Antenna Array Pattern Nulling via Convex Optimization

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Abstract. This paper introduces a novel approach for achieving antenna array pattern nulling through the application of convex optimization techniques. Nulling, the suppression of unwanted interference sources or jamming signals, is a critical aspect of modern wireless communication and radar systems. Leveraging the power of convex optimization, we propose an approach that offers precise control over null placement while maintaining the main lobe and suppressing the sidelobe to enhance the resilience and reliability of such systems.

Keywords: Array Pattern Synthesis · Pattern Nulling · Convex Optimization · Beamforming · Interference Suppression

1 Introduction

Wireless communication and radar systems are continually evolving to meet the demands of a connected world and the challenges of modern warfare. Among the many critical aspects of these systems, the ability to suppress interference and jamming signals is of paramount importance. Antenna arrays, comprising multiple radiating elements, have proven to be a powerful tool for achieving this goal. However, the efficient nulling of undesired sources in the received signal pattern remains a complex and critical task [1–3].

Recently, the design of antenna arrays for interference suppression relied on metaheuristics or nature-inspired optimization such as bat algorithms [4–6], particle swarm optimization [7], grey wolf optimization [8]. When the issue sizes are moderate, these global optimization strategies are extremely flexible and produce good results [9], but as the issue sizes grow, their processing time increases noticeably [10]. Besides, convex optimization has emerged as a transformative tool for addressing a wide range of optimization problems, including those related to antenna array pattern synthesis [11]. Convex optimization (CO) techniques offer a mathematically rigorous and systematic approach to pattern nulling. By formulating the nulling problem within the framework of convex optimization, we can leverage powerful algorithms to achieve precise control over the nulls in the antenna pattern. This approach enables us to adapt rapidly to

N. Thi Dieu Linh et al. (Eds.): ADHOCNETS 2023, LNICST 558, pp. 55–62, 2024.
https://doi.org/10.1007/978-3-031-55993-8_5

changing interference scenarios, optimize null placement, and mitigate unwanted signals effectively [12, 13].

In this paper, we explore the concept of antenna array pattern nulling via convex optimization. We delve into the theoretical foundations of convex optimization and its application to the nulling problem. Through a series of case studies, we demonstrate how convex optimization techniques can be employed to create robust and adaptive nulling solutions that meet stringent interference suppression requirements.

Furthermore, we highlight the advantages of this approach, including its capacity to handle various interference scenarios, accommodate dynamic environments, and optimize array parameters for nulling while preserving desired signal strength. As we embark on this journey, our aim is to provide researchers and engineers with a comprehensive understanding of the principles, methodologies, and practical implementations of antenna array pattern nulling via convex optimization. The innovations in this domain hold the potential to significantly enhance the resilience and reliability of wireless communication and radar systems in the face of ever-evolving interference challenges.

2 Antenna Array Pattern Nulling Formulation

In this paper, uniform rectangular arrays with half-wavelength dipoles has been investigated in Fig. 1. The array pattern can be expressed as [14]:

