A Stochastic Traffic Model for Congestion Detection in Multi-lane Highways

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Abstract. Vehicular Ad Hoc Networks (VANETs) represent a significant leap forward in the deployment of intelligent transport systems. These networks enable vehicles to instantly exchange traffic information with the aim of smoothing traffic flows and intensifying drivers comfort. In this context, this study addresses the issue of traffic congestion description and detection in multi-lane highways. By making use of collected information, a Markov chain based mobility model is proposed to predict the future road traffic states. Based on the obtained stationary distribution probabilities, performance criteria in steady-state are inferred and computed for different road configurations. The numerical results validate the model demonstrated in the paper.

Keywords: VANETs · Mobility model · Vehicular traffic · Markov chain

1 Introduction

The advancement of wireless network infrastructure has led to the emergence of a new type of networks dedicated to supporting intelligent traffic management, called VANETs. These networks would allow real-time communication and information exchange between moving vehicles equipped with wireless communication devices. Therefore, VANETs can offer a solution to many transportation issues affecting the efficiency of transport operations such as traffic congestion, safety, and fuel consumption. Considering that congestion is such a potential threat to public safety, economy and air quality, there is a strong need for faster detection of future congestion and saturation points. This depends heavily on the availability of accurate and realistic traffic forecasts. The classical methods for traffic prediction such as Google Maps and Waze depend primarily on centralized approaches to calculate traffic jams using historical and real-time traffic data. The major drawbacks of these methods is that they perform poorly in areas with little or no activity and they require a large data storage capacity. With the benefits granted by VANETs, a shift to a fully distributed traffic estimation becomes feasible. Based on locally collected traffic information via V2V or V2I

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communications, vehicles can automatically calculate estimates of future traffic distributions and reacts accordingly.

As such, the main focus of this study is to develop a decentralized traffic estimation approach in a multi-lane highway environment. Principles of Markov chain theory are used to build a model able to describe the temporal evolution of the traffic state as a random process. Markov chain theory has been widely used in various fields of research [1–3], to study systems characterized by uncertainty. As events in the system occur in continuous time, the model is constructed as a continuous-time Markov chain (CTMC) with a finite number of states. The stationary distribution will be obtained by applying a numerical approach [3]. Following that, performance indicators such as average density, congestion rate and average sojourn time will be described and computed. The network performance is evaluated in terms of the derived performance metrics by varying different parameters including arrival rate, average speed, and the number of lanes.

The remainder of this paper is structured as follows. Section 2 provides a brief literature review. Section 3 comprises the proposed model for highway traffic forecasting along with the definition of steady-state probabilities. In Sect. 4, the performance measures are defined and the numerical results of the model are demonstrated for two different scenarios. Our concluding and suggestions for future researches are given in Sect. 5.

2 Related Work

In this section, the body of related work that is available on vehicular mobility modelling is reviewed.

Generally, there are two classes of vehicular mobility models [4]: trace based mobility models (TBMM) and synthetic mobility models. The trace based models derive mobility patterns of moving vehicles from real world traces [5–7]. These models provide more credible spatial and temporal measurements and are more suitable for large scale and complex scenarios. However, they highly depend on the availability of large datasets of recorded vehicle trajectories. Several projects have been conducted to collect mobility traces such as OpenStreetMap [8] and CRAWDAD [9]. Still, these small-sized GPS datasets require the involvement of many participants to be more useful in the future.

Synthetic models describe the mobility of vehicles based on mathematical approaches, and are classified into five categories:

Stochastic models are simplistic models that describe the movements of nodes as a random process, where speed and paths are usually randomly chosen. A number of studies have presented stochastic mobility models such as City section [4], Freeway and Manhattan [10]. Nagel and Schreckenberg [11] addressed the issue of vehicular mobility in single lane-freeways. This method was extended in [12] to take into account drivers' reactions to velocity and headway distance. These models are widely applied because of their limited data demands and their easy implementation since they model traffic dynamics with a minimal level of details. However, they require further improvements to mimic real-world driving behaviors such as lane change, queues forming and stop-and-go phenomenon.

