



RSS Fingerprint Based Signal Source Localization Using Singular Value Decomposition

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Abstract. As the technique determines the position of a target device based on radio frequency (RF) fingerprint, received signal strength (RSS) fingerprint source localization technology is attracting increasing attention due to its numerous applications. In this paper, we propose a novel RSS Fingerprint Based Signal Source Localization algorithm. We use multi-dimensional interpolation to establish a fingerprint database, and singular value decomposition (SVD) is utilized to extract effective information from the fingerprint data. We divide the fingerprint database into multiple sub-fingerprint databases according to the location area and k -nearest neighbor (KNN) algorithm. Moreover, we adjust the fingerprints in each sub-fingerprint database to complete the offline training phase. In the online positioning phase, in order to improve the positioning efficiency, we use the k -dimensional (KD) tree algorithm to predict the fingerprint database using the fingerprint data received from the test point and determine the final position of the target source. Extensive measurements are carried out and it is proved that the proposed method is superior to existing ones.

Keywords: Radio frequency fingerprint · Signal source localization · Singular value decomposition

1 Introduction

With the continuous development of modern society, both daily life and public utilities are inseparable wireless communications [1]. Therefore, it is necessary to monitor, to locate and to standardize radio signals. Radio signal sources are affected by the reflection, refraction, diffraction and other radial propagation of obstacles. The current radio signal source positioning mainly relies on GPS systems or fixed monitoring stations or mobile monitoring vehicles [2], using Time of Arrival (TOA), Time Difference of Arrival (TDOA) or Angle of Arrival (AOA) and other ranging methods for positioning. However, these methods have higher requirements for hardware equipment and consume corresponding costs. The use of radio frequency (RF) fingerprint positioning

technology can obtain higher positioning accuracy in an environment with complex radio signal propagation characteristics [3], which can basically meet the needs. Moreover, this technology uses the received signal strength (RSS) fingerprint and the corresponding location coordinate information to calculate the positioning accuracy. It does not involve the location information of the access point (signal transmitter) and save working time [4].

The RSS was widely utilized as a feature in localization [5], as the RSS can be obtained easily. In the offline database building stage, the fingerprint signal collected in a large area are relatively sparse, and the RF signal are affected by attenuation and multipath effects during propagation. Therefore, the collected RSS fingerprints are directly used, which will cause a decrease in positioning accuracy. To solve this problem, some scholars have proposed preprocessing and clustering of fingerprint data. For instance, Fang [6] proposed the fingerprint feature extraction method of the signal projection, which projected the collected fingerprint signals to the relevant physical space. In literature experiment part, the principal components analysis (PCA) method was used to prove that this method can reduce the influence of the external environment on data and reduce the positioning error. When Youssef released the Horus positioning system [7], he proposed clustering to improve the accuracy of this positioning system. The specific method is to first divide the fingerprints into several areas after the establishment of the fingerprint database. And in the positioning stage, after obtaining the fingerprint vector of the points to be located, this method determines the point to be located in a certain area. Finally, in this area the accuracy of the position is determined. The method greatly reduces the positioning complexity in a large scale in environment and effectively improves the positioning accuracy.

And the research team led by Castor designed and completed a RF fingerprint positioning system called “Nibble” [8]. Nibble uses the signal noise ratio (SNR) of a RF signal reception as the location signal. In the literature [9], the author improved the accuracy of RF fingerprint by selecting the channel impulse response (CIR) as the fingerprint, and proved the superiority of fingerprint as the signal of physical layer. However, with underlying physical information they rely on specialized software and hardware to receive them [10]. The additional hardware leads to an increase in cost, which greatly limits the development of these fingerprints. Besides, with the rapid breakthrough of machine learning in various fields, more and more scholars are considering using machine learning to solve this problem, such as semi-supervised learning method [11, 12]. Researcher Pan proposed a semi-supervised learning database construction method [13], which uses part of the RSS fingerprints with marked locations and part of the RSS fingerprints without marked locations to build a fingerprint database through a manifold model, thereby reducing the workload.