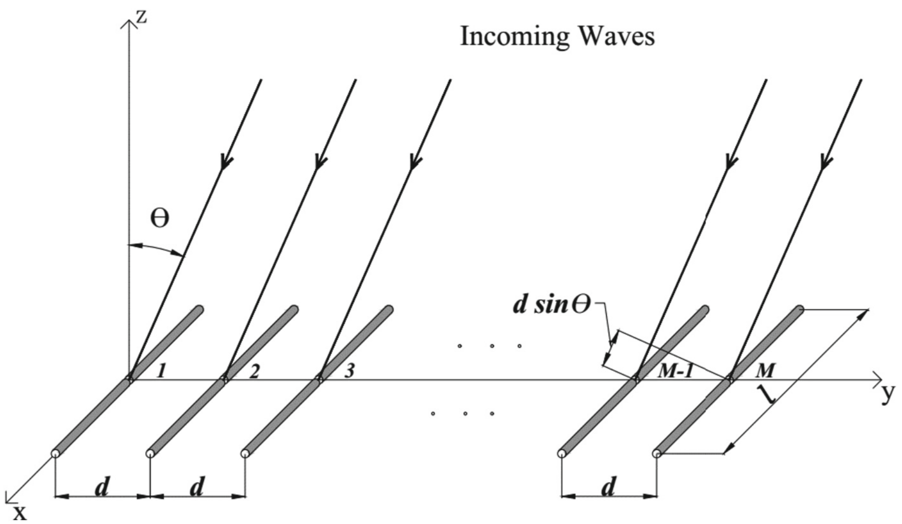

Fig. 1. A linear half-wave dipole array with uniformly spaced elements

$$P(\theta) = EF(\theta) \cdot AF(\theta) = EF(\theta) \sum_{m=1}^{M} w_m e^{j(m-1)kd\sin(\theta)} \tag{1}$$

where

- EF is the element factor of the dipole and AF is the array factor of the array
- w_m is the complex weight at the element $(m)^{th}$

Vectorization form of this pattern can be described as:

$$P(\theta) = EF(\theta)\mathbf{s}(\theta)\mathbf{w} \tag{2}$$

where:

s: steering vectors

w: complex weights

To obtain patterns nulling, **w** can be found by solving optimization problems described with typical mathematical forms as [15]:

$$\begin{aligned} &\text{minimize } f_0(\boldsymbol{x}) \\ &\text{subject to } f_i(\boldsymbol{x}) \leq b_i, i = 1, \ldots, I, \end{aligned} \tag{3}$$

where: the variable of the problem is the vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_{Nx})$, fitness function is $f_0(\boldsymbol{x}) : \mathbb{R}^{Nx} \to \mathbb{R}$, and the functions $f_i(\boldsymbol{x}) : \mathbb{R}^{Nx} \to \mathbb{R}$ are the (inequality) constraint functions with the constant limits, or bounds $b_1, b_2, \ldots, b_I$ corresponding to each constraint function. The vector $\boldsymbol{x}*$ is an optimal solution to the problem (3) if and only if among all the vectors that satisfy the constraints, it has the smallest fitness value: for any $\boldsymbol{u}$ with $f_1(\boldsymbol{u}) \leq b_1, \ldots, f_I(\boldsymbol{u}) \leq b_I$, then $f_0(\boldsymbol{u}) \geq f_0(\boldsymbol{x}*)$. In the later section, a convex optimization-based approach will be proposed to find optimized weights to conduct required pattern nulling.

3 Proposed Antenna Array Pattern Nulling

The optimal weight vector for the problem (3) utilizing CO-based approach is expressed as:

$$\mathbf{w_o} = \mathbf{w_{ref}} - \boldsymbol{\Delta}, \tag{4}$$

$\mathbf{w_{ref}}$: reference weight vector, such as weights using Chebyshev method

$\mathbf{w_o}$: optimal weight vector

$\boldsymbol{\Delta}$: the perturbation of the weight vector

Optimized pattern with imposed null, maintained main lobe and suppressed sidelobes as presented as:

$$P_o(\theta) = EF(\theta)\mathbf{s}(\theta)\mathbf{w_o} = EF(\theta)\mathbf{s}(\theta)(\mathbf{w_{ref}} - \boldsymbol{\Delta}) \tag{5}$$

$$\Leftrightarrow P_o(\theta) = EF(\theta)\mathbf{s}(\theta)\mathbf{w_{ref}} - EF(\theta)\mathbf{s}(\theta)\boldsymbol{\Delta} \tag{6}$$

$$\Leftrightarrow P_o(\theta) = EF(\theta)AF_{ref}(\theta) - EF(\theta)\mathbf{s}(\theta)\mathbf{\Delta} \quad (7)$$

