Chaotic Sea Clutter Modeling Based on Gray Wolf Algorithm

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Abstract. This paper mainly verifies the chaotic characteristics of sea clutter and studies the prediction of chaotic sea clutter. The chaotic phase space was reconstructed by calculating the delay time and embedding dimension pairs. This paper provide a sea clutter prediction method based on the Grey Wolf Optimizer (GWO). The iterative results of the Grey Wolf algorithm were used as the weights of the RBF neural network. The simulation shows the predicted value of GWO-RBF network is close to the real value, and the prediction error is small, so as to verify the effectiveness of GWO-RBF network for chaotic sea clutter prediction.

Keywords: Chaos · Sea clutter · Phase space reconstruction · Grey Wolf algorithm · RBF neural network

1 Introduction

Sea clutter is the backscattered echo of a small area of ocean covered by the transmitted radar signal, and this signal is a time series with chaotic properties. Chaotic time series are a part of chaos theory and a representation of the time change of some states of a chaotic system. Thus the properties of chaotic time series are also properties of chaos theory. Using chaotic time series to reconstruct chaotic system is the first step to study chaotic system. Phase space is reconstructed according to Takens theorem [1], and chaotic system is reconstructed by combining neural network, deep learning and other methods, so as to realize the analysis of chaotic system. Therefore, the study of chaotic time series becomes one of the important research directions in the field of radar signal processing [2].

Leng et al. [3], found for the first time that the sea clutter in the X-band had chaotic characteristics by calculating the correlation dimension of sea clutter in 1990. Later, a large number of literatures showed that sea clutter had chaotic characteristics. In 2013, R. A. Simon et al. [4] proved that the randomness of sea clutter data was generated by deterministic chaos rather than random process through a large number of experiments. Meanwhile, Lyapunov exponent is calculated, and the result is greater than 0, which further verifies the chaotic characteristics of sea clutter.

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In recent years, through in-depth study of sea clutter, it is found that sea clutter is not a complete random time series. For the most part, it contains some features that are evident in chaotic systems. Because chaotic systems belong to deterministic systems, the trajectories of chaotic attractors can be predicted in a very short time [5, 6]. Early researchers considered sea clutter as a random signal and proposed a variety of modeling methods. The common problems of these modeling methods are complex establishment process, poor universality and low accuracy. These traditional sea clutter prediction methods adopt statistical process, which essentially cannot make accurate prediction. The new method combines neural network, machine learning and deep learning methods to model sea clutter using chaotic characteristics. Therefore, through in-depth research on the chaotic sea clutter can provide a new development direction for sea clutter prediction and sea target detection. Neural network and some other nonlinear algorithms have also been applied to target detection in the marine environment [7, 8].

At present, most of the research on sea clutter mainly focuses on the construction of the probability model of sea clutter [9], or the classification of clutter and target by using neural network classifier [10]. The application of these new technologies and algorithms makes the target detection technology under the background of sea clutter develop continuously and innovate, and makes the detection level reach a new height.

In 2013, Mirjalili et al. [11], put forward the Gray Wolf algorithm based on the cooperative hunting mechanism of simulated wolves in accordance with the inspiration of group hunting of gray wolves. Grey Wolf algorithm has a rigorous hierarchical system, adaptive convergence factor and information feedback system, and can effectively improve global search and local optimization. Therefore, this algorithm has a good solution effect in terms of accuracy and convergence speed.

2 Chaotic Characteristics of Sea Clutter

2.1 Phase Space Reconstruction

The first step in studying chaos theory is to reconstruct phase space. In the basic theory of phase space reconstruction, the change of any variable in space is associated with other variables, and they interact with each other. According to the theory proposed by Packard et al. [12], the phase space can be reconstructed by using the delay coordinate of a variable in the dynamical system. Takens proved that under appropriate embedding dimensions [1], the dimensions of the prime dynamic system can be restored.

