



Dynamics of a Single Population Model with Non-transient/Transient Impulsive Harvesting and Birth Pulse in a Polluted Environment

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Abstract. In this paper, we present a single population model with non-transient/transient impulsive harvesting and birth pulse in a polluted environment. The sufficient conditions for system permanence is presented.

Keywords: Single population system · Non-transient/transient impulses · Birth pulse · Permanence

1 Introduction

Many investigations [1–4] devoted into impulsive equations. Clack [5] has studied the logistic equation with optimal harvesting. The environmental toxicant decreases the carrying capacity in polluted environments [6, 7]. They are assumed that the inputting toxicant was continuous. Liu et al. [8] considered that the environmental toxicant is often emitted with regular pulse. In this paper, we do notation as $N = n\tau$ and $L = l\tau$.

2 The Model

In this work, we present

Supported by National Natural Science Foundation of China (11761019, 11361014).

$$\left\{ \begin{array}{l} \frac{dx}{dt} = -(d_1 + \beta_1 c_o)x + c_1 x^2, \quad t \in (N, (N+L)], \\ \Delta x = -\mu x, \\ \Delta c_0 = 0, \\ \Delta c_{e1} = 0, \\ \Delta c_{e2} = 0, \end{array} \right\} t = (N+L), \\
\left\{ \begin{array}{l} \frac{dx}{dt} = -(d_2 + E + \beta_2 c_o(t))x + c_2 x^2, \quad t \in ((N+L), (N+1)], \\ \Delta x = bx, \\ \Delta c_0 = 0, \\ \Delta c_{e1} = D(c_{e2} - c_{e1}) + \mu_1, \\ \Delta c_{e2} = D(c_{e1} - c_{e2}) + \mu_2, \end{array} \right\} t = (N+1), \\
\left\{ \begin{array}{l} \frac{dc_o}{dt} = f c_{e1} - (g + m)c_o, \\ \frac{dc_{e1}}{dt} = -h_1 c_{e1}, \\ \frac{dc_{e2}}{dt} = -h_2 c_{e2}, \end{array} \right\} t \in (N, (N+1)]. \tag{1}$$

The biological meanings of the varies and parameters can reference [8–10].

3 The Dynamics

For (1), one subsystem of (1) is

$$\left\{ \begin{array}{l} \frac{dx}{dt} = -d_1 x + c_1 x^2, \quad t \in (N, (N+L)], \\ \Delta x = -\mu x, \quad t = (N+L), \\ \frac{dx}{dt} = -(d_2 + E)x + c_2 x^2, \quad t \in ((N+L), (N+1)], \\ \Delta x = bx, \quad t = (N+1). \end{array} \right. \tag{2}$$

Integrating (2), we get

$$x(t) = \begin{cases} \frac{d_1 x(N^+) e^{-d_1(t-N)}}{d_1 + c_1 x(N^+) (1 - e^{-d_1(t-N)})}, & t \in (N, (N+L)], \\ \frac{(d_2 + E)x((N+l)^+) e^{-(d_2+E)(t-N)}}{(d_2 + E) + c_2 x((N+l)^+) (1 - e^{-(d_2+E)(t-N)})}, & t \in ((N+L), (N+1)]. \end{cases} \tag{3}$$

Stroboscopic map for (2) is

$$x((n+1)\tau^+) = \frac{M_1}{M_2}, \quad (4)$$

where $M_1 = (1+b)(1-\mu)d_1(d_2+E)e^{-[d_1l+(d_2+E)(1-l)]\tau}x(N^+)$, $M_2 = d_1(d_2+E) + [(d_2+E)c_1(1-e^{-d_1l}) + (1-\mu)d_1c_2e^{-d_1l\tau}(1-e^{-(d_2+E)(1-l)\tau})]x(N^+)$.

We rewrite (4) as

$$x((N+1)^+) = \frac{A_0x(N^+)}{d_1(d_2+E) + B_0x(N^+)}. \quad (5)$$

where

$$A_0 = (1+b)(1-\mu)d_1(d_2+E)e^{-[d_1l+(d_2+E)(1-l)]\tau},$$

$$B_0 = (d_2+E)c_1(1-e^{-d_1l}) + (1-\mu)d_1c_2e^{-d_1l\tau}(1-e^{-(d_2+E)(1-l)\tau}).$$

We get points $G_1(0)$ and $G_2(x^*)$ of (4), and

$$x^* = \frac{d_1(d_2+E)[(1+b)(1-\mu)e^{-(d_1l+(d_2+E)(1-l))\tau} - 1]}{(d_2+E)c_1(1-e^{-d_1l}) + (1-\mu)d_1c_2e^{-d_1l\tau}(1-e^{-(d_2+E)(1-l)\tau})}, \quad (6)$$

$$(1+b)(1-\mu)e^{-(d_1l+(d_2+E)(1-l))\tau} > 1.$$

Condition $(1+b)(1-\mu)e^{-(d_1l+(d_2+E)(1-l))\tau} < 1$ is made as (C_1) , and condition $(1+b)(1-\mu)e^{-(d_1l+(d_2+E)(1-l))\tau} > 1$ is made as (C_2) .

