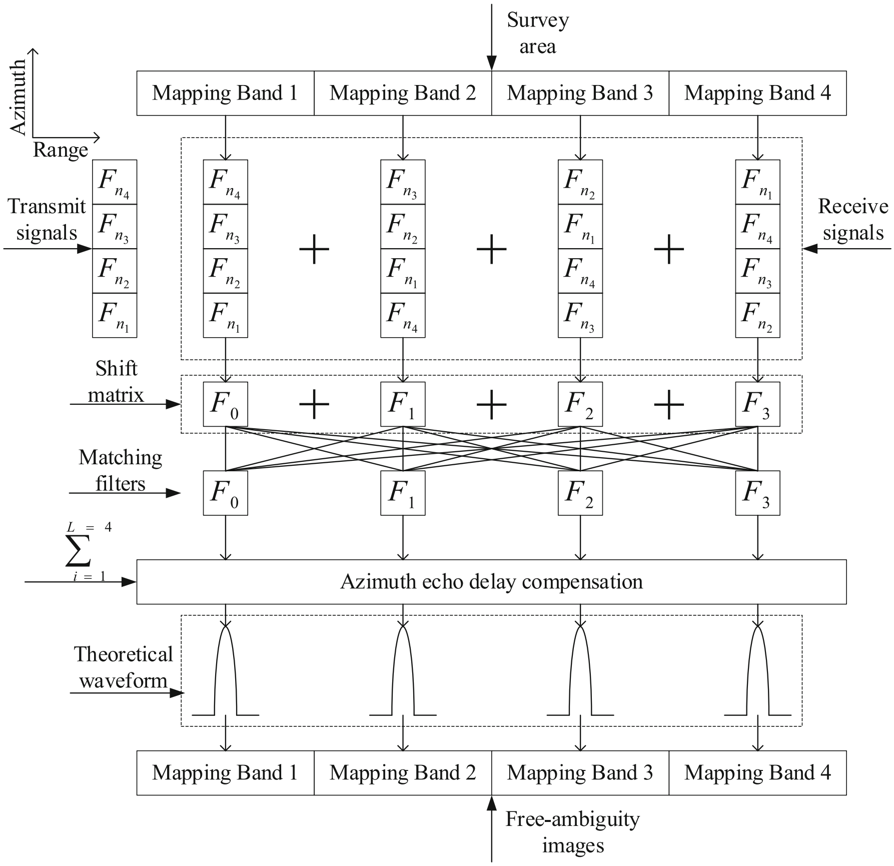

Fig. 3. Signal processing flowchart based on Frank code with L = 4

In Fig. 3, the processing flow of using the $L = 4$ Frank code to suppress the range ambiguity is given. The observation area is divided into four sub-mapping zones. It is assumed that the maximum distance of the radar to region 1 is the maximum unambiguity distance, which means the distance is that the radar signal travels in one-half pulse repetition time. In this case, use the traditional LFM signal will obtain ambiguity images. To avoid this case, choose the Frank-modulation signals. In the azimuth direction, transmit alternately the signal of phase-modulation by F_{n_i}. Suppose the platform motion model is in stop-and-go mode, take the azimuth time when transmitting F_{n_1} as an example, the received signals include F_{n_1} form the band 1, F_{n_4} from the band 2, F_{n_3} from the band 3 and F_{n_2} from the band 4. These signals are represented in the time domain as different time shifts of the reference signal, which can be regard as $\sum_{i=1}^{L} F_{n_i}$ after phase compensation in the frequency domain. Then think about the echo signals from band 1 along the azimuth direction, which can be regard as F_1. In summary, the echo signals can be expressed as $\sum_{i=1}^{L} F_i$. After that, the range compression is carried out by selecting the different matching filters to obtain different bands' unambiguity images. Taking the processing process of the final mapping band 1 image as an example, after the range matching filtering and azimuth echo delay compensation, the final result can be expressed as $\tilde{\Re}(F_0) + \tilde{\Re}(F_0, F_1) + \tilde{\Re}(F_0, F_2) + \tilde{\Re}(F_0, F_3)$. According to the conclusions and derived above, the theoretical waveform can be obtained as shown in the figure. In the same way, free-ambiguity imaging of mapping bands 2, 3, and 4 can be obtained. It can be concluded that the width of free-ambiguity zone will be enlarged 4 times, which means that the extended range of the unambiguity band is equal to the number of Frank code sets. However, using the method based on Frank code will reduce the azimuth sampling rate which may result spectral aliasing in azimuth. The time-domain duty cycle of Frank code is small, which can improve the anti-occlusion performance of Frank code.

