A Novel Method of Combined Window Function Construction Filter Applied to F-OFDM System

Mingxin Liu[1(✉)], Wei Xue[1,3], Yidong Xu[1,3], Wenjian Chen[2], and Sergey B. Makarov[4]

[1] College of Information and Communication Engineering, Harbin Engineering University, Harbin 150001, China
{liumx,xuewei,xuyidong}@hrbeu.edu.cn

[2] College of Underwater Acoustic Engineering, Harbin Engineering University, Harbin 150001, China
chenwenjian@hrbeu.edu.cn

[3] Yantai Research Institute of Harbin Engineering University, Yantai 264000, China

[4] Institute of Physics, Nanotechnology and Telecommunications, Higher School of Applied Physics and Space Technologies, Peter the Great St. Petersburg Polytechnic University (SPbPU), Polytechnicheskaya, 29, 195251 St. Petersburg, Russia
makarov@cee.spbstu.ru

Abstract. The Orthogonal Frequency Division Multiplexing (OFDM) systems use rectangular pulse as baseband signal, and a main problem of the rectangular pulse is that its main lobe energy concentration is low and the out-of-band emissions (OOBE) is too high. The filtered orthogonal frequency division multiplexing (F-OFDM) system can make the F-OFDM system have a less energy radiation because of filtering in the time domain. Applying filtering operations can prevent signals from different communication scenarios from interfering with each other at the same time. In general, the filter used in the F-OFDM system is by applying a finite pulse to multiply the sinc function. In this study, a functional model is used, which can obtain a pulse that exhibits fast attenuation features in the frequency domain. The energy of this window function is mainly concentrated in the main pulse of the energy spectral density. This window function is combined with Hanning window and Blackman window to form a new window function form, truncating the sinc function, and constructing a filter. This filter has less energy leakage in F-OFDM system than filters constructed by other combined finite pulses to multiply the sinc function.

This research work was supported by International Science & Technology Cooperation Program of China (2014DFR10240) ,the Fundamental Research Funds for the Central Universities (3072021CF0802), the Key Laboratory of Advanced Marine Communication and Information Technology, Ministry of Industry and Information Technology (AMCIT2101-02), and Sino-Russian Cooperation Fund of Harbin Engineering University (2021HEUCRF006).

S. Liu and X. Ma (Eds.): ADHIP 2021, LNICST 416, pp. 280–291, 2022.
https://doi.org/10.1007/978-3-030-94551-0_23

Keywords: F-OFDM · Functional model · Combination window · Energy leakage

1 Introduction

For broadband channels, multi-carrier transmission schemes are used. Among them, OFDM system is the first choice [1,2]. OFDM system has many advantages, such as anti-multipath channel interference [3,4]. However, the problem of excessive energy leakage has always been the focus of research. Therefore, many emerging multi-carrier modulation systems have been derived. Among them, generalized frequency division multiplexing (GFDM) and filter bank multi-carrier (FBMC) schemes have been widely studied, and both of them also show excellent performance in terms of energy leakage [5–7]. However, the large amount of calculation of them and the inability to directly apply some mature algorithms of the OFDM system are the main problems. The F-OFDM system only adds a filtering operation, and the amount of calculation does not increase significantly. Most of the algorithms of the OFDM system can also be directly applied [8,9]. On the other hand, energy leakage can also be controlled in a small range through filtering [8]. First proposed the F-OFDM system, which used the Hanning pulse to multiply the sinc function to construct a filter, which effectively reduced the energy leakage. In [10], Blackman-Harris pulse is used in F-OFDM system. Compared with F-OFDM system based on Hamming pulse and Hanning pulse, this scheme effectively controls energy leakage. In [11], the traditional pulses were used to construct fiters, and the sinc window function is truncated to construct the filter. Compared with applying the window function alone, the combined window makes the energy leakage of the F-OFDM system lower. The window function method used in the above [8–11] is based on the commonly used window function truncated sinc function to construct the filter. In essence, the most important point of this method is the choice of window function, that is, different pulses will result in different energy leakage. In this study, a functional model is used, which can obtain a window function that exhibits fast attenuation characteristics in the frequency domain. The energy of this window function is mainly concentrated in the main part of the energy spectral density. Combine this pulse with Hanning window, Blackman window, etc. to form a new pulse form, multiplying the sinc function, and construct a filter. The results show that the new filters in the F-OFDM system compared with other combined window functions, the filter has less energy leakage.