The first step in reconstructing phase space is to calculate two parameters, One is the embedding dimension and the other is the delay time. The mutual information method is used to calculate the delay time and the False Nearest Neighbors method is used to calculate the embedding dimension [13]. Set time series $x(i) i = 1, 2, 3, 4, \ldots, N$, the delay time is τ, and the embedding dimension is m, then the phase space reconstruction vector is:

$$Y(i) = \{x(i), x(i+\tau), \cdots, x(i+(m-1)\tau)\} \quad (1)$$

2.2 Delay Time

Let the sampling data set $X(j) = x(t_0 + jT)$, where t_0 is the sampling time, T is the sample period, $j = 1, 2, \cdots, N$. Then its correlation function is:

$$C(\tau) = \frac{\sum_{j=1}^{N} [x(t_0 + jT + \tau) - \langle x \rangle][x(t_0 + jT) - \langle x \rangle]}{\sum_{j=1}^{N} [x(t_0 + jT) - \langle x \rangle]^2} \quad (2)$$

Where,

$$\langle x \rangle = \frac{1}{N} \sum_{j=1}^{N} x(t_0 + jT) \quad (3)$$

If the autocorrelation function has a zero crossing at time τ, then τ can be used as the optimal solution. Otherwise, the first local minimum value is used as the best delay time.

2.3 Embedding Dimension

In an m-dimensional phase space, the vector of the phase space attractor is $X(j) = \{x(j), x(j+\tau), \ldots, x(j+(m-1)\tau)\}, j = 1, 2, \cdots, N$. Chaotic time series can be regarded as the projection of the trajectory of the attractor in the high-dimensional space into the low-dimensional space, and the two adjacent points projected are false adjacent points. Therefore, when the dimension m changes to $m+1$, it is necessary to investigate which adjacent points of the trajectory line $x(j)$ are real adjacent points and which are false adjacent points. Set $X^{NN}(j)$ is the neighbor of $X(j)$, and the distance between them be:

$$R_m(j) = \left\| X(j) - X^{NN}(j) \right\| \quad (4)$$

When the dimension is incremented from m to $m+1$, the $R_{m+1}(j)$ is:

$$R_{m+1}(j) = \sqrt{R_l^2(j) + \|X(j+\tau m) - X^{NN}(j+\tau m)\|} \quad (5)$$

$$a_1(j, m) = \frac{|X(j+\tau m) - X^{NN}(j+\tau m)|}{R_m(j)} > R_r \quad (6)$$

$$a_2(j, m) = \frac{\left\| X_{m+1}(j) - X_{m+1}^{NN}(j) \right\|}{\left\| X_m(j) - X_m^{NN}(j) \right\|} \quad (7)$$

If $R_{m+1}(j)$ is much larger than $R_m(j)$, it can be considered that these two points are not the nearest neighbors, but the higher-dimensional attractors are projected onto the lower-dimensional trajectories. So they're not really the nearest neighbor.

Among Eq. (6) (7), $X_m(j)$ and $X_m^{NN}(j)$ of m dimensional space of J th vector and its adjacent points, $X_{m+1}(j)$ and $X_{m+1}^{NN}(j)$ of m + 1 dimensional space of the J th vector and its adjacent points, $R_r \in [10, 50]$. Define:

$$\begin{cases} E(m) = \frac{1}{N-m\tau} \sum_{i=1}^{N-m\tau} a_2(i,m) \\ E_2(m) = E(m+1)/E(m) \end{cases} \tag{8}$$

If the time series is deterministic, then $E(m)$ will unchanged after m is greater than a value. If the time series is a random, then $E(m)$ should increase gradually. However, in the application, it is difficult to judge whether E(m) of finite time series changes slowly or has been stable, so supplement to a condition:

$$\begin{cases} E(m) = \frac{1}{N-m\tau} \sum_{i=1}^{N-m\tau} X(j+\tau m) - X^{NN}(j+\tau m) \\ E_2(m) = E(m+1)/E(m) \end{cases} \tag{9}$$

For a deterministic time series, the correlation between data points is dependent on the change of embedding dimension m values, and there are always some m values where $E_2(m)$ does not equal 1.