4 Simulation

In order to verify the effectiveness of the wide-swath imaging method based on Frank code to suppress the range ambiguity of the SAR system, the fuzzy scene has been constructed in the simulation below. The SAR system adopts the strip imaging mode and the imaging algorithm adopts the improved RD algorithm. The imaging results using the LFM signal and the phase coding signal based on Frank code are simulated and compared, and the simulation parameters are shown in the following Table 1:

In Fig. 4, the distribution of target points on different mapping zones is shown, and it can be seen that the location of the target points constructed can meet the requirements of fuzzy scenes.

In Fig. 5, the LFM signal was used to image the fuzzy scene, and it was found that the target echoes of different mapping bands were mixed together in the same imaging interval, resulting in range ambiguity.

In Fig. 6, the Frank code of $L = 4$ is used to phase encode the transmitted signal, and the free-ambiguity imaging of four mapping bands has been obtained, which verifies the

Table 1. Main SAR system parameters

Parameters	Values
Carrier frequency	5 GHz
Available velocity	250 m/s
Bandwidth	200 MHz
The pulse duration of LFM	1 us
Sample frequency	800 MHz
Azimuth antenna length	5 m
PRF	2500 Hz
Centre line distance of the imaging area	60 km
The rank of the Frank matrix	4

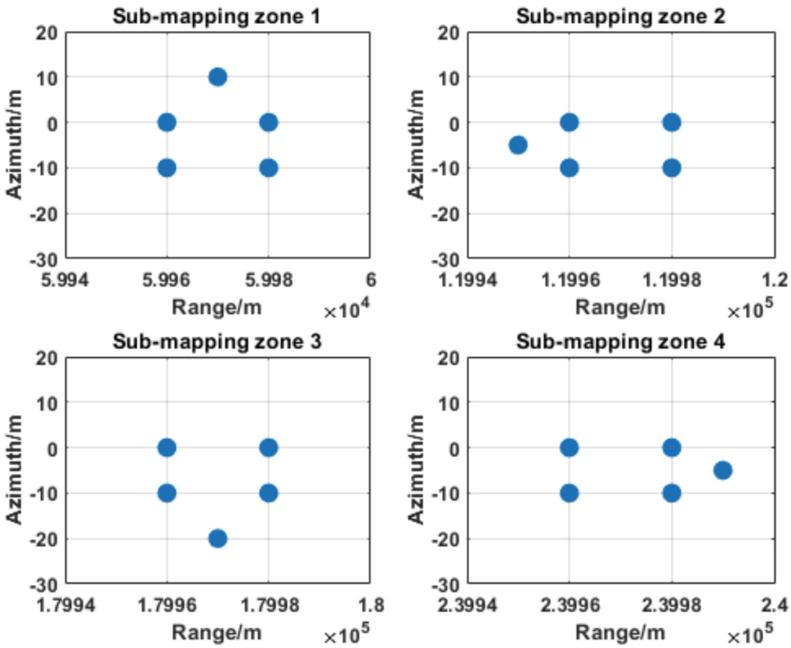

Fig. 4. Distribution of target points in different mapping zones

effectiveness of the method of suppressing the range ambiguity in SAR system based on Frank code.

In Fig. 7, it is a stitch image of four sub-mapping zone in Fig. 6.