The remaining sections of this study are arranged as follows:

The Sect. 2 models the F-OFDM system and the functional model with less energy radiation. Section 3 restricts the functional model and carries out numerical simulation. The window function obtained in Sect. 4 is combined with Hanning window and Blackman window to form new window function forms, truncating the sinc function, constructing the filters and applying them to F-OFDM system, and discussing the performance. Conclusions in Sect. 5.

2 System Model

2.1 F-OFDM System Model

The F-OFDM filters the entire full-band signal, which makes the implementation of the F-OFDM system not as complicated as FBMC. Here, only one embranchment part of the F-OFDM system is taken for a brief analysis. Figure 1 shows the structure of a embranchment of the F-OFDM system.

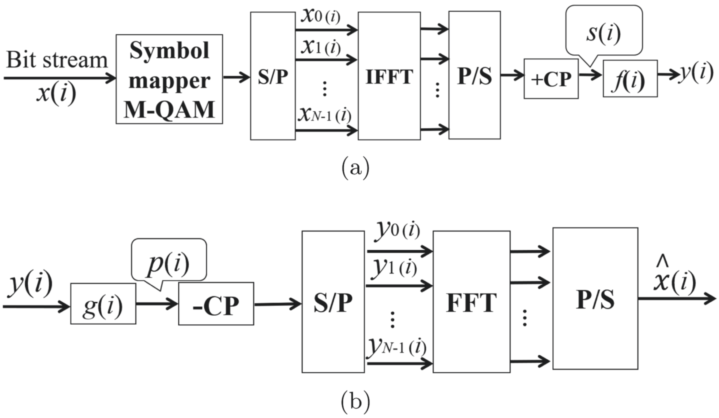

Fig. 1. A embranchment part of the F-OFDM system. (a) Transmitting end embranchment of the F-OFDM system. (b) Receiving end embranchment of the F-OFDM system.

In Fig. 1, the length of an F-OFDM signal is N, and the length of CP is N_{CP}. It can be seen that the length of the whole F-OFDM signal is $N_{F-OFDM} = N + N_{CP}$. Further, the baseband signal of F-OFDM is expressed as:

$$s(i) = \frac{1}{N} \sum_{k=0}^{N-1} x_k(i) \exp \frac{j2\pi k(i - N_{CP})}{N}, \tag{1}$$

Filtering $s(i)$, in fact, is $s(i)$ convolved with filter $f(i)$:

$$y(i) = \frac{1}{N} \sum_{l=-\infty}^{l=\infty} f(i-l) \sum_{k=0}^{N-1} x_k(i) \exp\{\frac{j2\pi k(i - N_{CP})}{N}\}. \tag{2}$$

The filter $f(i)$ is used on the transmission end, and the matched filter $g(i)$ can be used for equalization on the receiving end.

The traditional filter $f(i)$ is obtained by Hanning pulse multiplying the sinc function.

The expression of Hanning pulse is:

$$w_H(i) = 0.5\{1 - [\frac{\cos(2\pi i)}{N-1}]\}, -\frac{(N-1)}{2} \le i \le \frac{(N-1)}{2} \tag{3}$$

The form of the sinc function:

$$\sin c(i) = \frac{\sin(\pi B i T_s)}{\pi B i T_s}, -\frac{(N-1)}{2} \leq i \leq \frac{(N-1)}{2}, \tag{4}$$

where 1/B is the width of the main part of the sinc function, T_s is the sampling period, the length of the signal is N.