Traffic stream models describe traffic flow as a hydrodynamic phenomenon and they involve the three major macroscopic characteristics of traffic stream: speed, density and flow. The most extensively used model within this category is Lighthill-Whitham-Richards (LWR) [13,14]. Based on kinematic waves, the authors consider that the traffic flow is a continuous function of density and predict the evolution of traffic flow along arterial roads and near shock waves. The LWR framework was used as a bedrock in [15] and [16] to study the dynamics of queues at intersections. Traffic stream models manage to capture the overall traffic behavior with low details. However, bringing a solution to these models is time consuming and cumbersome when incoming flow varies continuously in time.

Car following models take into account measures such as the headway distance and the speed of nearby vehicles to display the behavior of each individual vehicle. Authors in [17] introduced a velocity threshold to maintain the safety distance from the leading car in single-lane roads. The previous work was extended in [18] to tackle the issue of lane changing. Another model was introduced in [19] to emulate the lane-change behavior in multiple-lane roads by using Bayesian reasoning. Generally, these models are more precise and provide an accurate description of traffic which increases the computational complexity, especially for large scale simulations. Besides, the overall dynamics of the system cannot be captured.

Behavioural models were inspired from the fields of biological physics and artificial intelligence. They are built based on behavioral rules conducted by social influences, rational decisions or actions following a stimulus-reaction process. Legendre *et al.* [20] introduced the first behavioral model to describe human mobility. Later, the model was improved to fit in the case of vehicular traffic. The work proposed in [21] involves describing the behaviour of the driver such as the response to the abrupt braking of the preceding vehicle. An improved behavioural model was proposed in [22] to emulate various mobility characteristics that can't be inferred from recorded driving traces. The model allows to generate simulation measures consistent with real traces. Although the aforementioned models appear to successfully provide a mobility description close to the actual real world, they cannot be applied for large-scale scenarios as they require complex calculations.

Queue models are constructed to study queue lengths and delays of vehicles waiting in queues by applying basic probabilistic distributions. Gawron [23] was the first to introduce a queuing model dedicated to vehicular traffic. In [24], a queuing model was developed for signal control optimization at isolated intersections. Cremer and Landenfeld [25] developed a model for signalized intersections while capturing the effects of spill-back phenomena. Authors in [26] presented a model that takes into account the streets topology to describe mobility at intersections and along the streets. The main reason why queue models are used is their ability to model complex systems. Moreover, they can be easily deployed as they are based on restrictive assumptions. Their major drawback is that the waiting queue in each street are described independently. Thus, they overlook the importance of coordinating the vehicles crossing the intersection and lane changing.

Our approach differs from other works as we aim to address the congestion issue in multiple-lane free-flow highways in the context of VANETs by implementing a stochastic model that describes and predicts traffic density and travel time based on traffic information collected by vehicles at the start point of the highway. The lane change behavior is also described based on the density of the lanes. The proposed model is to be implemented in vehicles and has shown its effectiveness to cover all aspects of mobility in highway environments, as it is demonstrated in Sect. 4 through analytical results of different scenarios.

3 Proposed Model

In this section, the continuous time Markov chain model for future traffic state prediction is first presented, then the steady state probabilities are derived.

In order to formulate the proposed CTMC, the notations which will be used throughout the rest of the paper are introduced in Table 1.

Table 1. Notations

$\mathcal{N}$	Set of road lanes
N	Total number of lanes
Len	Length of a highway section
Sp	Average speed on a highway section
i	Index value for lane i
C	Lane capacity
α_i	Weight of lane i
D_i	Density of lane i
R	Lane change probability matrix
R_{ij}	Lane change probability from lane i to j
λ	Arrival rate of vehicles
λ_i	Arrival rate of vehicles at lane i
μ	Service/Departure rate of vehicles
μ_i	Service/Departure rate of vehicles at lane i
t	Temporal variable
$X(t)$	Multivariate random vector representing the traffic condition of the system at time t
$X_i(t)$	Random variable representing the traffic condition of lane i at time t

(contniued)

Table 1. *(contniued)*

Ω	State space
x,y,z	Values of X(t)
P	Transition probability matrix
p_{xy}	Transition probability from state x to state y
π	Steady-state probability distribution
L_i	Average number of vehicles at lane i
W_i	Average time spent in lane i
VC_i	Average volume-to-traffic ratio of lane i