Positioning is divided into receiving end positioning and signal transmitting source positioning. Since both species are based on radio signal strength, there is interoperability. Therefore, this paper draws on receiving end positioning, adopting RF fingerprint location technology to carry out signal source localization research and improve the offline database building stage and online positioning stage. We propose a novel RSS Fingerprint Based Signal Source Localization algorithm. First, in the offline training phase, we perform multi-dimensional interpolation on the RSS data collected from the

field to obtain all the fingerprints in the positioning area. The SVD processing of the obtained data not only retains useful information but also reduces the impact of external factors on fingerprints. At this time, we complete the establishment of the fingerprint database. We divide the database into multiple intersecting sub-fingerprint databases according to the location area, and use the KNN algorithm to adjust the data in the sub-fingerprint databases. In the online positioning phase, we use the KD tree algorithm to predict the fingerprint database using the fingerprint data received from the test point and determine the final position of the target source by the improved weighted k -nearest neighbor (WKNN) algorithm.

2 RF Fingerprint Location Process

In this section, we introduce the process of RF fingerprint location technology, including the offline phase and the online phase.

RF fingerprint positioning is to establish the corresponding relationship between the fingerprint signal space and the geographic location space to achieve location determination [14]. This process is divided into two phases: the offline phase of fingerprint database construction and the online phase of location positioning, as shown in Fig. 1. In the first phase, we build a fingerprint database that stores the RF fingerprint signals and the corresponding location coordinates. In the next phase, the received signal characteristics of the text point to be located are matched with the fingerprints' in the database. After the matching fingerprint is obtained, the position coordinates can be determined.

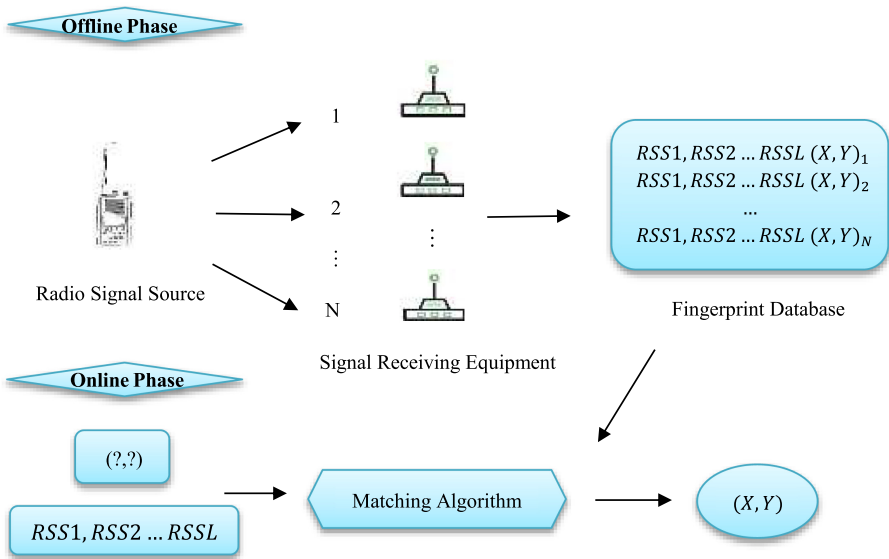


Fig. 1. Principle of RF fingerprinting positioning

The choice of matching algorithm for radio signal source RF fingerprint positioning should be reasonable because the choice of algorithm has a great influence on positioning accuracy. Commonly used location fingerprint location algorithms mainly include NN, KNN and WKNN method.

2.1 Nearest Neighbor

Suppose the characteristic value of the signal strength at reference points received by receiving devices in the offline phase is (1)

$$RSS_i = (RSS_{1i}, RSS_{2i}, RSS_{3i} \dots RSS_{Li}) \quad (1)$$

Among them, L is the number of collection nodes, RSS_{ji} represents its signal strength of the reference point i received by the j th collection node, and the signal strength of each reference point received by the collection node is stored in the fingerprint database. For the measured signal strength characteristic value of the test point is (2)

$$RSS = (RSS_1, RSS_2, RSS_3 \dots RSS_L) \quad (2)$$

For each reference point, the vector Euclidean distance between the test point and the reference point i with respect to the signal strength is obtained by formula (3) as

$$D_i = \sqrt{(RSS_1 - RSS_{1i})^2 + (RSS_2 - RSS_{2i})^2 + \dots + (RSS_L - RSS_{Li})^2} \quad (3)$$

Take the estimated point coordinates corresponding to the minimum signal strength distance as the output coordinates of the positioning result.

