



# Evaluation Method for Water Network Connectivity Based on Graph Theory

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**Abstract.** Water network connectivity plays a pivotal role in water security and ecological civilization, which also has been viewed as an important part for water comprehensive improvement. Using graph theory, the water network can be modeled as an undirected graph with multiple interconnected elements which represent rivers by edges and junctions by nodes. In this paper, the weight on edge represents the simplified river cross-section area which can be equivalent to flow capacity between two adjacent nodes. In order to obtain the flow capacity between any two nodes, a maximum flow evaluation scheme is designed by using Boykov-Kolmogorov algorithm. Then by using the maximum flow between nodes before and after dredging based on node degree features, a solution is assayed for average flow capacity of each node and water network connectivity. The proposed approach has been tested on the river network of Xiacheng District in Hangzhou with dredging data in year 2016. The result shows that the water network connectivity is improved by 7.89% over the whole area and more than 40% in part region by dredging, and significant influenced by intersect density and dredging extent.

**Keywords:** Water network · Graph theory · Connectivity · Dredging

## 1 Introduction

In the recent studies of water resources and hydrological cycle, water networks represent an important and fundamental class due to their simple structures. The connectivity of the water networks has an important impact on the regional flood and drought disaster resilience, water resources allocation and river-lake health protection, and be significant for water resources management. The distribution of natural rivers generally appears to be a network or treelike shape, and water can flow through the channel with the function of flood prevention, water resources regulation and improvement of water eco-environment [1]. Deficiency of network connectivity caused by river channel shrinkage and disappearance may lead to serious problem for water shortage and series disasters. However, numbers of engineering measures as dredging and artificial drainage can be used to improve water connectivity, which can be helpful to optimize water resources allocation, improve water supper, and promote water ecological civilization [2, 3].

Numbers of methods including graph theory method, hydrologic and hydraulic method, landscape ecology method, biological method, and comprehensive index method are used to evaluate water network connectivity. Graph theory is helpful to simplify water system, which comprises of nodes for features at river interchange and links for river or pipe between nodes [4–6]. It can be utilized to calculate connectivity of the water network by analysis connectivity of graph with edge weight given by specify physical attributes such as river length, river width or hydrodynamic factor. From Xia Jihong's summarize, the meaning of edge weight between any two nodes could be guarantee rate of streamflow from one node to another under certain water level and discharge [7]. Gao Yuqin used graph theory to develop a methodology for calculating connectivity with edge weight by dynamic flow passing capacity [8]. Another evaluation method for river network connectivity was proposed by Xu Guanglai which can be calculated by edge weight allocated with the reciprocal of hydraulic resistance, thus flow capacity of any two nodes, average flow capacity of each node, and connectivity of river network can be obtained [9, 10]. It was proved to be suitable for assessment of connectivity of the water network which influenced by hydraulic engineering as dredging and artificial river. On the other hand, water network was considered as internet of things due to the function of water supply and delivery by perception and intelligent dispatching based on the connected structure [11]. Two important ways to increase water supply and delivery are links added and flow capacity added. Compare to links added by artificial river, flow capacity added by dredging appears a more practical measure to increase the connectivity of water network.

In this paper, flow capacity and connectivity change by dredging will be detail analysis, where graph is established by rivers of Xiacheng District in Hangzhou, while edge weight references to flow capacity by river cross-section area, average flow capacity of node references to ability for one node to other nodes, connectivity of river network references to average flow capacity of all nodes.

## 2 Methodology

### 2.1 Graph of Water Network

According to graph theory, water network is represented as a mathematical graph  $G = G(V, E, W)$  in which  $V$  is the set of all graph nodes with  $n$  elements,  $E$  is the set of graph edges with  $m$  elements and  $W$  is the set of graph edge weights between any two nodes which can be defined as adjacency matrix  $W = (w_{ij})_{n \times n}$ . It is common to represent rivers or pipes by graph edges  $E$ , junctions such as pipe intersections, reservoirs and consumers by graph nodes  $V$  and edge weights by flow capacity  $W$ . Here  $w_{ij}$  has the meaning of flow capacity from node  $i$  to  $j$ , and  $(i, j) \in E$  is out-degree neighbors for  $j$  to  $i$ , otherwise  $(j, i) \in E$  is in-degree neighbors for  $i$  to  $j$  which means node  $i$  can supply water for nose  $j$ . It is equal for out-degree and in-degree for undirected graph. With such representations, connectivity of the water network can be evaluated by connectivity of graph [12].