In general, the important design problem for F-OFDM system is how to design filter $f(i)$. By multiplying the sinc function with the Hanning pulse can obtain the filter, and the specific form is as follows:

$$f_H = w_H(i) \sin c(i) \tag{5}$$

At the receiving end, $g(i) = f(-i)$; when $f(i)$ is set as a real-valued even function, and $f(i) = f(-i)$, there, $g(i) = f(i)$. The receiving end signal $p(i)$ can be written:

$$p(i) = y(i) * g(i), \tag{6}$$

where the $*$ is the convolution symbol. After deleting the CP, serial-to-parallel conversion, fast Fourier transform (FFT) conversion, and parallel-to-serial conversion, the signal $\hat{x}(i)$ can be written as:

$$\hat{x}(i) = \frac{1}{N} \sum_{l=-\infty}^{l=\infty} p(i-l) \sum_{k=0}^{N-1} y_k(i) \exp\{\frac{j2\pi k(i - N_{CP})}{N}\} \tag{7}$$

2.2 Functional Model with Less Energy Leakage

The functional model with less energy leakage is established as [12]:

$$J = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\omega) |S(\omega)|^2 d\omega, \tag{8}$$

In (8), $g(\omega)$ is a function which restricts specific waveform and characteristics of the target pulse. The choice of $g(\omega)$ will determine the properties of the pulse $w(t)$. $S(\omega)$ is the Fourier transform of the pulse $w(t)$, and $|S(\omega)|^2$ is the energy spectral density of $w(t)$. First assume that the pulse $w(t)$ is an even function and the length is T, that is, $t \in [-T/2, T/2]$.

Specifically, $g(\omega)$ is expressed as $g(\omega) = \omega^{2n}$, n=1,2,. . . . . ., n is the variate. And $g(\omega)$ is symmetric about the ordinate and has a fast rising trend. Therefore, in order for J to satisfy the convergence property, $|S(\omega)|^2$ must have a rising speed faster than $g(\omega)$ The rate of decline. Therefore, it can be explained that $|S(\omega)|^2$ has the ability to obey the minimum out-of-band radiation criterion.

According to literature [12], formula (8) can be rewritten as:

$$J = (-1)^n \int_{-T/2}^{T/2} w(t) w^{(2n)}(t) dt, \tag{9}$$

where $w^{(2n)}(t)$ is the $2n$-order derivative of $w(t)$.

Therefore, to solve the pulse $w(t)$ is transformed into when its energy spectral density $|S(\omega)|^2$ obeys the minimum out-of-band energy radiation criterion, J obtains the minimum value, and the problem of the form $w(t)$ is solved.

3 Constraints and Numerical Simulation

3.1 Restrictions

In order to obtain a pulse with lower energy leakage, three constraints are added.

Restriction 1. The boundary constraints of the pulse:

$$w^{(n-1)}(\pm T/2) = \cdots = w'(\pm T/2) = w(\pm T/2) = 0, \tag{10}$$

Here, the value of the length T of the window function is a normalized value, that is, $T = 1$. Restriction 1 can make the less out-of-band radiation in the frequency domain.

Restriction 2. Energy restriction conditions of pulse:

For $w(t)$, according to the setting of its time domain length, its energy needs to be a certain value, that is, its energy cannot be infinite. The energy limit of a single symbol signal is expressed as:

$$E = \int_{-T/2}^{T/2} w^2(t)dt \tag{11}$$

here, the energy value of a pulse is taken as the normalized value.

Restriction 3. Energy spectrum constraints:

In order to further improve the efficiency of spectrum utilization, and then we add constraints in the frequency domain. The expressions are as:

$$|S(f_1)|^2 = X_1; |S(f_2)|^2 = X_2 \tag{12}$$

where f_1 is a point of the main lobe function, relative to the f_1, and the value of spectral density is X_1. In addition, f_2 is a point of the first side lobe, relative to the f_2, and the value of spectral density is X_2. The Lagrangian multiplier method can be used to control the values of these two points.

According to the above constraints, the Lagrangian function can be expressed as:

$$G = J + \alpha[\int_{-T/2}^{T/2} a^2(t) - E] + \beta[|S(f_1)|^2 - X_1] + \gamma[|S(f_2)|^2 - X_2] \tag{13}$$

where α, β, and γ are Lagrangian coefficients.