2.2 K-Nearest Neighbor

The KNN method is an improvement of the NN. Considering that the NN method may have a great influence on the result due to the selection of reference points and the inaccuracy of the characteristic data measured by the test points, we consider adding some points. The specific method is: instead of taking the smallest distance, the distance obtained in the previous step is to take the k smallest distances from small to large in turn. For the coordinates corresponding to these k distances, the average value is calculated as the output, and the two-dimensional coordinate system the following is expressed as:

$$(x, y) = \frac{1}{k} \sum_{k=1}^k (x_k, y_k) \quad (4)$$

2.3 Weighted K-Nearest Neighbor

The WKNN method is an improvement of the KNN [15]. The k minimum distances selected by the KNN method contribute the same to the final positioning result, and the average value is taken. But the smaller the distance is and the greater the contribution

to the coordinate of the result is. Therefore, weights are assigned to the k minimum distances. The positioning result in the two-dimensional coordinate system is expressed as:

$$(x, y) = \sum_{k=1}^k w_k(x_k, y_k) \quad (5)$$

Among them, the sum of w_k is 1.

$$w_k = \frac{1/D_k}{\sum_{k=1}^k 1/D_k} \quad (6)$$

3 Proposed Method

In the offline process, one problem is that the fingerprint signal collection is time-consuming and labor-intensive. The traditional fingerprint collection work is to first determine a large number of reference points in the area to be tested, then collect and store the RF signal reception intensity value for a period of time at each reference point in the database. When the collection area is relatively large, the establishment of database will consume huge manpower and material resources. Another problem is the preprocessing of fingerprint signals. Directly using the collected fingerprints for positioning will cause the dropping of the positioning accuracy. In the online phase, due to the large positioning space, the calculation complexity of positioning process is relatively high, the calculation amount is large, and the efficiency of determining the unit is low. In order to solve the above problems, this section proposes a novel RSS Fingerprint Based Signal Source Localization algorithm that introduces some new methods and processes in two phases.

3.1 Offline Training Phase

Fill in Fingerprint Database Data. In order to solve the problem of high database construction cost, we use multi-dimensional interpolation to reduce the number of sampling points and the cost of database construction. In the fingerprint collection stage, only a few numbers of fingerprints need to be interpolated to obtain a sufficient number of fingerprints.

Using MATLAB 2016a $F = scatteredInterpolant(x, y, v)$ to create a two-dimensional interpolation of $v = F(x, y)$, it can be understood as the RSS at a point (x, y) in the space. The basic idea is to first establish an interpolation class based on the existing training samples, and then evaluate the interpolation points. We can calculate the F value at a set of query points (for example (xq, yq)) to obtain the inserted value $vq = F(xq, yq)$. After multiple interpolations, the fingerprints received by each receiving device from the transmitting source can be obtained [16] (Fig 2).

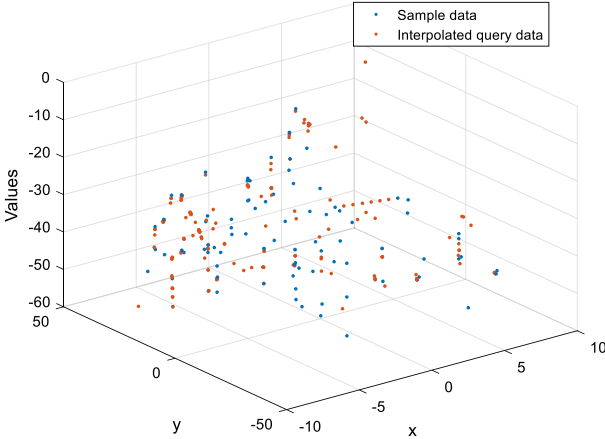


Fig. 2. Two-dimensional interpolation result

Extraction of Effective Fingerprint Information. In order to solve the problem of low positioning accuracy when directly using the collected RSS fingerprints for positioning, SVD is used to extract effective information from the complete data after the multi-dimensional interpolation.