The developed model considers a highway of N lanes, where it's assumed that all lanes are of equal capacity C. The set of lanes will be denoted $\mathcal{N}$. Each lane i is assigned a weight α_i relative to its position on the road, such as $\alpha_i > \alpha_j$ if lane j is at the left side of lane i, and $\sum_{i \in \mathcal{N}} \alpha_i = 1$. The weight of a lane i is defined by the following formula:

$$\alpha_i = \frac{N - i + 1}{N!} \tag{1}$$

Figure 1 illustrates an example of a single direction highway, while Fig. 2 represents the highway diagram as a queue system.

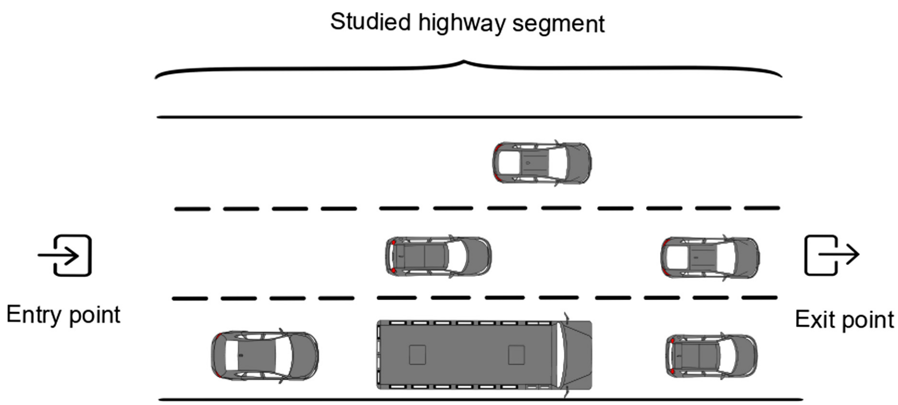

Fig. 1. An example of a highway road segment.

Similarly to works in [1,2,26], we assume arrivals of vehicles at the start point occur at a rate λ according to a Poisson process and service times have an exponential distribution with a rate parameter $\mu = \frac{Sp}{Len}$, where Len is the length of the highway segment and Sp the average speed of vehicles. Arrival rates at

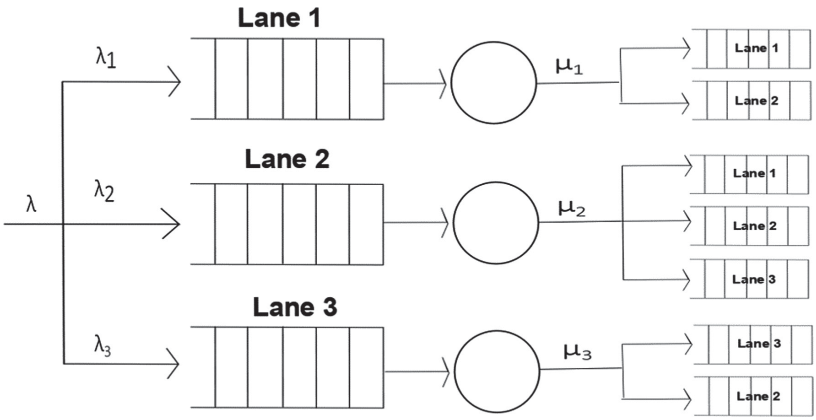

Fig. 2. A highway road segment as a queue system.

each lane are defined by their position on the road, such that the right lane has a higher arrival rate than the left lane. This was expressed as the following formula:

$$\lambda_i = \lambda \times \alpha_i \tag{2}$$

Similarly to the arrival rates, the service rates are defined in a way that the right lane has a lower service rate than the one on the left of the road. This condition can be expressed as:

$$\mu_i = \mu \times \frac{1}{\alpha_i} \tag{3}$$

The traffic state of the studied segment is represented by a vector of multiple random variables corresponding to the number of vehicles on each lane as follows:

$$X(t) = \{X_1(t), X_2(t), ..., X_N(t)\} \tag{4}$$

As each lane can only handle a limited number of vehicles, the following constraint must be satisfied:

$$\forall i \in \mathcal{N}, 0 \le X_i(t) \le C \tag{5}$$

The collection of possible values of $X(t)$ forms the state space Ω of the Markov chain.