2.4 Correlation Dimension

The correlation dimension describes the distribution of points in dynamic space. The correlation dimension is usually calculated by the G-P (Grassber-Procaccia) method. $X_1, X_2, \ldots, X_N$ is the point in the phase space, and given a distance r, the proportion of the phase points whose two points are more than r is:

$$C(r) = \frac{1}{N(N-1)} \sum_{i=1, i \neq j}^{N} \sum_{j=1}^{N} H(r - \|X_i - X_j\|) \tag{10}$$

Among them, $\|\cdot\|$ is Euclidean norm, H is Heaviside step function:

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{11}$$

So, D_2 is:

$$D_2 = \lim_{r \to 0} \frac{\ln(C(r))}{\ln(r)} \tag{12}$$

When, $r \to 0$ $C(r) \approx r^{D_2}$:

$$\lim_{r\to 0} C(r) = \lim_{r\to\infty} \frac{1}{N(N-1)} \sum_{i=1, i\neq j}^{N} \sum_{j=1}^{N} H(r - \|X_i - X_j\|) \tag{13}$$

2.5 Lyapunov Exponents

Lyapunov exponent is a quantitative index to describe the state evolution of a dynamical system. It measures the average divergence or convergence rate of adjacent orbits in the phase space of the system. Small data volume method is a method to calculate Lyapunov exponent with less data and low complexity. Its basic principle is as follows: after the reconstruction of time series, look for the nearest neighbor $X_{\widehat{j}}$ of the JTH point X_j on the orbit, that is:

$$d_j(0) = \min\left\|X_j - X_{\widehat{j}}\right\|, \ \left|j - \widehat{j}\right| > p \tag{14}$$

Among them, p is the period of X_j. And calculate the average dispersion of each point to get largest Lyapunov exponent. So largest Lyapunov exponent is:

$$\lambda_1(i) = \frac{1}{i\Delta t} \frac{1}{(N-i)} \sum_{j=1}^{N-i} \ln \frac{d_j(i)}{d_j(0)} \tag{15}$$

Where, Δt is the sample period, $d_j(i)$ is the distance of the JTH pair of nearest neighbors on the orbit after i discrete time steps. The above equation is rewritten as follows:

$$\lambda_1(i,k) = \frac{1}{k\Delta t} \frac{1}{(N-k)} \sum_{j=1}^{N-k} \ln \frac{d_j(i+k)}{d_j(i)} \tag{16}$$

Where, K is constant,

$$d_j(i) \approx C_j e^{\lambda_1 (i\cdot\Delta t)}, C_j = d_j(0) \tag{17}$$

$$\ln(d_j(i)) = \ln(C_j) + \lambda_1(i\cdot\Delta t), j = 1, 2, \cdots, N-(m-1)\tau \tag{18}$$

The value of the largest Lyapunov exponent can be obtained by approximating the slope of Eq. (17) by the least square method:

$$x(j) = \frac{1}{\Delta t}\left\langle \ln(d_j(t)) \right\rangle \tag{19}$$

Where, $\langle\cdot\rangle$ is taking the average of all about j points.

3 Grey Wolf Algorithm

3.1 Grey Wolf Algorithm

In 2013, Mirjalili proposed a new group intelligence algorithm based on the hunting and inter-population relations of gray wolves. The Gray Wolf optimization algorithm mainly imitates the hunting behavior of natural Gray wolves to solve complex optimization problems [11, 14].

In the Gray Wolf algorithm, there are four different levels of Wolf α, β, δ, ω, and the different levels of Wolf represent different solutions. α represents the optimal solution, β represents second optimal solution, δ represents the third optimal solution, ω Wolf is the solution that updates α, β, δ.