Theorem 1. *i)* Suppose (C_1) holds $G_1(0)$ of (4) is globally asymptotically stable;

ii) Suppose (C_2) holds, $G_2(x^*)$ is globally asymptotically stable.

Proof. Marking x^n as $x(n\tau^+)$, we rewrite (4) as

$$x_{n+1} = F(x_n) = \frac{Ax_n}{d_1(d_2+E) + Bx_n}. \quad (7)$$

i) Suppose (C_1) holds, then,

$$\frac{dF(x)}{dx} \Big|_{x=0} = (1+b)(1-\mu)e^{-(d_1l+(d_2+E)(1-l))\tau} < 1. \quad (8)$$

Then, locally stable $G_1(0)$ exists, Furthermore, it is globally asymptotically stable.

ii) Suppose (C_2) holds, concerning to $G_1(0)$, we get

$$\begin{aligned} \frac{dF(x)}{dx} \Big|_{x=0} &= \frac{Ad_1(d_2+E)}{[d_1(d_2+E) + Bx]^2} \Big|_{x=0} \\ &= \frac{A}{d_1(d_2+E)}. \\ &= (1+b)(1-\mu)e^{-(d_1l+(d_2+E)(1-l))\tau} > 1, \end{aligned} \quad (9)$$

then, $G_1(0)$ is unstable.

Concerning $G_2(x^*)$, we also get

$$\begin{aligned} \frac{dF(x)}{dx} \Big|_{x=x^*} &= \frac{Ad_1(d_2 + E)}{[d_1(d_2 + E) + Bx]^2} \Big|_{x=x^*} \\ &= \frac{1}{(1+b)(1-\mu)e^{-(d_1l+(d_2+E)(1-l))\tau}} < 1. \end{aligned} \quad (10)$$

Then, locally stable $G_2(x^*)$ exists. Furthermore, it is globally asymptotically stable.

Similarly with Reference [6], the following lemma can easily be proved.

Theorem 2. *i)* Suppose (C_1) holds, 0 of (1) is globally asymptotically stable; *ii)* Suppose (C_2) holds, $\widetilde{x(t)}$ of (1) is globally asymptotically stable, and

$$\widetilde{x(t)} = \begin{cases} \frac{d_1 x^* e^{-d_1(t-N)}}{d_1 + c_1 x^* (1 - e^{-d_1(t-N)})}, t \in (N, (N+L)], \\ \frac{(d_2 + E)x^{**} e^{-(d_2+E)(t-N)}}{(d_2 + E) + c_2 x^{**} (1 - e^{-(d_2+E)(t-N)})}, t \in ((N+L), (N+1)]. \end{cases} \quad (11)$$

here x^* is by (6), and x^{**} is by $x^{**} = \frac{(1-\mu_1)d_1 x^* e^{-d_1 l \tau}}{d_1 + c_1 x^* (1 - e^{-d_1 l \tau})}$.

For (1), another subsystem of system (1) is obtained as

$$\left. \begin{cases} \frac{dc_o}{dt} = fc_{e_1} - (g+m)c_o, \\ \frac{dc_{e1}}{dt} = -h_1 c_{e1}, \\ \frac{dc_{e2}}{dt} = -h_2 c_{e2}, \end{cases} \right\} t \neq N, \quad (12)$$

$$\left. \begin{cases} \Delta c_o = 0, \\ \Delta c_{e1} = D(c_{e2} - c_{e1}) + \mu_1, \\ \Delta c_{e2} = D(c_{e1} - c_{e2}) + \mu_2, \end{cases} \right\} t = N.$$

Integrating (12), we get

$$\left\{ \begin{aligned} \widetilde{c}_o &= c_o(N^+)e^{-(g+m)(t-N)} \\ &\quad + \frac{fc_{e1}(N^+)(1 - e^{-(h-g-m)(t-N)})}{h-g-m}, t \in (N, (N+1)], \\ \widetilde{c}_{e1} &= c_{e1}(N^+)e^{-h_1(t-N)}, t \in (N, (N+1)], \\ \widetilde{c}_{e2} &= c_{e2}(N^+)e^{-h_2(t-N)}, t \in (N, (N+1)]. \end{aligned} \right. \quad (13)$$