The main application of matrix singular value decomposition in statistics is PCA, which is a data analysis method used to find “patterns” hidden in large amounts of data [17]. The collected fingerprint data is formed into matrix A . For the matrix $A_{m \times n}$, there are $U_{m \times m}$, $V_{n \times n}$, $S_{m \times n}$, satisfying $A = U * S * V'$. Except for the singular value of A , the elements on the diagonal of matrix S are all 0, and these values on the diagonal are arranged from large to small. The singular value is the weight, and the larger the weight is, the greater the information is. In many cases, the sum of the top 10% or even 1% of the values accounts for more than 99% of the sum of all. Therefore, a singular value of the first r can be used to approximate the matrix.

$$C = \sqrt{(S_{11})^2 + (S_{22})^2 + \dots + (S_{nn})^2} \tag{7}$$

$$C_r \sqrt{\sum (S_{rr})^2} \quad r = 1, 2, \dots, n \tag{8}$$

where $S_{11}, S_{22}, S_{33}, \dots, S_{nn}$ are singular values, when $C_r/C \geq 80\%$, r can be obtained.

$$A_{m \times n} \approx U_{m \times r} \sum_{r \times r} V_{r \times n}^T \tag{9}$$

where $\sum_{r \times r}$ is a diagonal matrix formed by $S_{11}, S_{22}, S_{33}, \dots, S_{rr}$. $A_{m \times n}$ is the result after extracting valuable information.

Currently, the common data preprocessing methods include arithmetic average filtering method and median average filtering method. For the same experimental environment, the same collection node, reference point, test point layout, we use these methods to process the collected fingerprint data and establish fingerprint databases, and we perform SVD processing on the data on the basis of the above methods, and then compare

it in the same online positioning algorithm (KNN). The test point data is positioned and analyzed in different fingerprint databases. The estimated position errors are obtained, as shown in Table 1.

From the data in the Table 1, it can be seen that when the fingerprint data are processed by two commonly used data preprocessing methods and then by SVD processing, the test points have a better positioning effect when online positioning is performed in the fingerprint database established than the one without SVD processing.

Table 1. The positioning errors (m) of the test points in the fingerprint databases established after the data are processed by the original data preprocessing methods and the SVD processing on the basis of these methods.

Serial numbers of test points	Arithmetic average filtering method	SVD processing afterwards	Median average filtering method	SVD processing afterwards
1	6.35	6.35	8.29	6.08
2	8.70	5.62	10.36	7.34
3	15.03	13.41	16.25	15.99
4	22.06	18.98	21.38	18.20
5	11.43	9.13	10.64	10.02
6	21.90	21.90	20.45	17.28
7	15.80	14.33	17.51	15.03
8	16.52	13.09	14.47	12.62
9	29.71	29.19	28.52	28.52
10	27.18	23.08	26.91	21.21

Establish Sub-fingerprint Database. After obtaining the complete fingerprint database, the database is divided into multiple intersecting sub-fingerprint databases according to the geographic location area, and the RSS fingerprint in each sub-fingerprint database is adjusted according to the KNN algorithm.

Taking one of the sub-fingerprint databases as an example, each reference point is used as a test point for online positioning in this database. The matching algorithm used is the KNN algorithm, and k is obtained by multiple cross-validation. At this time, we set the threshold which is the Euclidean distance between predicted positioning coordinates and actual coordinates. When the result of a certain test point is larger than the threshold, this point and the fingerprint information containing that will be deleted from database and transferred to the pending all the data in this database meet the requirements. After performing the above process on all data in sub-fingerprint databases, we filter out the repeated data from the pending area, and return to their previous databases to perform online positioning again. If the positioning result still does not meet the requirements, remove it from the fingerprint database. Finally, the

information contained in the fingerprint database are the position coordinates of each reference point, the RSS fingerprint data and the location of the sub-fingerprint database. The above is a new process improved based on the offline phase of traditional RF fingerprint positioning, and the flowchart is shown in Fig. 3.