A change in the system's state can be caused by one of the following events:

- A vehicle arriving at the start point of the highway;
- A vehicle arriving at the end point of the highway;
- A vehicle changing lanes.

P is used to denote the transition matrix of the Markov chain, where its elements p_{xy} is the transition probability of state x to state y. In order to define

P, R the lane change probability matrix is first expressed. Given D_i the density of lane i which represents the actual number of vehicles travelling on lane i, R_{ij} the probability a vehicle changes from lane i to an adjacent lane j is defined as below:

$$R_{ij} = \begin{cases} 1 - \dfrac{D_j}{\sum\limits_{k \in \mathcal{N}} D_k . \mathbb{1}_{(i-k=1 || i-k=-1)}} & \text{if } i \text{ and } j \text{ are adjacent lanes} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Given the arrival, service and lane change rates, the transition probability from each state x to state y ($(x, y) \in \Omega^2$ and $x \neq y$) can be established:

$$p_{xy} = \begin{cases} \lambda_i & \text{if a vehicle arrives at the start point of lane } i \\ \mu_i & \text{if a vehicle arrives at the end point of lane } i \\ R_{ij} & \text{if a vehicle changes from lane } i \text{ to lane } j \\ -\sum\limits_{z \in \Omega_{\setminus \{x\}}} p_{xz} & \text{if x = y} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

The stationary distribution vector π is defined by the steady-state statement such as $\pi P = \pi$. By applying the rate-conservation principle, π can be obtained. Given the transition probability matrix P defined previously and the equilibrium vector π, the balance equation is expressed as follows:

$$\pi_x . \sum_{y \in \Omega_{\setminus \{x\}}} (\sum_{i \in \mathcal{N}} \lambda_i . \mathbb{1}_{(p_{xy} = \lambda_i)} + \sum_{i \in \mathcal{N}} \mu_i . \mathbb{1}_{(p_{xy} = \mu_i)} + \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}_{\setminus \{i\}}} R_{ij} . \mathbb{1}_{(p_{xy} = R_{ij})} =$$
$$\sum_{y \in \Omega_{\setminus \{x\}}} \pi_y (\sum_{i \in \mathcal{N}} \lambda_i . \mathbb{1}_{(p_{yx} = \lambda_i)} + \sum_{i \in \mathcal{N}} \mu_i . \mathbb{1}_{(p_{yx} = \mu_i)} + \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}_{\setminus \{i\}}} R_{ij} . \mathbb{1}_{(p_{yx} = R_{ij})})$$

To compute the stationary vector of the transition rate matrix P, the Grassmann-Taksar-Heyman (GTH) algorithm [27] is applied.

4 Performance Indicators and Numerical Results

To demonstrate the validity of the proposed approach, numerical solutions are presented in this section.

4.1 Performance Indicators

From the obtained vector π, L_i the long-run average number of vehicles on lane i is first computed using the following formula:

$$L_i = \sum_{x \in \Omega} \pi_x . x_i \tag{8}$$

The average volume-to-capacity ratio VC_i of lane i can be inferred from L_i as below:

$$VC_i = L_i/C \tag{9}$$

Based on Little's law formula, W_i the average sojourn time a vehicle spends on lane i is calculated as follow:

$$W_i = \frac{1}{\lambda_i}.\sum_{x \in \Omega} \pi_x.x_i \tag{10}$$

4.2 Numerical Results

The values used to find the numerical solution are based on samples extracted from highD dataset [28] that offers measurement data collected at German highways. Table 2 summarizes the values of the settings used.

Table 2. Values used for numerical resolution

Parameter	Scenario 1	Scenario 2
Highway length	400 m	200 m
Number of lanes	3	Varies from 1 to 4
Per-lane capacity	18	8
Average speed	33 m/s	25-30-35 m/s
Time unit	1 min	1 min
Arrival rate	Varies from 0 to 10	5 vehicles/time unit

First, different values of arrival rates are tested to analyse the impact of this parameter on the performance of a highway section of 400 m long. The studied highway is composed of three lanes and only one travel direction is considered similarly to the configuration previously presented in Fig. 1.