During each iteration, the position of ω is actively updated, replacing the corresponding α, β, δ with the position of ω only when the solution of ω is better than that of α, β, δ. The position update formula of the three wolves is as follows:

$$d_\alpha = |Y_\alpha \cdot C_1 - Y|, d_\beta = \left|Y_\beta \cdot C_2 - Y\right|, d_\delta = |Y_\delta \cdot C_3 - Y| \tag{20}$$

$$Y_1 = Y_\alpha - d_\alpha \cdot A_1, Y_2 = Y_\beta - d_\beta \cdot A_2, Y_3 = Y_\delta - d_\delta \cdot A_3 \tag{21}$$

$$Y(t+1) = (Y_1 + Y_2 + Y_3)/3 \tag{22}$$

Where, Y_α, Y_β, Y_δ are the position vectors of α, β and δ, and Y is the position vector of ω in iteration t, $Y(t+1)$ is the position vector of ω in iteration $t+1$, The formula for A and C is:

$$A = \mathbf{r_1} \cdot 2\alpha - \alpha, \quad C = 2 \cdot \mathbf{r}_2 \tag{23}$$

$\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are random numbers in [0, 1], as the number of iterations of the algorithm increases, α gradually decreases from 2 to 0.

$x_i^j(t)$ is jth dimensional position value of the ith wolf in the current iteration, x_{max} is maximum position, x_{min} is minimum position.

3.2 RBF Neural Network Based on Gray Wolf Optimization Algorithm

RBF neural network has excellent performance characteristics such as simple structure, self-learning, self-organization, self-adaptation, easy to overcome local minimum, etc. [8], and has been widely used in online identification, model approximation and parameter learning of nonlinear systems. Because of this, this paper will use RBF neural network as the basis to achieve the prediction of chaotic sea clutter. Meanwhile, Gray Wolf algorithm was used to train the parameters of RBF neural network to improve the prediction accuracy.

In Fig. 1, the input vector of the RBF network is $\boldsymbol{x} = [x_1, x_2, \cdots, x_i]$ input of the network. Let RBF neural network have N hidden layers, the output vector of the hidden layer is h, $h = [h_1, h_2, \cdots, h_N]$, the calculation formula of h_i is:

$$h_j = \exp(\frac{\left\| \mathbf{x} - \varphi_j \right\|^2}{2\theta_j^2}), j = 1, \cdots, N \tag{24}$$

$$y_i = \sum_{j=1}^{N} p_{ij} h_j, j = 1, \cdots, N \tag{25}$$

Where, $\boldsymbol{x}$ is input value, $\boldsymbol{\varphi}_j$ is central vector of the JTH hidden layer neuron, $\boldsymbol{\varphi}_j = \left[\varphi_{1,j}, \varphi_{2,j}, \cdots, \varphi_{r,j}\right]$. N central vectors constitute a central vector matrix $\boldsymbol{\Phi} = [\varphi_1, \varphi_2, \cdots, \varphi_N]$, θ_j is the Gaussian RMS width value of the jth hidden layer neuron $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_N]^{\mathrm{T}}(\theta_j > 0)$, Y is final output of RBF network $Y = [y_1, y_2, \cdots, y_n]$, p_{ij} is transmission weights between neurons in the JTH hiding layer and the ITH output neuron and weight matrix is $\boldsymbol{p} = [p_1, p_2, \cdots, p_N]^T$.

In the GWO-RBF network, $\boldsymbol{\theta}$, $\boldsymbol{\Phi}$ and $\boldsymbol{p}$ are the wolf position vectors, and the optimal objective function is selected as follows:

$$E = \frac{1}{2n} \sum_{i=1}^{n} (y_i - y_{mi})^2 \tag{26}$$

Where, the number of training samples is N, y_{mi} is the expected output value of the ith $\boldsymbol{Y_m} = [y_{m1}, y_{m2}, \cdots, y_{mn}]$, y_i is the actual value of the ITH $\boldsymbol{Y} = [y_1, y_2, \cdots, y_n]$. The steps of chaotic sea clutter prediction by GWO-RBF network are as follows:

Step1: The network input data is reconstructed sea clutter data.
Step2: Compared to the expected value and output data.
Step3: Output data put in GWO.
Step4: Repeat Step1 $\sim$ Step3.until the end of the iteration.