The stroboscopic map of (12) is

$$\left\{ \begin{array}{l} c_o((N+1)^+) = c_o(N^+)e^{-(g+m)\tau} \\ \quad + \frac{fc_{e1}(N^+)(1-e^{-(h-g-m)\tau})}{h-g-m}, t \in (N, (N+1)], \\ c_{e1}((N+1)^+) = (1-D)c_{e1}(N^+)e^{-h_1\tau} + Dc_{e2}(N^+)e^{-h_2\tau} + \mu_1, \\ \quad t \in (N, (N+1)], \\ c_{e2}((N+1)N^+) = Dc_{e1}(N^+)e^{-h_1\tau} + (1-D)c_{e2}(N^+)e^{-h_2\tau} + \mu_2 \\ \quad t \in (N, (N+1)]. \end{array} \right. \quad (14)$$

A unique fixed point of (14) is

$$\left\{ \begin{array}{l} c_o^* = \frac{f[\mu_1(1-(1-D)e^{-h_2\tau}) + \mu_2De^{-h_2\tau}](1-e^{-(h-g-m)\tau})}{M_3}, \\ c_{e1}^* = \frac{\mu_1[1-(1-D)e^{-h_2\tau}] + \mu_2De^{-h_2\tau}}{M_4}, \\ c_{e2}^* = \frac{\mu_2[1-(1-D)e^{-h_1\tau}] + \mu_1De^{-h_1\tau}}{M_4}, \end{array} \right. \quad (15)$$

where $M_3 = (h-g-m)(1-e^{-(h-g-m)\tau})[(1-(1-D)e^{-h_1\tau})(1-(1-D)e^{-h_2\tau}) - D^2e^{-(h_1+h_2)\tau}]$ and $M_4 = [1-(1-D)e^{-h_1\tau}][1-(1-D)e^{-h_2\tau}] - D^2e^{-(h_1+h_2)\tau}$.

Writing (14) as a map, and defining it as $F: R_+^3 \rightarrow R_+^3$

$$\left\{ \begin{array}{l} F_1(c) = c_o(N^+)e^{-(g+m)\tau} \\ \quad + \frac{fc_{e1}(N^+)(1-e^{-(h-g-m)\tau})}{h-g-m}, t \in (N, (N+1)], \\ F_2(c) = (1-D)c_{e1}(N^+)e^{-h_1\tau} + Dc_{e2}(N^+)e^{-h_2\tau} + \mu_1, \\ F_3(c) = Dc_{e1}(N^+)e^{-h_1\tau} + (1-D)c_{e2}(N^+)e^{-h_2\tau} + \mu_2. \end{array} \right. \quad (16)$$

Lemma 3. Suppose $D > \frac{1}{2}$ holds, $F(c_o^*, c_{e1}^*, c_{e2}^*)$ of (16) is globally asymptotically stable.

Proof. We mark $(c_o^n, c_{e1}^n, c_{e2}^n)$ as $(c_o(n\tau^+), c_{e1}(n\tau^+), c_{e2}(n\tau^+))$. Rewriting linearity of (16) as

$$\begin{pmatrix} c_o^{n+1} \\ c_{e1}^{n+1} \\ c_{e2}^{n+1} \end{pmatrix} = M \begin{pmatrix} c_o^n \\ c_{e1}^n \\ c_{e2}^n \end{pmatrix}. \quad (17)$$

Obviously, the stabilities of $F(c_o^*, c_{e1}^*, c_{e2}^*)$ is by eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of M , which are less than 1.

Suppose $D > \frac{1}{2}$ holds, $0 < e^{-h_i} < 1 (i = 1, 2)$, $F(c_o^*, c_{e1}^*, c_{e2}^*)$ exists, and

$$M = \begin{pmatrix} e^{-(g+m)\tau} \frac{f(1-e^{-(h-g-m)\tau})}{h-g-m} & 0 \\ 0 & (1-D)e^{-h_1\tau} & De^{-h_2\tau} \\ 0 & De^{-h_1\tau} & (1-D)e^{-h_2\tau} \end{pmatrix}. \quad (18)$$