## 2.2 Edge Weight of Water Network

Edge weight  $w$  of graph is represented by the degree of patency within a river channel can be estimated by the reciprocal of hydraulic resistance in the plain as slope of river bed is usually neglected. Hydraulic resistance of trapezoid channel can be expressed as a function of channel length  $l$ , roughness  $n$  and bottom width  $b$ , water depth  $h$  and slope coefficient  $m$  [9], the formula can express as

$$H = \ln \left[ \frac{(b + mh)h}{b + 2h\sqrt{1 + m}} \right]^{-\frac{2}{3}} \quad (1)$$

Then edge weight follows as  $w = 1/H$ .

For the rectangular channel,  $m$  equals zero and river cross-section area added  $\Delta a$  by dredging can be computed by

$$\Delta a = \Delta hb = \frac{q}{l}, \quad (2)$$

where  $\Delta h$  represents added depth and  $q$  represents river dredging volume.

As length and roughness is constant for the same river, ratio of the degree of patency within a river channel  $w'$  after dredging to  $w$  before dredging is calculated as

$$\frac{w'}{w} = \left[ 1 + \frac{\Delta hb}{h(b + 2h + 2\Delta h)} \right]^{\frac{2}{3}} \quad (3)$$

In the condition of the flat land, roughness can be ignored, the degree of patency within a river channel can be instead by flow capacity in simple, more easy formula is adopted for evaluating ratio of flow capacity  $w'$  after dredging to  $w$  before dredging as [1]

$$\frac{w'}{w} \approx \frac{a + \Delta a}{a} = \frac{h + \Delta h}{h}, \quad (4)$$

which only need the parameter of the increased river cross-section area or increased water depth to compute connectivity change.

## 2.3 Connectivity of Water Network

Connectivity algorithms for graph usually refer to node connectivity and edge connectivity which means the graph will be disconnected by removing with fewest nodes or edges. However, it usually appears as a sparse network graph while water network is not so dense compared with personal networks and power network, the result of node connectivity and edge connectivity often show 1 which cannot be utilized to evaluation water network connectivity exactly. As the graph of water network can clearly express river channel and intersect, node degree could be obtained with number of river channel connected with intersect, and higher node degree implies the more important position in the network. Thus, it is easy to get average node degree for assessing the connectivity of water network as a basic method with meaning of higher average node degree better connectivity.

Graph edge weights  $w$  with flow capacity for adjacent nodes can be represented by river cross-section area, and non-adjacent nodes can be expressed by maximum flow by given the hypothesis that indirectly connected nodes can be reached to each other through middle nodes. Maximum flow of two nodes can be obtained by the principle that flow capacity will be limited by the smallest flow capacity of middle links. There is a calculation example for the directed graph with a given graph by 5 nodes and 6 edges (see Fig. 1). According to this graph, there are 3 paths from node 1 to node 5 refer to ①  $v_1-v_2-v_4-v_5$ , ②  $v_1-v_2-v_3-v_5$  and ③  $v_1-v_3-v_5$ , and the maximum flow of path is limited by the minimum edge weight which represents flow capacity of link. Thus, the maximum flow through path ① is 3, path ② is 2, and path ③ is 2. Considering path ② and path ③ share edge  $v_3-v_5$ , so the maximum flow can pass is limited by edge weight  $v_3-v_5$  which equals 2. Finally, maximum flow from node 1 to node 5 equals 5. In addition, for the undirected graph of double bidirectional flow may contains more paths, but it also follows the basic principle that the maximum flow between two nodes is limited by the minimum flow capacity of the edge along the path and the flow of each edge does not exceed the capacity.