Figure 3 illustrates the average number of vehicles on each highway section lane. Generally, left lanes are usually used for passing or when traffic on right lanes is congested. It can be observed from the results that density decreases from the right lane to left one, which is a clear proof that the information obtained reflects the realistic behavior of traffic.

The chart in Fig. 4 represents the congestion rates corresponding to each lane. The curves show the same results as the previous one. It can be seen that the congestion rate on the first lane increases rashly compared to the other two lanes. The congestion rate of the upper lane grows more slowly. The average sojourn time per lane is depicted in Fig. 5 and shows that the average driven time grows as the traffic density rises. It can also be noticed that the travel speed considerably slows down on the most congested lane.

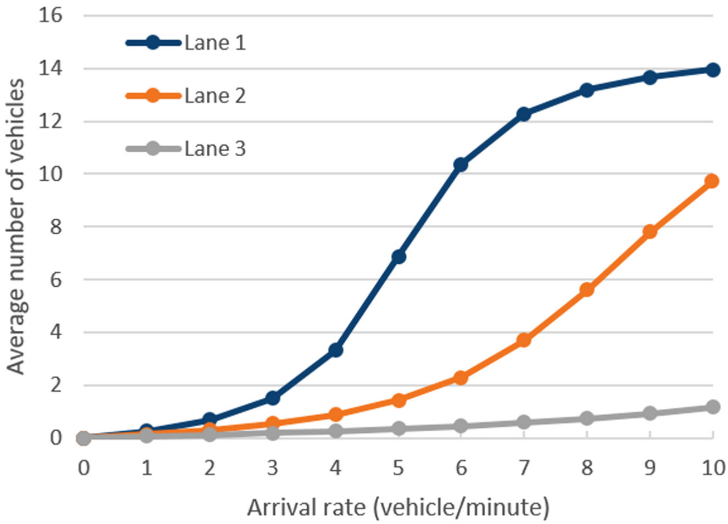

Fig. 3. Average number of vehicles vs. Arrival rate.

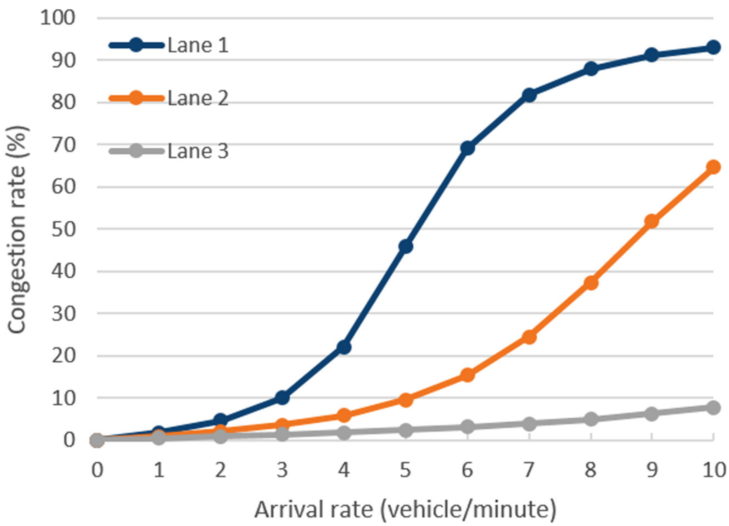

Fig. 4. Congestion rate vs. Arrival rate.

In a different test, we consider a highway section 200 m that handles at most 8 vehicles at the same time. We derive the performance measures for different numbers of lanes and average speeds to capture how these settings affect the performance. The estimated average number of vehicles versus the number of lanes is shown in Fig. 6, for three different values of the average speed. It's noted that having more lanes increases the capacity of the road allowing to handle