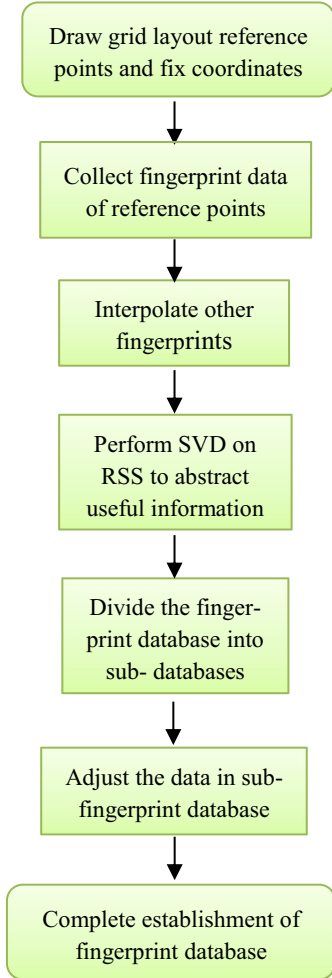


Fig. 3. Offline training phase

3.2 Online Positioning Stage

At this stage, we put the RSS fingerprint data collected from the test point into the fingerprint database for SVD processing to extract useful information, and use the KD

tree algorithm to predict the location of the sub-fingerprint database. At last, we apply the improved WKNN algorithm for the predictive positioning.

Determine the Location of the Test Point. In order to improve the efficiency of searching, we consider using a special structure to store training data to reduce the number of distance calculations. Using the KD tree can save the search for most data points, thereby reducing the amount of search calculations.

The KD tree is a tree data structure that stores instance points in a k -dimensional space for quick retrieval [18]. It is also a binary tree, which represents a division of the k -dimensional space. Constructing the KD tree is equivalent to continuously dividing the space with a hyperplane perpendicular to the coordinate axis to form a series of k -dimensional hyper rectangular regions. Each node of the KD tree corresponds to a k -dimensional super rectangular area.

Using this method to determine the location of the sub-fingerprint database where the test point is located includes three steps. The first step is to build a tree, the second is to search for the nearest neighbor, and the last step is to predict. KD tree building uses the variance of the values of n features from the n -dimensional features of m samples to be calculated, where m corresponds to the number of reference points in the fingerprint database, and n corresponds to the number of receiving devices. We use the j th dimension feature n_j with the largest variance as the root node. For this feature, we choose the sample corresponding to the median n_{jv} of the value of n_j as the division point. For all samples with the value of the j th feature less than n_{jv} , we divide it into the left subtree, and for the j th dimension feature for samples with a value greater than or equal to n_{jv} , we divide it into the right subtree. For the left subtree and the right subtree, we use the same method as before to find the feature with the largest variance to change the node, and recursively generate the KD tree (Fig. 4).

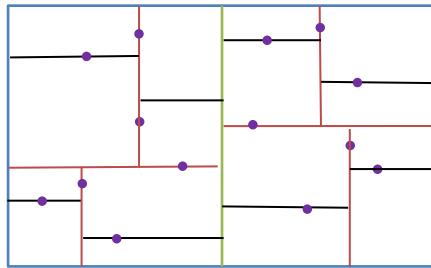


Fig. 4. Schematic diagram of KD tree space division for 2D data

For a test point, we first find the leaf node that contains the test point in the KD tree, take the test point as the center and the distance from the modified point to the sample instance of the leaf node as the radius to obtain a hypersphere. The nearest neighbor must be inside the hypersphere. Then return to the parent node of the leaf node, we should check whether the super rectangular body contained in another child node intersects the super sphere. If it intersects, we can go to this child node to find if there is a closer neighbor, and update the nearest neighbor. If they do not intersect, we directly return to

the parent node and continue to search for the nearest neighbor in another subtree. When backtracking to the root node, the algorithm ends, and the nearest neighbor node saved at this time is the final nearest neighbor.