We can express $w(t)$ in the form of Fourier series.

$$w(t) = \frac{a_0}{2} + \sum_{k=1}^{m} a_k \cos(\frac{2\pi kt}{T}), t \in [-\frac{T}{2}, \frac{T}{2}], \tag{14}$$

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} w(t)dt;\ a_k = \frac{2}{T}\int_{-T/2}^{T/2} w(t)\cos(\frac{2\pi kt}{T})dt \tag{15}$$

Applying Variations theory to solve the pulse $w(t)$, the Lagrange function needs to satisfy the following equations:

$$\frac{\partial G}{\partial a_k} = 0, k = 1, .., m; \frac{\partial G}{\partial \alpha} = 0; \frac{\partial G}{\partial \beta} = 0; \frac{\partial G}{\partial \gamma} = 0. \tag{16}$$

3.2 Numerical Simulation

As a demonstration, this article chooses the rising index $n = 2$, 4, and 6, and the maximum number of Fourier series is a_6. The following window function expressions are obtained through numerical simulation:

$$\begin{aligned} w_{2,6}(i) = 1.6613/2 &+ 0.7866\cos(\frac{2\pi i T_s}{N-1}) - 0.0340\cos(\frac{4\pi i T_s}{N-1}) \\ &+ 0.0066\cos(\frac{6\pi i T_s}{N-1}) - 0.0021\cos(\frac{8\pi i T_s}{N-1}) \\ &+ 0.0009\cos(\frac{10\pi i T_s}{N-1}) - 0.0004\cos(\frac{12\pi i T_s}{N-1}) \end{aligned} \tag{17}$$

$$\begin{aligned} w_{4,6}(i) = 1.4780/2 &+ 0.9362\cos(\frac{2\pi i T_s}{N-1}) + 0.1763\cos(\frac{4\pi i T_s}{N-1}) \\ &- 0.0168\cos(\frac{6\pi i T_s}{N-1}) + 0.0031\cos(\frac{8\pi i T_s}{N-1}) \\ &- 0.0008\cos(\frac{10\pi i T_s}{N-1}) + 0.0003\cos(\frac{12\pi i T_s}{N-1}) \end{aligned} \tag{18}$$

$$\begin{aligned} w_{6,6}(i) = 1.3579/2 &+ 0.9982\cos(\frac{2\pi i T_s}{N-1}) + 0.3386\cos(\frac{4\pi i T_s}{N-1}) \\ &+ 0.0299\cos(\frac{6\pi i T_s}{N-1}) - 0.0046\cos(\frac{8\pi i T_s}{N-1}) \\ &+ 0.0009\cos(\frac{10\pi i T_s}{N-1}) - 0.0002\cos(\frac{12\pi i T_s}{N-1}) \end{aligned} \tag{19}$$

Blackman pulse and Kaiser pulse are respectively expressed as:

$$w_B(i) = \{0.42 - 0.5\cos(\frac{2\pi i}{N-1}) + 0.8\cos(\frac{4\pi i}{N-1})\} \tag{20}$$

where $t = iT_s$ is used for sampling, the window length is N. Among them, T_s is the sampling period.

4 Performance Analysis of F-OFDM System Under Different Filters

4.1 Combination of Optimized Pulse and Traditional Pulse

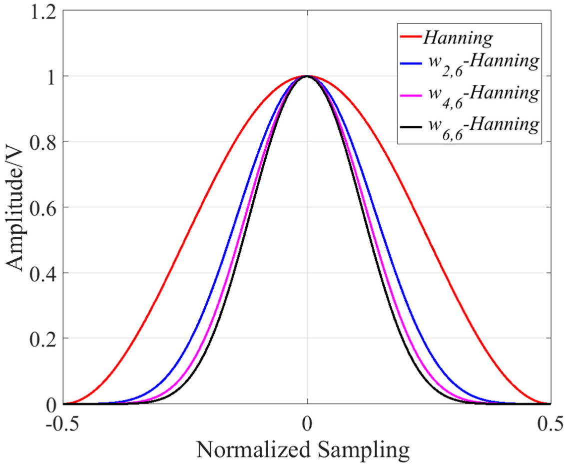

Fig. 2. The plots of different pulses under the normalized sampling in time domain.