To impose K nulls in the directions of $\theta_k = [\theta_1, \ldots, \theta_K]$ with $k = 1, \ldots, K$, $P_o(\theta_k)$ is set equal to zero, and the resultant equations are then written as:

$$\mathbf{S}\Delta = \mathbf{v_{ref}} \quad (8)$$

where

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}(\theta_1)_1 & \cdots & \mathbf{s}(\theta_1)_M \\ \mathbf{s}(\theta_2)_1 & \cdots & \mathbf{s}(\theta_2)_M \\ \vdots & \ddots & \vdots \\ \mathbf{s}(\theta_K)_1 & \cdots & \mathbf{s}(\theta_K)_M \end{bmatrix}.$$

$$\mathbf{\Delta} = [\Delta_1, \ldots, \Delta_M]^{\mathrm{T}}$$

$$\mathbf{v_{ref}} = \left[AF_{ref}(\theta)_1, \ldots, AF_{ref}(\theta_K)\right]^{\mathrm{T}}$$

Then, optimization problem can be expressed as:

$$\begin{aligned} &\text{minimize } \|\mathbf{\Delta}\| \\ &\text{subject to } \mathbf{S}\Delta - \mathbf{v_{ref}} = 0 \end{aligned} \quad (9)$$

This problem can be solved by CVX toolbox in [16] to obtain the optimal weight of the desired pattern.

4 Numerical Results

The performance of the proposed approach for sidelobe control is evaluated in this section through several scenarios. The proposal is introduced and evaluated for the receiver, and for the transmitter, it will also go through a similar development process. All scenario simulations use the following parameters if not specified:

The array includes 20 half-wavelength dipoles. The approach based on the Nature-Inspired Algorithm (NIO) is used as the one to compare to our proposed approach. The NIO-based approach utilizes the bat algorithm as in the paper [6]. The important parameters for the bat algorithm: the population is 500; the maximum number of iterations is 100; and the penalty parameter is 10000. The reference pattern is calculated by using the Chebyshev method-based weights with the side lobe level (SLL) of −30 dB. The illustrative results for all scenarios are taken as an average of over 50 simulations in MATLAB 2023a with an Intel® Xeon® Intel® Gold 5115 processor.

4.1 Pattern Nulling Ability

In this scenario, the anti-interference ability of the proposed solution will be verified. First, we assume interference occurs at 14°. The optimal radiation pattern shown in Fig. 2 indicates that the main beam and side beam levels are preserved approximately as well as the reference radiation pattern while placing a strong null point at 14°. The null point depth of −232.5 dB for the proposed approach is about 150 dB deeper than the NIO-based approach. To achieve that optimal radiation pattern, the convex optimization approach requires 6 iterations to converge with the fitness function value of 0.089 as shown in Fig. 3. For the proposed approach, the time required to find the optimal solution for this fitness function is 0.441 s. Meanwhile, the NIO-based approach requires 11.8 s, more than 26.5 times the calculation time of the proposed approach. This is also possible because the computational complexity of convex optimization is $O\left((M)^{3.5}\, log\left(\varepsilon^{-1}\right)\right)$, where ε is the tolerance of the primal-dual interior-point method, while that of NIO is $O(maxIter \times pop \times range \times M)$, where *maxIter* is the maximum number of iterations, *pop* is the population size, *range* is the number of degrees used in fitness function (e.g., 181 when the angle range is from −90° to 90° with step 1°).

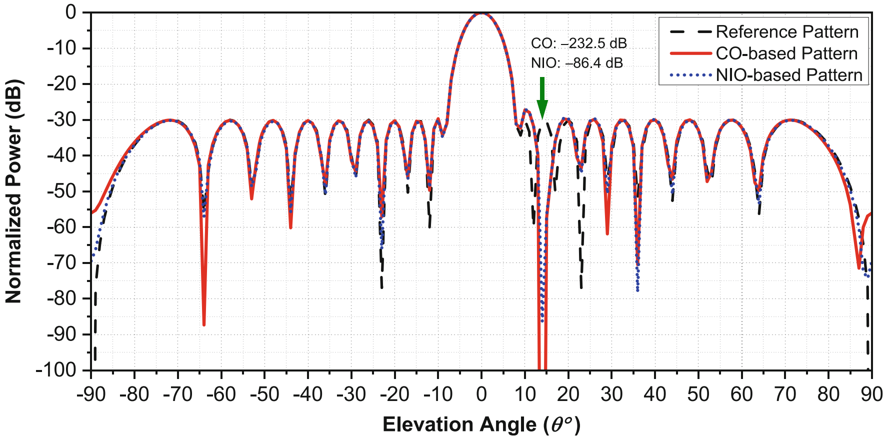

Fig. 2. Normalized patterns with a single null.