4 Simulation and Modeling

The validity of the proposed GWO-RBF network for sea clutter prediction is verified through a group of comparative experiments. The first group of experiments used RBF neural network to predict the sequence, and the second group of experiments used GWO-RBF network to predict the sequence, and the data were all sea clutter data.

The sea clutter data came from IPIX radar data collected by McMaster University in Canada in 1993, which spanned 14 range cells. We select the data in one of the unit doors as the test data.

The experimental data is affected by a variety of errors, and the data should be preprocessed before analysis to reduce these errors and eliminate the influence of different attributes of data with different orders of magnitude.

Figure 2 shows the embedding dimension. The intersecting point of $E_1(d)$ and $E_2(d)$ is the embedding dimension calculated by CAO method, and $E_1(d)$ gradually

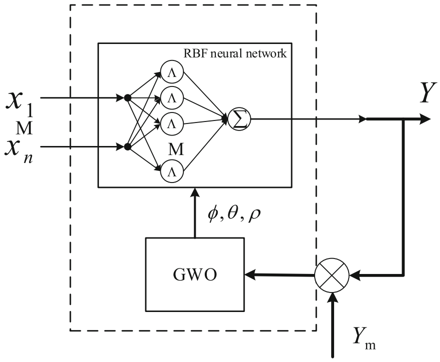


Fig. 1. RBF-GWO network structure

approaches to 1 with the increase of dimension, while $E_2(d)$ slowly changes around 1 with the increase of dimension. According to the figure, the embedding dimension is 8.

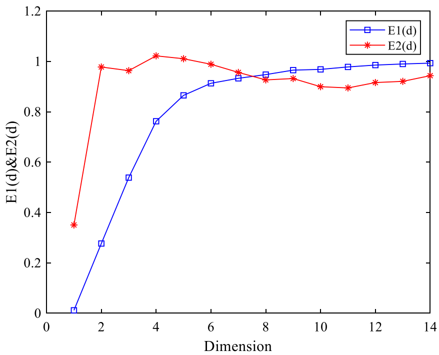
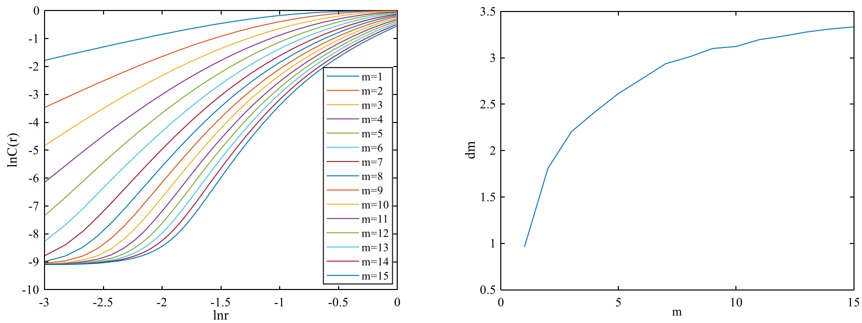


Fig. 2. Embedding dimension

Figure 3 shows the correlation dimensions under different embedded dimension change curve of the figure shows, with the increase of critical distance $log(r)$ correlation integral $log\,(C(r))$ increase gradually and tends to a stable value, the correlation integral every curve fitting, the slope is the correlation dimension of different number of embedded, correlation dimension drawing curve as shown in Fig. 3(a). As the embedding dimension increases, the correlation dimension is gradually becoming larger and tends to be stable. This stable value is the correlation dimension requested. In Fig. 3(b), the correlation dimension is approximately 4. When the image slope is a positive value, the correlation dimension is a chaotic invariant, which proves that one characteristic of sea clutter sequence is chaos.