On the basis of KD tree search for nearest neighbors, we select the first nearest neighbor sample and set it as selected. In the second round, we ignore the selected samples and reselect the nearest neighbors. In this way, we run k times to get the k nearest neighbors of the target. According to the majority voting method, it is predicted that the category with the largest number of categories in neighbors can determine the location of the sub-fingerprint database where the test point is located.

Improved Positioning Algorithm. It is necessary to improve the vector Euclidean distance formula for calculating the RSS of the test point and the reference point to reduce the positioning error [19]. $RSS_L - RSS_{Li}$ is the absolute error, and the deviation from the true value is reflected in the same unit dimension, but due to the different magnitudes, the contribution to D_i may be inconsistent and cause errors. Therefore, the absolute error is converted into a relative error. The relative error is expressed as a percentage, it is a dimensionless value. Generally speaking, the relative error can better reflect the credibility of the data, so it is changed to the relative error to improve the above possible drawbacks. Therefore, the formula for obtaining D_i under the same intensity becomes:

$$D_i = \sqrt{\left(\frac{RSS_1 - RSS_{1i}}{RSS_{1i}}\right)^2 + \left(\frac{RSS_2 - RSS_{2i}}{RSS_{2i}}\right)^2 + \dots + \left(\frac{RSS_L - RSS_{Li}}{RSS_{Li}}\right)^2} \quad (10)$$

The improved positioning method in the online phase uses the improved KNN ($K = 4$) and WKNN ($K = 5$) algorithms. The results are shown in the Figs. 5 and 6.

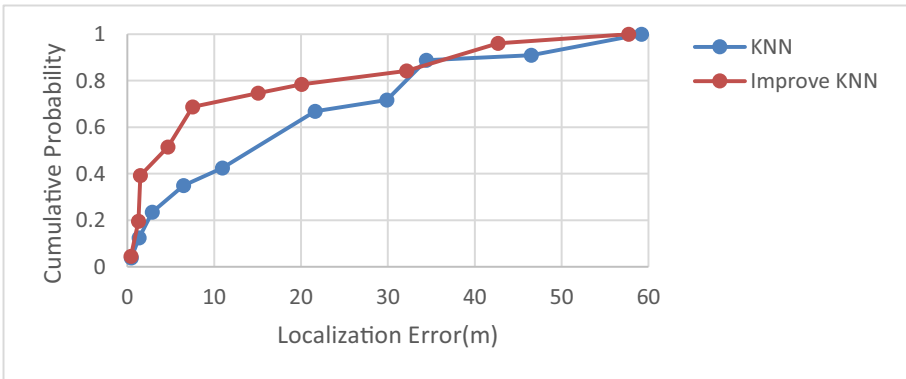


Fig. 5. Comparison of positioning results between KNN and improved KNN in the online phase of traditional method

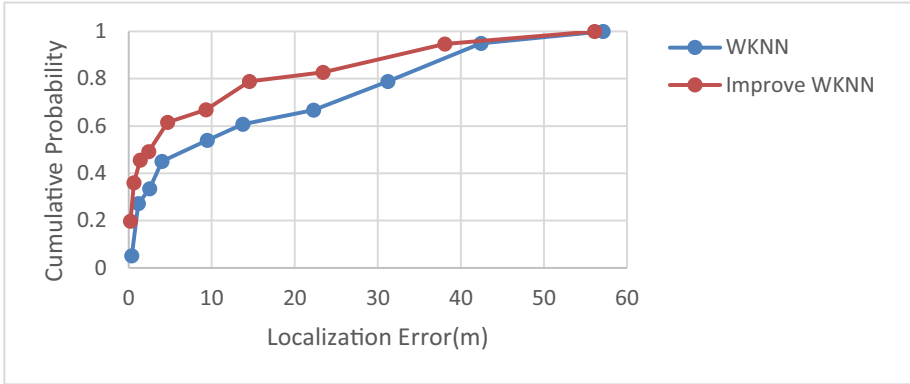


Fig. 6. Comparison of positioning results between WKNN and improved WKNN in the online phase of traditional method