The combined forms of the optimized pulses and Hanning pulse (Fig. 2):

$$\begin{aligned} w_{2,H}(i) &= w_{2,6}(i)w_H(i), \\ w_{4,H}(i) &= w_{4,6}(i)w_H(i), \\ w_{6,H}(i) &= w_{6,6}(i)w_H(i). \end{aligned} \tag{21}$$

The combined forms of the optimized pulses and Blackman pulse:

$$\begin{aligned} w_{2,B}(i) &= w_{2,6}(i)w_B(i), \\ w_{4,B}(i) &= w_{4,6}(i)w_B(i), \\ w_{6,B}(i) &= w_{6,6}(i)w_B(i). \end{aligned} \tag{22}$$

The feature of the pulse is that as the variate n increases, the peak is sharper (three optimized windows are all sharper than the Hanning pulse and the Blackman pulse).

4.2 Frequency Domain Performance of Filter

Use the finite pulse to multiply the sinc function to construct the filter as follows, where $-N/2 \leq i \leq N/2$, N is odd (Fig. 3).

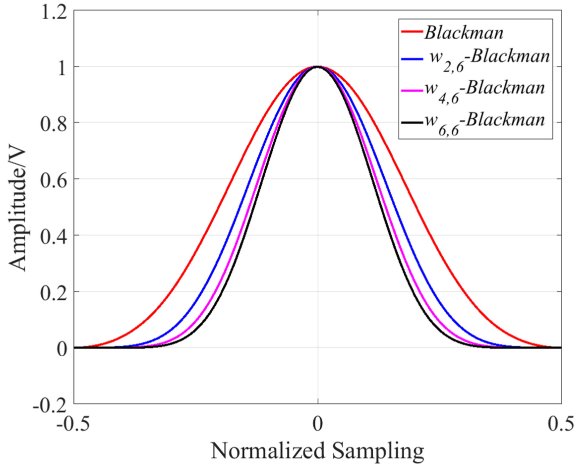

Fig. 3. Normalized PSD diagrams under different filters.

$$\begin{aligned}
f_{2,6-Hanning}(i) &= w_{2,H}(i) \sin c(i), \\
f_{4,6-Hanning}(i) &= w_{4,H}(i) \sin c(i), \\
f_{6,6-Hanning}(i) &= w_{6,H}(i) \sin c(i), \\
f_{2,6-Blackman}(i) &= w_{2,B}(i) \sin c(i), \\
f_{4,6-Blackman}(i) &= w_{4,B}(i) \sin c(i), \\
f_{6,6-Blackman}(i) &= w_{6,B}(i) \sin c(i).
\end{aligned} \tag{23}$$

The designed filter can solve the problem of high out-of-band radiation of the system. See the following figures for frequency normalization and amplitude normalized spectral density plots of the filters $f_{2,6-Hanning}(i)$, $f_{4,6-Hanning}(i)$, $f_{6,6-Hanning}(i)$, $f_{2,6-Blackman}(i)$, $f_{4,6-Blackman}(i)$, $f_{6,6-Blackman}(i)$.

For the parameter setting, according to the reference [11], the length of filter usually selected is $T_{filter} \leq 1/2T_{F-OFDM}$, where T_{filter} is the length of filter, and T_{filter} is the length of signal. Therefore, we choose the signal length to be 512, that is, the FFT/IFFT length is 512. The length of the filter selected in this study is 257, and the mapping method is 16QAM.

The normalized power spectral density (PSD) of the sinc-Hanning and the sinc-Blackman of the same length shown in Fig. 4 and Fig. 5.