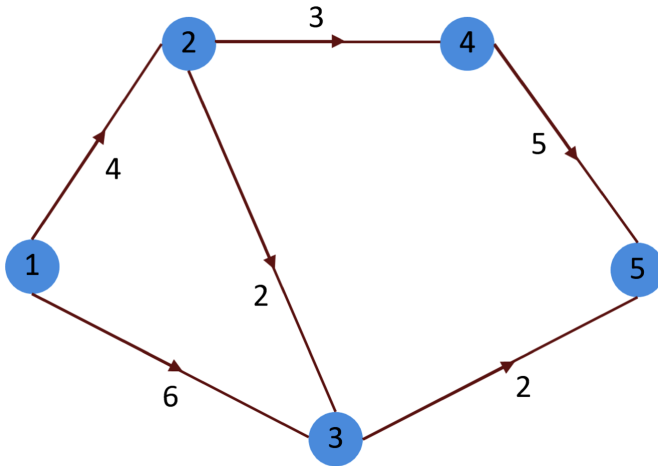


Fig. 1. Example of maximum flow between nodes

Connectivity of water network is calculated as follows:

Step 1: Boykov-Kolmogorov method is used to calculate maximum flow between any two nodes by MATLAB. The mathematical expression follows as [13]

$$(f_{ij})_{n \times n} = \text{maxflow } w_{ij}; i, j = 1, 2, \dots, n \tag{5}$$

Where,  $f_{ij}$  represents maximum flow from node  $i$  to node  $j$ , and the result appears the matrix of maximum flow by traveling node method.

Step 2: Average flow capacity of node  $D_i$  can be obtained by

$$D_i = \frac{1}{n-1} \sum_{i \neq j}^n f_{ij}; i, j = 1, 2, \dots, n \tag{6}$$

Step 3: Connectivity of water network  $D$  is actually the average maximum flow of all nodes as

$$D = \frac{1}{n} \sum_{i=1}^n D_i \tag{7}$$

The flowchart of connectivity algorithm is as follows (Fig. 2):

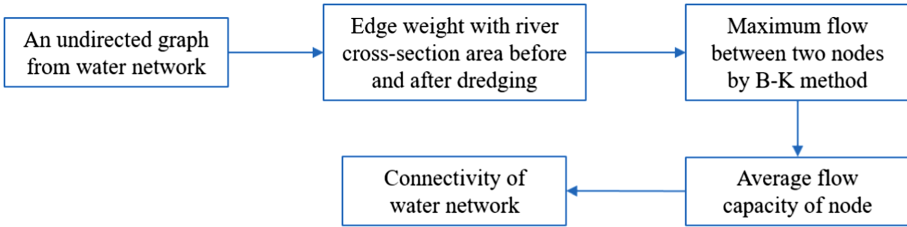


Fig. 2. The flowchart of connectivity algorithm

### 3 Application Study

#### 3.1 Region Introduction

Hangzhou is located in the northern part of Zhejiang Province and in the downstream region of the Qiantang River. It is the starting point of south for the Beijing-Hangzhou Grand Canal and considered as an important city of the Yangtze River Delta. It is a plain city with developed water system with river density of more than 10 km/km<sup>2</sup>. Benefit from abundant water resources, rivers play an important vector to meet the needs of irrigation, power generation, drainage, shipping, breeding, production and domestic water. According to the statistics, water network of Xiacheng District in Hangzhou consists of 31 main rivers with width from 10 to 100 m and depth from 2 to 3 m. Affected by urbanization, the cross-sections of most rivers appear as rectangle by artificial modification.

#### 3.2 Node Degree and Flow Capacity

Xiacheng District is so flatland that water network is suitable for undirected graph. It is a connected graph with 52 nodes and 71 edges (excluding beheaded river due to boundary restriction, see Fig. 3) by node connectivity and edge connectivity both equal 1. According to the statistics, there are 42 nodes with a degree value of 3 and a high percent of 80.77%, the maximum node degree is 4, the minimal node degree is 1, and the average node degree is 2.73. Nodes of degree 1 or 2 may be the beheaded river or boundary limit.

According to the data obtained, there are 8 rivers dredged with total 175,500 cubic meters over river length more than 19 km in the year 2016 in Xiacheng District. The cross-section area of dredged river increased by 2.92–28.93 m<sup>2</sup>, ratio of  $\Delta a/a$  ranges from 11.67% to 100.62%. As a result, a total of 18 graph edges have weight added (see Table 1).