In Fig. 8, by slicing the point target in Fig. 6 in the range and azimuth directions, it can be concluded that the peak sidelobe ratio of SAR imaging using Frank code is −

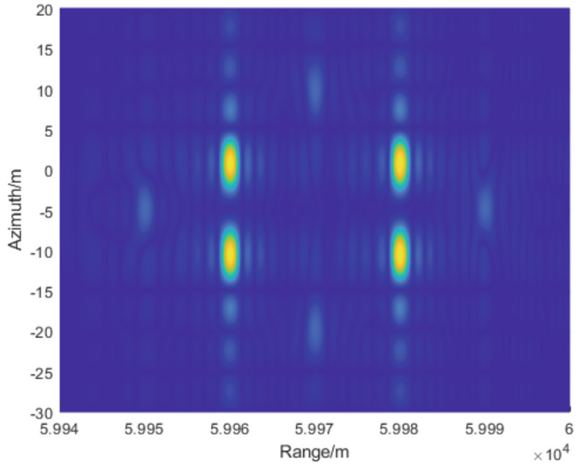

Fig. 5. Ambiguity image using LFM

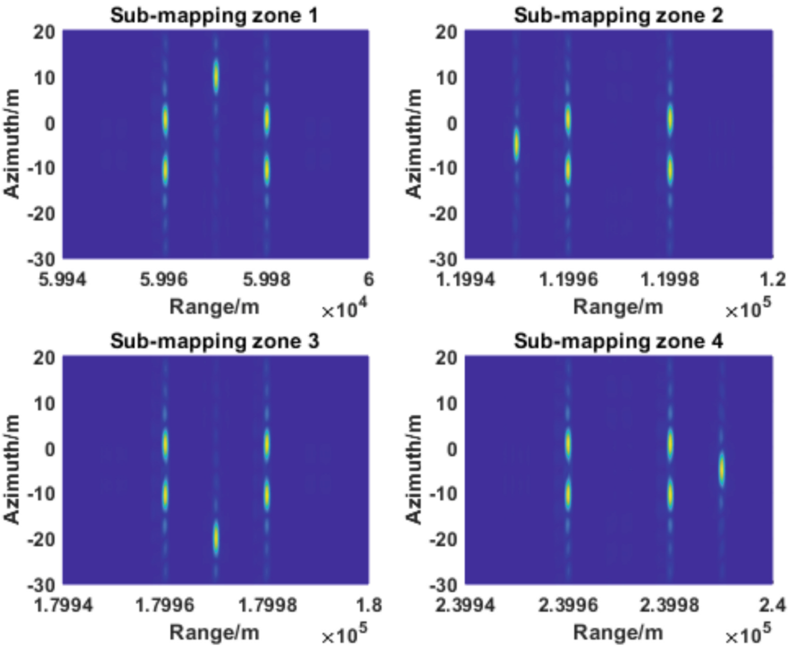

Fig. 6. Free-ambiguity image using Frank code

49 dB, which is significantly improved compared with the LFM signal. In addition, the range resolution is $c/2B$.