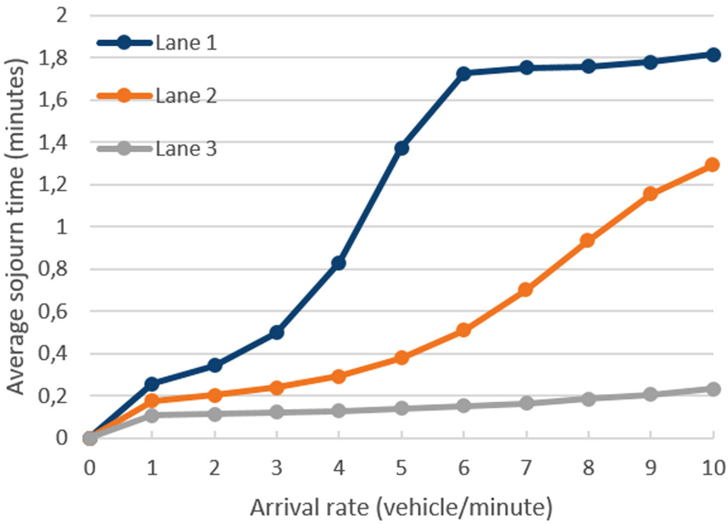

Fig. 5. Average sojourn time vs. Arrival rate.

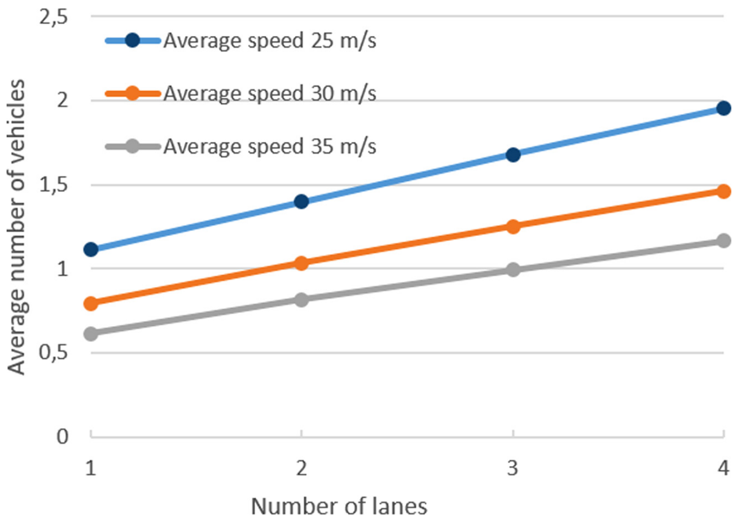

Fig. 6. Average number of vehicles vs. Number of lanes.

more vehicles. In addition, a significant reduction of density can be noticed when average speed increases specially for roads with multiple lanes.

As observed in Fig. 7, the overall congestion rate of the freeway is substantially higher for a single-lane highway than a multi-lane. From the results, it can also be deduced that speed is a major contributor to the congestion. A slight increase of the speed can allow to drop the congestion rate by 4% for single-lane highways. The average sojourn time depicted in Fig. 8 demonstrates that

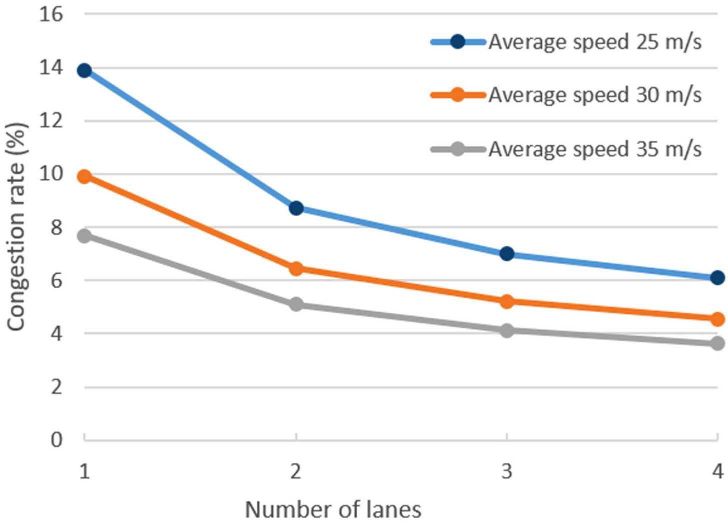

Fig. 7. Congestion rate vs. Number of lanes.