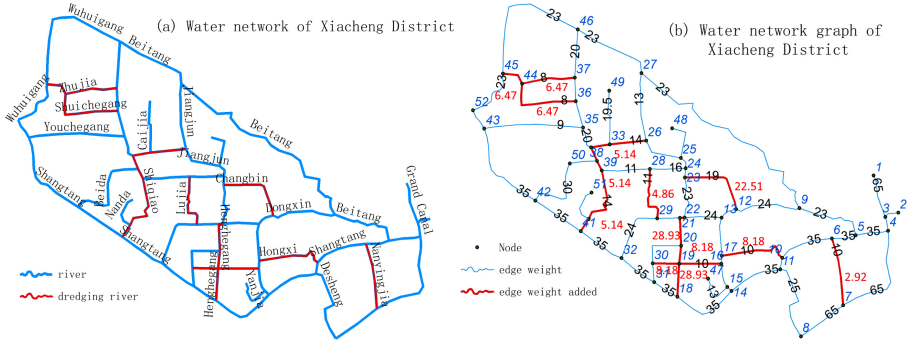


Fig. 3. Water network and graph

Table 1. Edge weight added by dredging

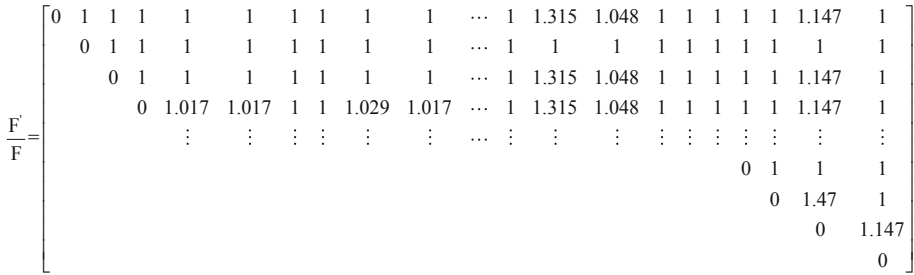
Name	l (m)	b (m)	h (m)	a (m <sup>2</sup> )	q (m <sup>3</sup> )	$\Delta a$ (m <sup>2</sup> )	E ( $\Delta a$ )
Nanyingjia	1200	10	2.5	25	3500	2.92	8
Changbin	1777	19	2.5	47.5	40000	22.51	18
Hongxi	3300	10	2.5	25	27000	8.18	15, 26, 30
Henghegang	1400	11.5	2.5	28.75	40500	28.93	27, 29, 32
Lujia	1850	11	2.5	27.5	9000	4.86	44
Shiqiao	7100	14	2.5	35	36500	5.14	42, 50, 53, 60, 62, 64
Zhujia	1546	8	2.5	20	10000	6.47	59, 70
Shuichegang	1513	8	2.5	20	9000	5.95	57

Note: E ( $\Delta a$ ) refers to edges with area added.

### 3.3 Average Flow Capacity and River Network Connectivity

The maximum flow between any two nodes is calculated as formula (1), and ratio matrix of maximum flow after dredging  $F'$  to maximum flow before dredging  $F$  shows in Fig. 4. As  $F$  is a symmetric matrix, where  $F = (f_{ij})_{n \times n}$ ,  $f_{ij} = f_{ji}$ , it can be expressed as upper triangular. Thus, the maximum flow calculation consists of 1326 node pairs, of which 765 increased and 561 unchanged. Among the increasing node pairs, there are 7 node pairs whose maximum flow  $f$  reaches 69.04%, including (12, 19), (14, 19), (18, 19), (19, 31), (19, 32), (19, 41), (19, 42). As node 19 with node degree value 4 is so important that each of the 7 node pairs with maximum flow increasing contain node 19. It is found 42 nodes pairs whose value  $f$  is equal or greater than 60%, and 40 of them has relationship with nodes 19 and 20, which is due to the higher node degree of 19 and mass dredging of the two nodes.

Average flow capacity added of each node is calculated by average maximum flow added for the point to all others. As shown in Table 2, there are 9 nodes with an increase of average flow capacity more than 10%, 27 nodes with an increase of 5% to 10%, 12 nodes with an increase of 0 (excluding 0) to 5%, and 4 nodes unchanged. According to data



**Fig. 4.** Ratio matrix of maximum flow between nodes dredging to original

analysis, Henghegang has the highest river cross-section area added by dredging, and the average flow capacity of the 4 middle nodes within it (18, 19, 20, and 21) increased by 6.93%, 40.42%, 44.01%, and 13.72%. Especially, the middle two nodes 19 and 20 which surrounded by dredging river links have the maximum increase. Average flow capacity of 4 nodes (2, 47, 48, 49) maintain zero while they are only connected with one edge and namely with node degree 1, and limited by it.