When interferences appear to fluctuate over an angular range, interference suppression over a wide range is really necessary. Figure 4 illustrates the ability to suppress interferences appearing in an angle range from −50° to −20°. The results show that the CO-based radiation pattern is capable of suppressing the sidelobe in the interference region mostly to less than 85 dB while it is only 55 dB for that based on NIO. The CO-based approach's ability to control the peak sidelobe level is also better than that based on NIO.

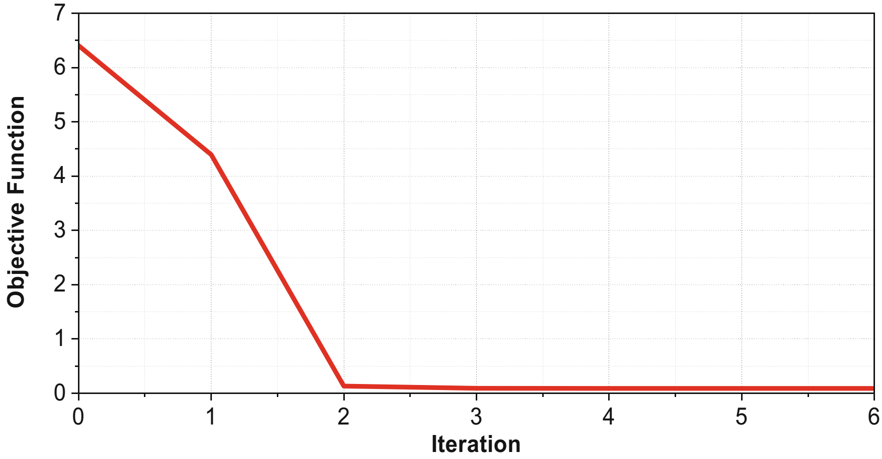

Fig. 3. The fitness function versus the number of iterations.

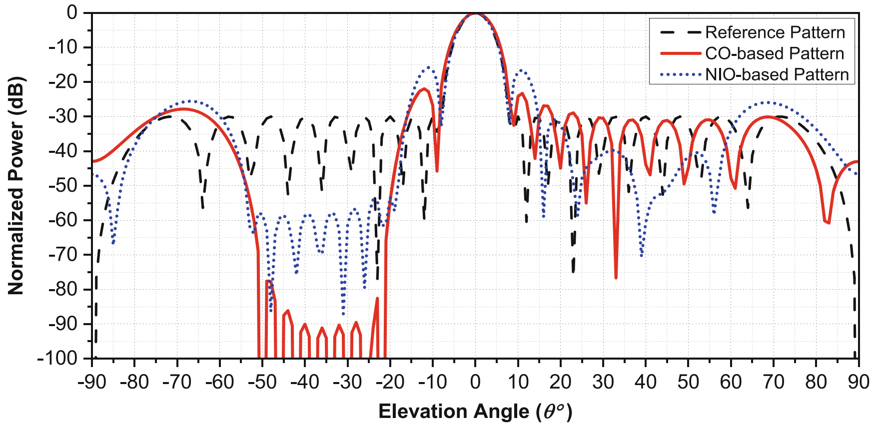

Fig. 4. Normalized patterns with a broad null from −50° to −20°.