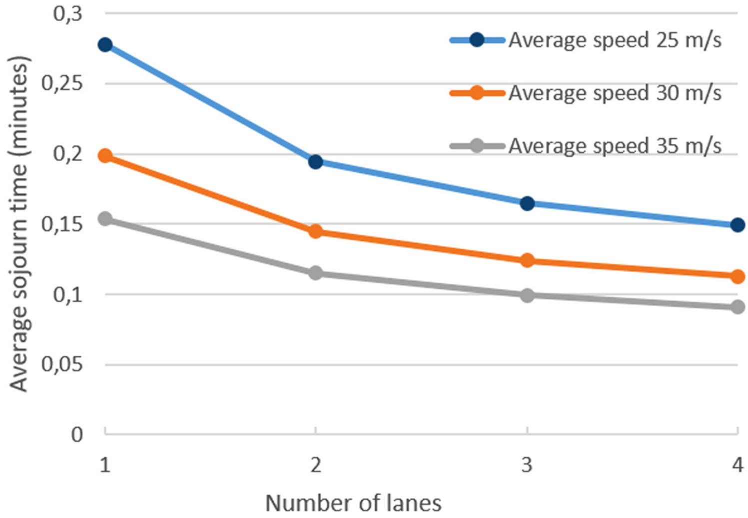

Fig. 8. Average sojourn time vs. Number of lanes.

multi-lane roads allow shorter journey times compared to single-lane roads. It can also be observed that the model accurately reproduces the travel delays brought on by low velocities.

From the presented results we can assume that the first lane is more seriously affected by congestion as more drivers tend to stay in the right most lane except in heavy traffic. The obtained estimates can be used to design lane change assistance to ease congested lanes and increase utilization of fast lanes. Moreover, intelligent speed assistance systems can be designed to adapt the velocity of vehicles depending on the estimated circumstances.

5 Conclusions

Congestion has become a serious issue that the majority of countries are facing. It does not solely affect the efficiency of transportation systems, but also the environmental and economic welfare. Thus, methods for congestion detection are vital to the improvement of transportation infrastructure. The availability of accurate estimates of future congestion points will make it easier to manage traffic flows. This study focuses on developing a mobility model for multilane highways using Markov chains. The proposed model allows to forecast the distribution of traffic over highway lanes and then derives synthetic measures that represent important performance indicators for congestion assessment. The evaluation results highlight the potential contribution of the proposed model to mitigating traffic congestion. This study was limited to highway traffic. For future work, it is planned to address the applicability of the model on urban environments.

References

1. Kafi, M.A., Ben-Othman, J., Mokdad, L., Badache, N.: Performance analysis and evaluation of REFIACC using queuing networks. Simul. Model. Pract. Theory **71**, 15–26 (2017)
2. Mokdad, L., Ben-Othman, J., Nguyen, A.T.: DJAVAN: detecting jamming attacks in vehicle ad hoc networks. Perform. Eval. **87**, 47–59 (2015)
3. Mokdad, L., Ben-Othman, J., Yahya, B., Niagne, S.: Performance evaluation tools for QoS MAC protocol for wireless sensor networks. Ad Hoc Netw. **12**, 86–99 (2014)
4. Davies, V.A.: Evaluating mobility models within an ad hoc network. Master's thesis, advisor: Tracy Camp. Department of Mathematical and Computer Sciences, Colorado School of Mines (2000)
5. Vetriselvi, V., Parthasarathi, R.: Trace based mobility model for ad hoc networks. In: Third IEEE International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob 2007), p. 81. IEEE, October 2007
6. Förster, A., Bin Muslim, A., Udugama, A.: TRAILS-A trace-based probabilistic mobility model. In: Proceedings of the 21st ACM International Conference on Modeling, Analysis and Simulation of Wireless and Mobile Systems, pp. 295–302, October 2018
7. Beiró, M.G., Panisson, A., Tizzoni, M., Cattuto, C.: Predicting human mobility through the assimilation of social media traces into mobility models. EPJ Data Sci. **5**(1), 1–15 (2016). https://doi.org/10.1140/epjds/s13688-016-0092-2
8. Haklay, M., Weber, P.: OpenStreetMap: "user-generated street maps.". IEEE Pervasive Comput. **7**(4), 12–18 (2008)
9. Kotz, D., Henderson, T.: CRAWDAD: a community resource for archiving wireless data at Dartmouth. IEEE Pervasive Comput. **4**(4), 12–14 (2005)
10. Bai, F., Sadagopan, N., Helmy, A.: The IMPORTANT framework for analyzing the Impact of Mobility on Performance Of RouTing protocols for Adhoc NeTworks. Ad Hoc Netw. **1**(4), 383–403 (2003)