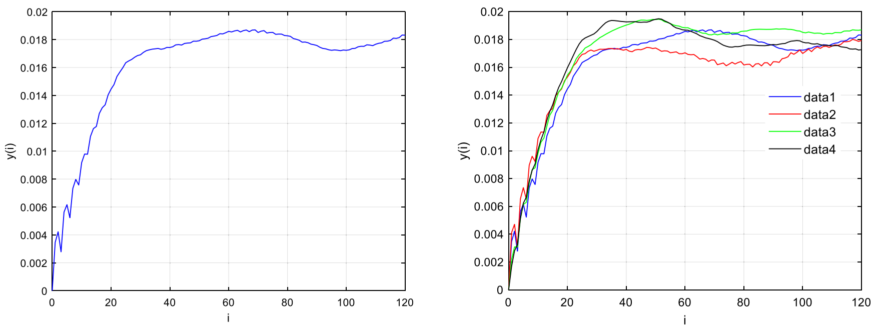
In this paper, we choose small data sets method as the method to calculate largest Lyapunov exponents, using formula (13)–(16) to calculate relationship between $\ln d_j(i)$ and $\lambda_1(i)$. The slope is calculated by the least square method. And then we get largest



(a).Correlation dimension change curve (b) Correlation dimension

Fig. 3. Correlation dimension

Lyapunov exponent is equal to 0.0181. In Fig. 4 (a), shown the largest Lyapunov exponent is 0.0181, and Fig. 4(b) is the Lyapunov exponent curve drawn by the sea clutter data of four different distances. As can be seen from the figure, the two graphs have certain upward fluctuations in the initial stage, which is caused by the chaotic superposition of the actual sea clutter books. But the overall trend is upward.



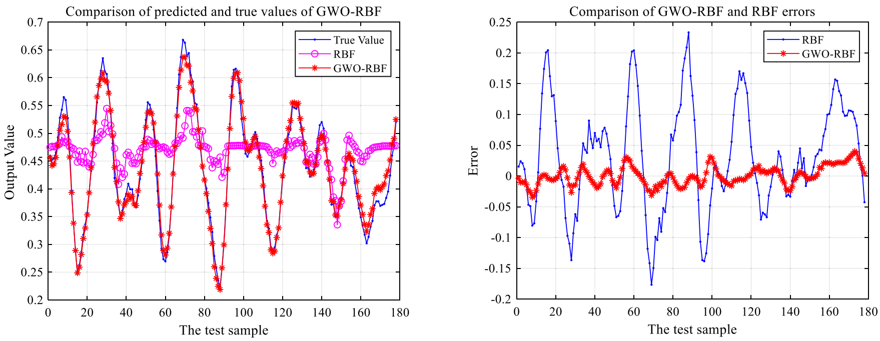
(a) Data1 Lyapunov exponents (b) four data Lyapunov exponents

Fig. 4. Lyapunov exponents

By combining the correlation dimension and the calculation results of the maximum A, it can be concluded that the clutter data has chaotic characteristics.

The reconstructed data were input to the input end of the neural network. The first 1800 data were selected as the test data, and the last 200 data were selected as the detection data.

Therefore, two sets of different results were obtained. Figure 5(a) is the comparison between the predicted and true sea clutter values of GWO-RBF network. The figure shows, Only using RBF neural network to predict sea clutter is not effective, but using GWO-RBF network predict sea clutter series, the predicted results are very close to the actual values.



(a). Predict comparison (b). Error comparison

Fig. 5. Prediction model and error

Figure 5(b) is the error comparison diagram of RBF and GWO-RBF network. According to the figure, the error of GWO-RBF network hovers around 0, while the error of RBF is a large fluctuation of neural network. In terms of error, the error of GWO-RBF network is smaller than that of RBF network. It is proved that the prediction result of GWO-RBF network is more accurate and the error is less when it is used in sea clutter prediction.