For

$$\lambda_1 = e^{-(g+m)\tau} < 1,$$

$$\begin{aligned} \lambda_2 &= \frac{(1-D)(e^{-h_1\tau} + e^{-h_2\tau}) + \sqrt{[(1-D)(e^{-h_1\tau} + e^{-h_2\tau})]^2 - 4(1-2D)e^{-(h_1+h_1)\tau}}}{2} \\ &= \frac{(1-D)(e^{-h_1\tau} + e^{-h_2\tau}) + \sqrt{[D(e^{-h_1\tau} + e^{-h_2\tau})]^2 + (1-2D)(e^{-(h_1-h_1)\tau})^2}}{2} \\ &< \frac{e^{-h_1\tau} + e^{-h_2\tau}}{2} < 1, \end{aligned}$$

$$\begin{aligned} \lambda_3 &= \frac{(1-D)(e^{-h_1\tau} + e^{-h_2\tau}) - \sqrt{[(1-D)(e^{-h_1\tau} + e^{-h_2\tau})]^2 - 4(1-2D)e^{-(h_1+h_1)\tau}}}{2} \\ &= \frac{(1-D)(e^{-h_1\tau} + e^{-h_2\tau}) - \sqrt{[(1-D)(e^{-h_1\tau} - e^{-h_2\tau})]^2 + 4D^2e^{-(h_1+h_1)\tau}}}{2} \\ &< \frac{(1-D) \min\{e^{-h_1\tau}, e^{-h_2\tau}\}}{2} < 1. \end{aligned}$$

That is, $\lambda_i < 1 (i = 1, 2, 3)$, then, $F(c_o^*, c_{e1}^*, c_{e2}^*)$ is locally stable. Then, it is globally asymptotically stable.

Similar to Reference [9], we get

Lemma 4. (12) has a unique globally asymptotically stable periodic solution $(\widetilde{c}_o, \widetilde{c}_{e1}, \widetilde{c}_{e2})$ of (12), and

$$\begin{cases} \widetilde{c}_o = c_o^* e^{-(g+m)(t-N)} \\ \quad + \frac{f c_{e1}^* (1 - e^{-(h-g-m)(t-N)})}{h-g-m}, t \in (N, (N+1)], \\ \widetilde{c}_{e1} = c_{e1}^* e^{-h_1(t-N)}, t \in (N, (N+1)], \\ \widetilde{c}_{e2} = c_{e2}^* e^{-h_2(t-N)}, t \in (N, (N+1)]. \end{cases} \quad (19)$$

where $c_o^*, c_{e1}^*, c_{e2}^*$ are by (15).

Remark 5. $m_o \leq c_o(t) \leq M_o$, $m_{e1} \leq c_{e1}(t) \leq M_{e1}$ and $m_{e2} \leq c_{e2}(t) \leq M_{e2}$ hold for t large enough, and $m_o = c_o^* e^{-(g+m)\tau} + \frac{fc_{e1}^*(1-e^{-(h-g-m)\tau})}{h-g-m} - \varepsilon > 0$, $M_o = c_o^* + \frac{fc_{e1}^*}{(h-g-m)} + \varepsilon$, $m_{e1} = c_{e1}^* e^{-h_1\tau} - \varepsilon > 0$, $M_{e1} = c_{e1}^* + \varepsilon > 0$, $m_{e2} = c_{e2}^* e^{-h_2\tau} - \varepsilon > 0$ and $M_{e2} = c_{e2}^* + \varepsilon > 0$.

Thinking (1), we get

$$\left\{ \begin{array}{l} \frac{dx}{dt} < -d_1x + c_1x^2, \\ \qquad \qquad \qquad t \in (N, (N+L)], \\ \Delta x = -\mu x, \\ \qquad \qquad \qquad t = (N+L), \\ \frac{dx}{dt} < -(d_2 + E)x + c_2x^2, \\ \qquad \qquad \qquad t \in ((N+L), (N+1)], \\ \Delta x = bx, \\ \qquad \qquad \qquad t = (N+1), \end{array} \right. \quad (20)$$

and

$$\left\{ \begin{array}{l} \frac{dx}{dt} > -(d_1 + \beta_1 M_o)x + c_1x^2, \\ \qquad \qquad \qquad t \in (N, (N+L)], \\ \Delta x = -\mu x, \\ \qquad \qquad \qquad t = (N+L), \\ \frac{dx}{dt} > -(d_2 + E + \beta_2 M_o)x + c_2x^2, \\ \qquad \qquad \qquad t \in ((N+L), (N+1)], \\ \Delta x = bx, \\ \qquad \qquad \qquad t = (N+L), \end{array} \right. \quad (21)$$