11. Nagel, K., Schreckenberg, M.: A cellular automaton model for freeway traffic. J. Phys. I **2**(12), 2221–2229 (1992)
12. Zhang, G., Xu, W.: Cellular automaton traffic model considering driver's reaction to velocity and headway distance with variable possibility of randomization. In: IOP Conference Series: Materials Science and Engineering, vol. 392, no. 6, p. 062024. IOP Publishing, July 2018
13. Lighthill, M.J., Whitham, G.B.: On kinematic waves II. A theory of traffic flow on long crowded roads. Proc. Roy. Soc. Lond. Ser. A Math. Phys. Sci. **229**(1178), 317–345 (1955)
14. Richards, P.I.: Shock waves on the highway. Oper. Res. **4**(1), 42–51 (1956)
15. Liu, H.X., Wu, X., Ma, W., Hu, H.: Real-time queue length estimation for congested signalized intersections. Transp. Res. Part C Emerg. Technol. **17**(4), 412–427 (2009)
16. Yang, H., Rakha, H., Ala, M.V.: Eco-cooperative adaptive cruise control at signalized intersections considering queue effects. IEEE Trans. Intell. Transp. Syst. **18**(6), 1575–1585 (2016)
17. Briesemeister, L.: Group membership and communication in highly mobile ad hoc networks (2001)
18. Krauß, S.: Microscopic modeling of traffic flow: investigation of collision free vehicle dynamics. Doctoral dissertation (1998)
19. Pop, M.D., Proştean, O., Proştean, G.: Multiple lane road car-following model using Bayesian reasoning for lane change behavior estimation: a smart approach for smart mobility. In: Proceedings of the 3rd International Conference on Future Networks and Distributed Systems, pp. 1–8, July 2019
20. Legendre, F., Borrel, V., de Amorim, M.D., Fdida, S.: Modeling mobility with behavioral rules: the case of incident and emergency situations. In: Cho, K., Jacquet, P. (eds.) AINTEC 2006. LNCS, vol. 4311, pp. 186–205. Springer, Heidelberg (2006). https://doi.org/10.1007/11930181_14
21. Gipps, P.G.: Behavioral car-following model for computer simulation. Transport. Res. **15**(2), 105–111 (1981)
22. Kharrazi, S., Almén, M., Frisk, E., Nielsen, L.: Extending behavioral models to generate mission-based driving cycles for data-driven vehicle development. IEEE Trans. Veh. Technol. **68**(2), 1222–1230 (2018)
23. Gawron, C.: An iterative algorithm to determine the dynamic user equilibrium in a traffic simulation model. Int. J. Mod. Phys. C **9**(03), 393–407 (1998)
24. Mirchandani, P.B., Zou, N.: Queuing models for analysis of traffic adaptive signal control. IEEE Trans. Intell. Transp. Syst. **8**(1), 50–59 (2007)
25. Cremer, M., Landenfeld, M.: A mesoscopic model for saturated urban road networks. In: Traffic and Granular Flow, vol. 97, pp. 169–180 (1998)
26. Mohimani, G.H., Ashtiani, F., Javanmard, A., Hamdi, M.: Mobility modeling, spatial traffic distribution, and probability of connectivity for sparse and dense vehicular ad hoc networks. IEEE Trans. Veh. Technol. **58**(4), 1998–2007 (2008)
27. Grassmann, W.K., Taksar, M.I., Heyman, D.P.: Regenerative analysis and steady state distributions for Markov chains. Oper. Res. **33**(5), 1107–1116 (1985)
28. Krajewski, R., Bock, J., Kloeker, L., Eckstein, L.: The highD dataset: a drone dataset of naturalistic vehicle trajectories on German highways for validation of highly automated driving systems. In: 2018 21st International Conference on Intelligent Transportation Systems (ITSC), Maui, HI, pp. 2118–2125 (2018). https://doi.org/10.1109/ITSC.2018.8569552