4 Experiments Validation

4.1 Experiments Setup

We choose an open space with fewer obstacles outdoors, this area is about $175 \times 100 \text{ m}^2$. Then we divide a grid every 5 m in this area, and place the reference point at the center of the grid. As shown in the Fig. 7, a total of 154 black points are reference points, which are the positions of the signal source when the RSS fingerprint is collected. The red points are a total of 52 test points, and 5 receiving devices. A, B, C, D, E are arranged on the roof around the area. All receiving equipment are placed in a position that is visible and encloses the entire area to ensure that the signals emitted at each reference point can be received by the equipment. Radio signals are transmitted at each set reference point, and 5 receiving devices simultaneously receive the signals and record the RSS fingerprints.

The signal source is Motorola GP328Plus walkie-talkie, the radio frequency is 430.11 MHz. Each receiving device consists of a receiving antenna and a Tektronix RSA306b receiver. This receiver is connected to a laptop and display multiple RSS values received in the same time period. After that, the data received by each receiving device is processed, the maximum value and the minimum value are removed, and the remaining data is averaged. Finally, the 5 received field strength values are sorted into a set of fingerprint data. In order to save the time of querying the IP address and realize the rapid positioning process, a laptop is connected and transmits data through the LAN composed of 3 Huawei Q2S routers. The instrument connection of a group of receiving equipment is shown in the Fig. 10. The fingerprint information of other reference points that are not marked is derived from multi-dimensional interpolation.

After the completing of the fingerprints in the database, we use SVD to preprocess the data to extract effective information to reduce the impact of the external environment. At this time, let $C_r/C = 90\%$.

As shown in the Fig. 7, the area is divided into 7 intersecting areas and the points are included in the corresponding sub-fingerprint database. All points from the red line to the left are included in the sub-fingerprint database 1, from the purple line to the bottom all points are included in the database 2 and the points from the blue line to the right are included in the database 3. The points between the green lines are included in the database 4 and between the orange lines are included in the database 5. Besides, the points to the upper left of the orange and purple lines are included in database 6, and points to the upper right of the green and purple lines are included in the database 7.

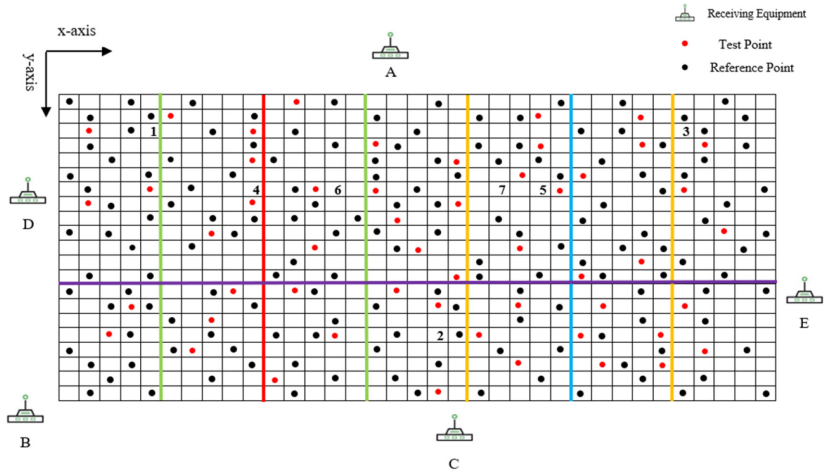


Fig. 7. The location of reference points and the connection of receiving instruments

According to the process of offline establishment of fingerprint database proposed in the previous section, the representative data of the fingerprint database are shown in the Table 2.

Table 2. Representative data in the fingerprint database.