Table 1. Error analysis

	Mean square error	Root of mean square error	Mean absolute error	Mean absolute percentage error
Before	0.0083811	0.091548	0.072779	19.56%
After	4.7054e-05	0.0068596	0.005162	1.16%

Table 1 is the error recording table of before and after optimization. It can be seen from the table that the mean square error before optimization is much larger than after optimization, and the mean absolute error is also greater than the result after optimization.

5 Conclusion

This paper mainly verifies whether sea clutter has chaotic characteristics. First of all, by calculating delay time (τ) and embedding dimension (m), reconstructing phase space, The second, calculating correlation dimension of sea clutter by G-P algorithm. On the basis of the slope of image correlation dimension is greater than 0. The last calculating the largest Lyapunov exponent by small data sets method, and largest Lyapunov exponent is 0.0181, According to the calculated correlation dimension and the maximum Lyapunov exponent, it can be concluded that the clutter sequence has chaotic characteristics.

The gray wolf algorithm is combined with RBF neural network to predict the sea clutter sequence after phase space reconstruction. The simulation results show that the GWO-RBF network can effectively reduce the prediction error, the prediction result of sea clutter sequence is very close to the real value, and the prediction effect is obviously better than that of the RBF neural network.

References

1. Takens, F.: Detecting strange attractors in turbulence. In: Rand, D., Young, L.S. (eds.) Dynamical Systems and Turbulence, Warwick 1980. Lecture Notes in Mathematics, vol. 898, pp. 366–381. Springer, Berlin, Heidelberg (1981). https://doi.org/10.1007/BFb0091924
2. Callaghan, D., Burger, J., Mishra, A.K.: A machine learning approach to radar sea clutter suppression. In: IEEE Radar Conference, pp. 1222–1227 (2017)
3. Leung, H., Haykin, S.: Is there a radar clutter attractor? Appl. Phys. Lett. **56**(6), 593–595 (1990)
4. Simon, R.A., Kumar, P.V.: A nonlinear sea clutter analysis using chaotic system. In: 2013 Fourth International Conference on Computing, Communications and Networking Technologies (ICCCNT) (2013)
5. Haykin, S., Puthusserypady, S.: Chaotic dynamics of sea clutter. Chaos **7**(4), 777–802 (1997)
6. Leung, H.: Applying chaos to radar detection in an ocean environment: an experimental study. IEEE J. Oceanic Eng. **20**(1), 56–64 (1995)
7. Shen, Y., Li, G.: The chaotic neural network is used to predict the sea clutter signal. In: 2009 International Conference on Artificial Intelligence and Computational Intelligence, pp. 25–30 (2009)
8. Haykin, S., Puthusserypady, S.: Chaos, sea clutter, and neural networks. In: Conference Record of the Thirty-First Asilomar Conference on Signals, Systems and Computers, pp. 1224–1227 (1998)
9. Jayaprakash, A., Reddy, G.R., Prasad, N.S.S.R.K.: Small target detection within sea clutter based on fractal analysis. Procedia Technol. **24**, 988–995 (2016)
10. Pathak, J., et al.: Using machine learning to replicate chaotic attractors and calculate Lyapunov exponents from data. Chaos **27**(12) (2017)
11. Mirjalili, S., Mirjalili, S.M., Lewis, A.: Grey Wolf optimizer. Adv. Eng. Softw. **69**, 46–61 (2014)
12. Packard, N.H., Crutchfield, J.P.: Geometry from a time series. Phys. Rev. Lett. 712–716 (1980)
13. Cao, L.: Practical method for determining the minimum embedding dimension of a scalar time series. Phys. D Nonlinear Phenom. **110**(1) (1997)
14. Mirjalili, S.: How effective is the Grey Wolf optimizer in training multi-layer perceptrons. Appl. Intell. **43**(1), 150–161 (2015)