Through Fig. 4 and Fig. 5, the normalized PSD of the filter constructed by the Hanning pulse and Blackman pulse converges to −162.7 dB and −171.6 dB, respectively; and the filter constructed by the combination of the optimized pulse and the Hanning pulse is normalized. The PSD of a single unit converges below

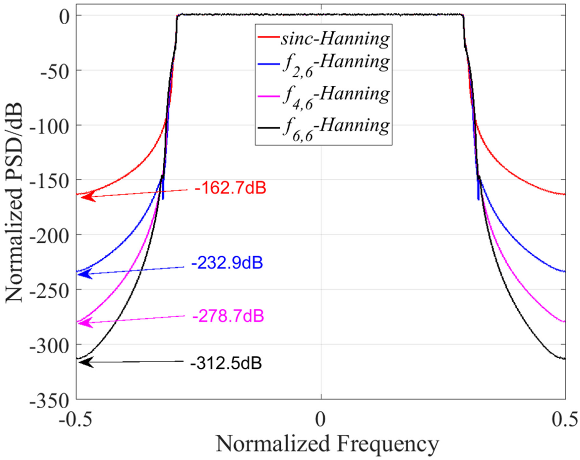

Fig. 4. Normalized PSDs under the different $f_{n,m-Hanning}(i)$ filter configurations.

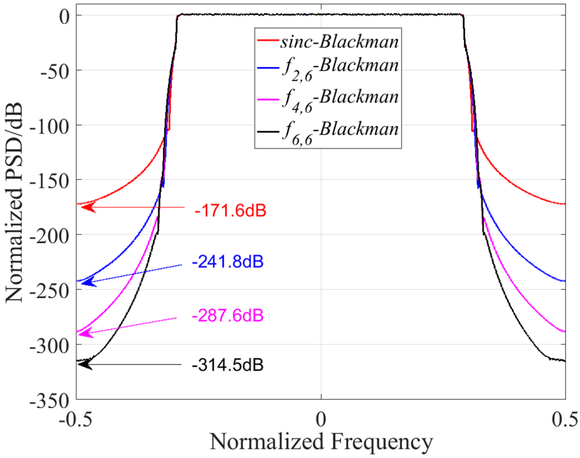

Fig. 5. Normalized PSDs under the different $f_{n,m-Blackman}(i)$ filter configurations.

−200 dB, and its out-of-band radiation is lower than that of a filter constructed with a single window function.

Choose the length of signal as 1024, that is, the FFT/IFFT length is 1024. The length of the filter selected in this study is 513, and the mapping method is 16QAM.

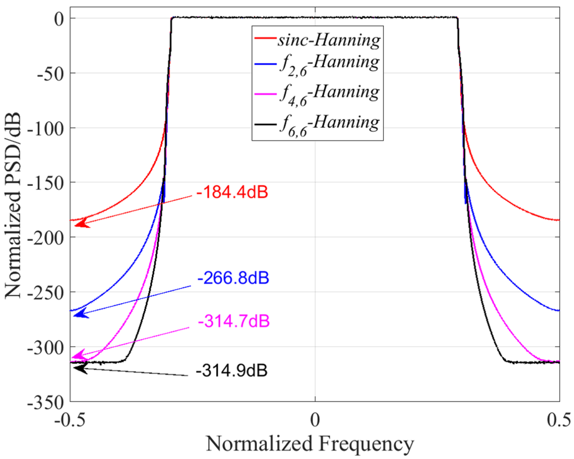

Fig. 6. Normalized PSDs under the different $f_{n,m-Hanning}(i)$ filter configurations.

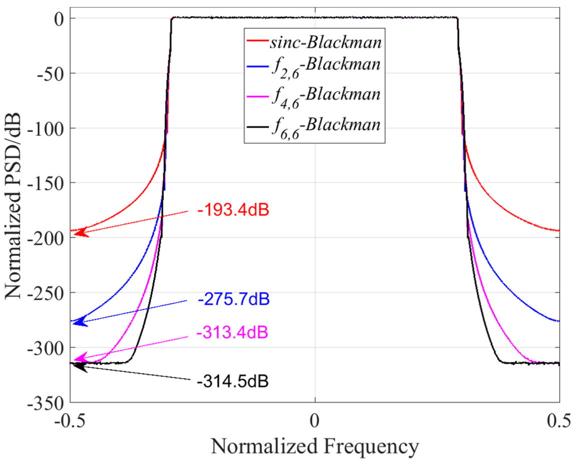

Fig. 7. Normalized PSDs of the F-OFDM system with different $f_{n,m-Blackman}(i)$ filter configurations.