4.2 Performance with Increasing Antennas

When we increased the number of antennas, we observed a noteworthy trend in the fitness function value in Fig. 5. Starting at the value of 2.13 and steadily decreasing, it reached its lowest point at 0.41 with 75 antennas before rising slightly again. This decrease in the fitness function value indicates a significant improvement in the efficiency of wireless power transfer system as more antennas are added. It suggests that the proposed approach effectively optimizes the power beamforming process, resulting in reduced energy losses and enhanced power transfer efficiency.

In parallel, Fig. 6 shows the fitness function versus the number of iterations. The number of iterations, representing the computational effort required, exhibited a pattern of stability after an initial decline. With 25 iterations for 20 antennas and then consistently at 7 iterations for higher antenna counts, it demonstrates that our approach maintains computational efficiency even with larger-scale systems. The proposed approach showcases impressive performance as the number of antennas increases. It not only achieves substantial gains in wireless power transfer efficiency, as evidenced by the decreasing fitness function values, but also maintains computational efficiency with a consistent and low number of iterations. These findings underscore the scalability and effectiveness of our approach in addressing the challenges of wireless power transfer in complex electromagnetic environments.

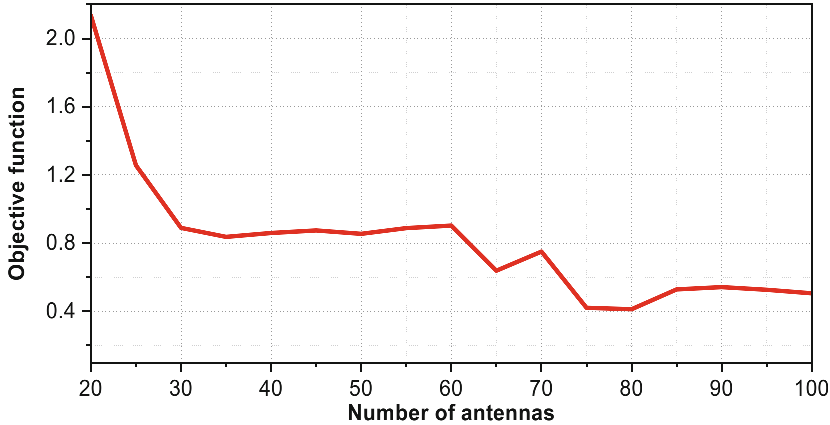

Fig. 5. The fitness function versus the number of antennas.

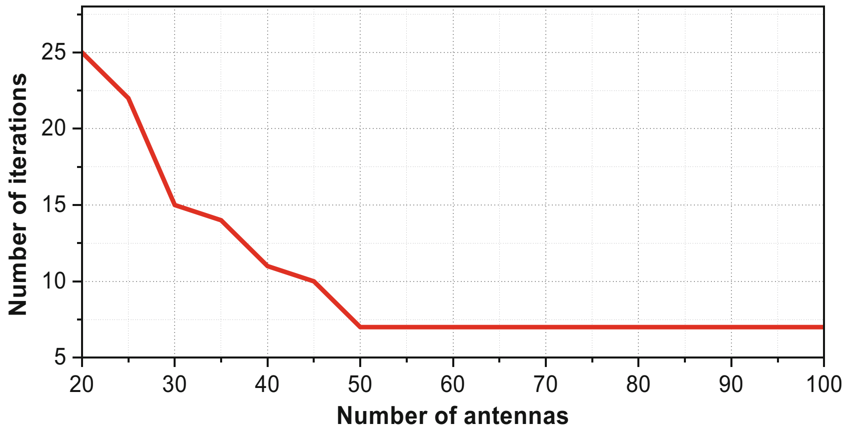

Fig. 6. The number of iterations versus the number of antennas.

5 Conclusion

In this paper, we have explored the application of convex optimization techniques for antenna array pattern nulling, shedding light on their efficacy in mitigating interference sources while maintaining the integrity of desired signals. The key takeaways from this study underscore the significance of convex optimization as a valuable tool in the realm of antenna array design.

Acknowledgment. This research is supported by Hanoi University of Industry [Grant number: 25–2023-RD/HĐ-ĐHCN].

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