Serial number	Coordinate(m)		RSS (dBm)					Sub-fingerprint database
	X	Y	A	B	C	D	E	
1	2.5	17.5	-38.43	-37.27	-37.21	-28.39	-38.87	1,6
2	2.5	22.5	-35.32	-41.26	-38.75	-31.67	-39.59	1
3	7.5	7.5	-37.53	-38.73	-37.32	-29.80	-39.25	1,6
4	7.5	12.5	-35.67	-40.58	-36.90	-27.89	-40.54	6
5	17.5	2.5	-27.98	-42.61	-40.67	-27.22	-45.27	1,6
6	17.5	7.5	-32.18	-43.02	-38.64	-28.36	-41.94	1
7	27.5	2.5	-32.55	-40.67	-40.56	-28.40	-41.34	1,4,6
8	37.5	27.5	-31.58	-40.03	-33.96	-24.27	-36.53	1,4,6
9	42.5	7.5	-33.14	-40.65	-45.00	-25.88	-37.78	4,6
10	62.5	22.5	-24.57	-45.10	-47.07	-32.49	-33.26	4,6

5 Experimental Results

Using the novel RSS Fingerprint Based Signal Source Localization algorithm proposed in this paper to predict positioning (the K in the KD tree and the improved WKNN algorithm are both obtained by cross-validation, $K = 6,5$) and compare to the previous methods. As shown in the Fig. 8, the new improved method has an error probability of

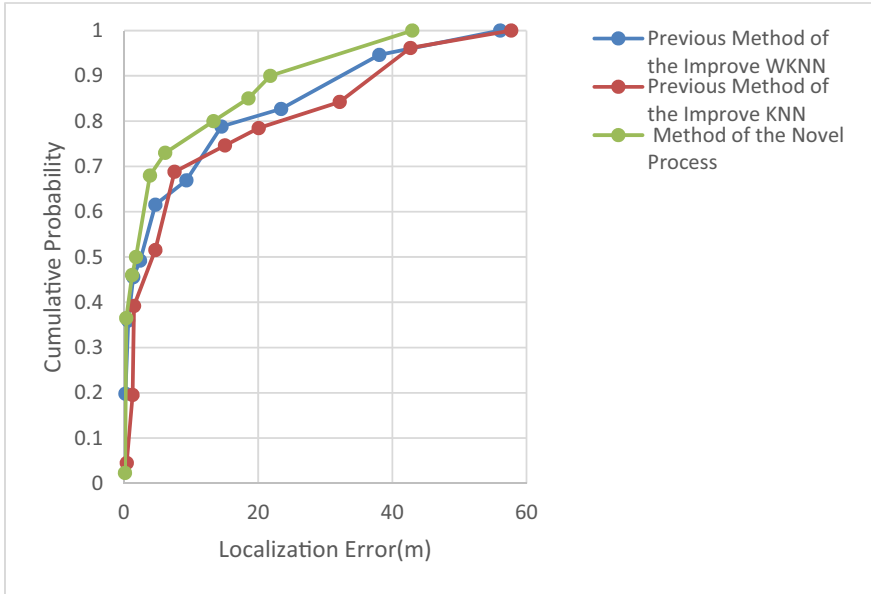


Fig. 8. Comparison of the positioning results of the new process method and the improved traditional methods

less than 13 m can reach 80%, less than 19 m can reach 85% and less than 22 m can reach 90%.

6 Conclusion

In short, the results of positioning using the novel RSS Fingerprint Based Signal Source Localization algorithm proposed in this paper are significantly better than the traditional positioning algorithm based on RF fingerprint technology. Using SVD to process the collected fingerprint data can reduce the impact of the external environment on the data. Besides, in the offline training phase, we use multi-dimensional interpolation to complete the fingerprints in the fingerprint database and KNN to adjust the fingerprints in the sub-fingerprint database. In the online positioning phase, we use the KD tree to determine the area of the test point and then use the improved WKNN algorithm to complete the positioning. Therefore, by using the combination with the improved positioning process that introduces other methods mentioned in this paper, the positioning accuracy can be greatly improved.

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