Through Fig. 6 and Fig. 7, the normalized PSD of the filter constructed by the Hanning pulse and Blackman pulse converges to −184.4 dB and −193.4 dB, respectively; and the filter constructed by the combination of the optimized pulse and the Hanning pulse is normalized in the F-OFDM system. The PSD of a single unit converges below −250 dB, and its out-of-band radiation is lower than that of a filter constructed with a single window function.

At the same time, it can be seen that the filter of the window structure combined with the Blackman pulse has a better out-of-band radiation capability than the Hanning pulse. And as the number of sub-carriers increases, the length of the filter can also increase, so the out-of-band radiation of the system will also be significantly reduced.

5 Conclusions

In this study, through the basic model of the F-OFDM system, the key technology is how to design filter. The filter is usually constructed by multiplying the known impulse function by the sinc function. The key to the design of the window function lies in its fast-decaying side-lobes features. We propose a functional model which has the less energy leakage criterion to obtain a pulse with features of the above requirements. The pulse is expressed as an even function in the form of a Fourier series. Changing the variate n in the functional model can obtain optimized pulses with different decay speeds. The optimized pulse and the traditional pulses are combined to form a new pulse, and the filter is constructed by multiplying the sinc function. In the time domain, the feature of the combination pulse is that as the variate n increases, its peak is steeper and the edges are smoother, and the optimized pulses have steeper peaks than the Hanning pulse and Blackman pulse. The combination pulse and the sinc function forming filter are applied to the F-OFDM system, and they both show the characteristics of less energy leakage. Therefore, it can be said that it is more inclined to use combination window to construct filters.

References

1. Weinstein, S.B.: The history of orthogonal frequency-division multiplexing. IEEE Commun. Mag. **47**, 26–35 (2009)
2. Mesleh, R.Y., Haas, H., Sinanovic, S., Ahn, C.W., Yun, S.: Spatial modulation. IEEE Trans. Veh. Technol. **57**, 2228–2241 (2008)
3. Brandes, S., Cosovic, I., Schnell, M.: Reduction of out-of-band radiation in OFDM systems by insertion of cancellation carriers. IEEE Commun. Lett. **10**, 420–422 (2006)
4. Cosovic, I., Brandes, S., Schnell, M.: Subcarrier weighting: a method for sidelobe suppression in OFDM systems. IEEE Commun. Lett. **10**, 444–446 (2006)
5. Farhang-Boroujeny, B.: OFDM versus filter bank multicarrier. IEEE Signal Process. Mag. **283**, 92–112 (2011)
6. Nissel, R., Schwarz, S., Rupp, M.: Filter bank multicarrier modulation schemes for future mobile communications. IEEE J. Sel. Areas Commun. **358**, 1768–1782 (2017)
7. Michailow, N., et al.: Generalized frequency division multiplexing for 5th generation cellular networks. IEEE Trans. Commun. **629**, 3045–3061 (2014)
8. Abdoli, J., Jia, M., Ma, J.: Filtered OFDM: a new waveform for future wireless systems. In: Proceedings of the IEEE International Workshop Signal Process. Advances Wireless Communication, pp. 66–70. IEEE, Stockholm, Sweden (2015). https://doi.org/10.1109/SPAWC.2015.7227001
9. Zhang, X., Jia, M., Chen, L., Ma, J., Qiu, J.: Filtered-OFDM - enabler for flexible waveform in the 5th generation cellular networks. In: Proceedings of the IEEE Global Communications Conference. IEEE, San Diego (2015). https://doi.org/10.1109/GLOCOM.2015.7417854

10. Yang, L.Z., Xu, Y.H.: Filtered-OFDM system performance research based on Nuttall's Blackman-Harris window. In: 2017 IEEE 17th International Conference on Communication Technology (ICCT). IEEE, Chengdu (2017)
11. Taher, M., Radhi, H., Jameil, A.: Enhanced F-OFDM candidate for 5G applications. J. Ambient Intell. Hum. Compt. **121**, 635–652 (2020). https://doi.org/10.1109/ICCT.2017.8359724
12. Xu, Y., Xue, W., Shang, W.: A pan-function model for the utilization of bandwidth improvement and PAPR reduction. Math. Probl. Eng. **2014**, 1–9 (2014)