**Table 2.** Average flow capacity added for nodes

<u>1-9</u>		<u>10-18</u>		<u>19-27</u>		<u>28-36</u>		<u>37-45</u>		<u>46-52</u>	
No	D <sub>i</sub> (%)	No	D <sub>i</sub> (%)	No	D <sub>i</sub> (%)	No	D <sub>i</sub> (%)	No	D <sub>i</sub> (%)	No	D <sub>i</sub> (%)
1	4.53	10	6.37	19	40.42	28	5.68	37	6.41	46	4.58
2	0.00	11	6.25	20	44.01	29	5.92	38	9.75	47	0.00
3	4.53	12	8.58	21	13.72	30	8.46	39	10.11	48	0.00
4	6.06	13	4.90	22	5.83	31	6.13	40	13.64	49	0.00
5	6.24	14	6.42	23	13.27	32	6.51	41	6.05	50	1.87
6	6.22	15	1.64	24	1.87	33	13.99	42	6.05	51	13.64
7	6.06	16	7.38	25	0.16	34	8.62	43	4.63	52	3.84
8	6.08	17	7.38	26	5.84	35	2.58	44	29.25		
9	5.46	18	6.93	27	3.93	36	6.10	45	6.56		

The distribution of the average flow capacity of each node before dredging is shown in Fig. 5(a). Nodes of the better average flow capacity before dredging are 4 to 13, which belong to the Grand Canal, Shangtang River, Beitang River, Desheng River and Nanyingjia River with large river cross-section area and high discharge passing capacity. Node of the worse average flow capacity before dredging are 4 nodes (47, 48, 49 and 51) with node degree 1, and node 44 with node degree 3. For node 44, although it is with node degree 3 but the total weight of the three connected edges is still one of the minimum values among the entire river network. The distribution of the average flow capacity added of each node is shown in Fig. 5(b). Henghegang region is with the highest increase of

the average flow capacity, and node 44 connected with Zhujia River and Shuichegang followed by. Although the average flow capacity of node 44 before dredging is worse than most nodes, but all of the three connected links are dredged with cross-section area added by 88.88%, which leads to a high increase. It also shows minor increase of Beitang River because there are no dredged rivers connected to it including itself. Analysis indicated that nodes with more dredging and higher node degree appear to be more the average flow capacity added. With the dredging data of the study region, the connectivity of water network is computed to add by 7.89%.

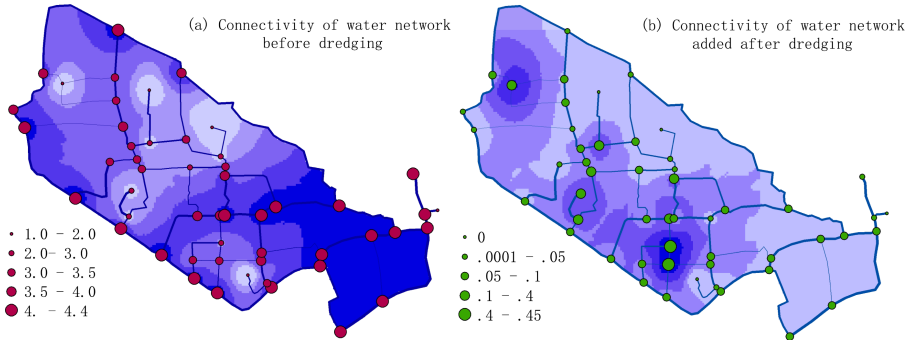


Fig. 5. Average flow capacity of node before dredging and its increase rate after dredging

## 4 Conclusion

Connectivity of water network declines with reduction of river cross-section by construction of urbanization, which may lead to problems as frequent flooding and bad quality of water. Dredge can be useful to solve the problem by improving river network patency.

- (1) This paper presents a method by using river cross-section area as edge weight to calculate average flow capacity of nodes and water network connectivity, which may be helpful to find a better dredging scheme.
- (2) The concept of network flow is applied to the water flow, namely maximum flow, which is used as river flow capacity between two nodes with the principle of the maximum flow of the path between two nodes is limited by the minimum flow capacity of the middle links in path and the flow of each link does not exceed its own capacity.
- (3) It will be a continues job to research on sensitivity, redundancy and robustness of edges in the water network, and simultaneously consider hydraulic engineering as an essential factor to evaluate flow capacity and connectivity in further study.

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