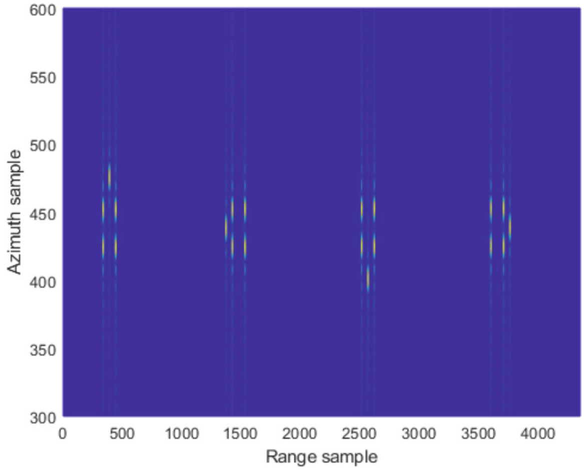

Fig. 7. A stitch image of four sub- mapping zone

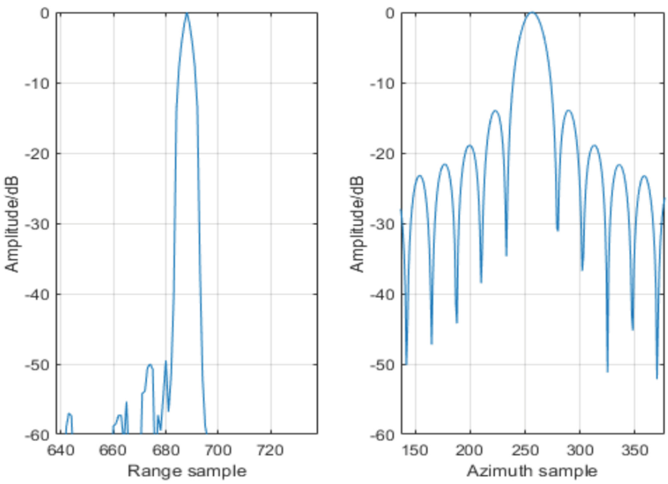

Fig. 8. Point target distance and azimuth slices using Frank code

5 Conclusions

How to suppress range ambiguity is the key to achieving SAR HRWS imaging. In this letter, a signal cyclic shifting method for suppressing range ambiguity in SAR systems based on Frank code is proposed to achieve it. There are some signals that also possess excellent correlation properties, such as signals modulated by up and down chirps and orthogonal signals obtained through genetic algorithms. These signals disperse ambiguous energy into the distance dimension instead of eliminating it, which could lead to a decrease in image quality. Using the method proposed by the letter, the Frank-modulation signals obtain excellent correlation properties by different zones' echo ambiguity energy cancellation. Besides, choose the orthogonal LFM signals encoded by azimuth phase coding (APC) technology couldn't suppress all zones ambiguity. Using the method proposed by the letter, the number of unambiguous range extensions that can be achieved is determined by the number of code sets. The method presented in this article can be applied to the following fields, such as the radar for detecting high-speed target, weather radar and high-frequency radar. In addition to this, when it is necessary to detect a weak target point next to a strong scattering point, the method is also effective.

References

1. Krieger, G., Gebert, N., Moreira, A.: Unambiguous SAR signal reconstruction from Nonuniform displaced phase center sampling. IEEE Geosci. Remote Sensing Lett. **1**, 260–264 (2004). https://doi.org/10.1109/LGRS.2004.832700
2. Krieger, G., Gebert, N., Moreira, A.: Multidimensional waveform encoding: a new digital beamforming technique for synthetic aperture radar remote sensing. IEEE Trans. Geosci. Remote Sensing. **46**, 31–46 (2008). https://doi.org/10.1109/TGRS.2007.905974
3. Cerutti-Maori, D., Sikaneta, I., Klare, J., Gierull, C.H.: MIMO SAR processing for multichannel high-resolution wide-swath radars. IEEE Trans. Geosci. Remote Sensing. **52**, 5034–5055 (2014). https://doi.org/10.1109/TGRS.2013.2286520
4. Mittermayer, J., Martinez, J.M.: Analysis of range ambiguity suppression in SAR by up and down chirp modulation for point and distributed targets. In: IGARSS 2003. 2003 IEEE International Geoscience and Remote Sensing Symposium. Proceedings (IEEE Cat. No. 03CH37477), pp. 4077–4079. IEEE, Toulouse, France (2003)
5. Wang, Y., Wang, W., Deng, Y., Zhang, Y., Zhao, P., Zhang, H.: A 2-D method based on nonlinear frequency modulation waveform and phase coding for range ambiguity suppression. IEEE Geosci. Remote Sensing Lett. **20**, 1–5 (2023). https://doi.org/10.1109/LGRS.2022.3233706
6. Xing, M., Wu, Y., Sun, G., Yang, J.: Range ambiguity suppression by azimuth phase coding in multichannel SAR systems. In: IET International Radar Conference 2013, p. 0576. Institution of Engineering and Technology, Xi'an, China (2013)
7. Chang, S., Deng, Y., Zhang, Y., Zhao, Q., Wang, R., Jia, X.: An advanced scheme for range ambiguity suppression of spaceborne SAR based on cocktail party effect. In: IGARSS 2022 - 2022 IEEE International Geoscience and Remote Sensing Symposium, pp. 2075–2078. IEEE, Kuala Lumpur, Malaysia (2022)
8. Liang, Z., Liu, Q., Long, T.: A novel subarray digital modulation technique for wideband phased array radar. IEEE Trans. Instrum. Meas.Instrum. Meas. **69**, 7365–7376 (2020). https://doi.org/10.1109/TIM.2020.2984417