with their comparative impulsive systems

$$\left\{ \begin{array}{l} \frac{dx_1}{dt} = -d_1x_1 + c_1x_1^2, \\ \qquad \qquad \qquad t \in (N, (N+L)], \\ \Delta x_1 = -\mu x_1, \\ \qquad \qquad \qquad t = (N+L), \\ \frac{dx_1}{dt} = -(d_2 + E)x_1 + c_2x_1^2, \\ \qquad \qquad \qquad t \in ((N+L), (N+L)], \\ \Delta x_1 = bx_1, \\ \qquad \qquad \qquad t = (N+1), \end{array} \right. \quad (22)$$

and

$$\left\{ \begin{array}{l} \frac{dx_2}{dt} = -(d_1 + \beta_1 M_o)x_2 + c_1 x_2^2, \\ \qquad \qquad \qquad t \in (N, (N + L)], \\ \Delta x_2 = -\mu x_2, \\ \qquad \qquad \qquad t = (N + L), \\ \frac{dx_2}{dt} = -(d_2 + E + \beta_2 M_o)x_2 + c_2 x_2^2, \\ \qquad \qquad \qquad t \in ((N + L), (N + 1)], \\ \Delta x_2 = b x_2, \\ \qquad \qquad \qquad t = (N + 1)\tau. \end{array} \right. \quad (23)$$

Marking $(1 + b)(1 - \mu)e^{-[(d_1 + \beta_1 M_o)l + (d_2 + E + \beta_2 M_o)(1 - l)]\tau} < 1$ as (C_3) and $(1 + b)(1 - \mu)e^{-[(d_1 + \beta_1 M_o)l + (d_2 + E + \beta_2 M_o)(1 - l)]\tau} > 1$ as (C_4) .

Similar to Theorem 2, we can get

Theorem 6. *i)* Suppose (C_3) holds, 0 of (23) is globally asymptotically stable;
ii) Suppose (C_4) holds, $\widetilde{x_2}(t)$ of (23) is globally asymptotically stable, and

$$\widetilde{x_2} = \left\{ \begin{array}{l} \frac{d_1 x^* e^{-d_1(t-N)}}{(d_1 + \beta_1 M_o) + c_1 x^*(1 - e^{-(d_1 + \beta_1 M_o)(t-N)})}, \\ \qquad \qquad \qquad t \in (N, (N + L)], \\ \frac{(d_2 + E + \beta_2 M_o)x^{**} e^{-(d_2 + E + \beta_2 M_o)(t-N)}}{(d_2 + E + \beta_2 M_o) + c_2 x^{**}(1 - e^{-(d_2 + E + \beta_2 M_o)(t-N)})}, \\ \qquad \qquad \qquad t \in ((N + l), (N + 1)]. \end{array} \right. \quad (24)$$

here x_2^* is by

$$x_2^* = \frac{M_4}{M_5}, (1 + b)(1 - \mu)e^{-[(d_1 + \beta_1 M_o)l + (d_2 + E + \beta_2 M_o)(1 - l)]\tau} > 1, \quad (25)$$

where $M_4 = (d_1 + \beta_1 M_o)(d_2 + E + \beta_2 M_o)[(1 + b)(1 - \mu)e^{-[(d_1 + \beta_1 M_o)l + (d_2 + E + \beta_2 M_o)(1 - l)]\tau} - 1]$, $M_5 = (d_2 + E + \beta_2 M_o)c_1(1 - e^{-(d_1 + \beta_1 M_o)l}) + (1 - \mu)(d_1 + \beta_1 M_o)c_2 e^{-(d_1 + \beta_1 M_o)l\tau}(1 - e^{-(d_2 + E + \beta_2 M_o)(1 - l)\tau})$ and $x_2^{**} = \frac{(1 - \mu_1)(d_1 + \beta_1 M_o)x_2^* e^{-(d_1 + \beta_1 M_o)l\tau}}{(d_1 + \beta_1 M_o) + c_1 x_2^*(1 - e^{-(d_1 + \beta_1 M_o)l\tau})}$.

Theorem 7. *i)* Suppose (C_1) and (C_3) hold, $(0, \widetilde{c_o}, \widetilde{c_{e1}}, \widetilde{c_{e2}})$ of (1) is globally asymptotically stable;

ii) Suppose (C_2) and (C_4) hold, (1) is permanent.

Proof. *i)* From condition (C_1) , (22), Theorem 2, and the theorem of the impulsive equation, we can have $x(t) \leq x_1(t) = x_1(t) \rightarrow 0$ as $t \rightarrow +\infty$. From condition (C_3) (23), Theorem 6, and the theorem of the impulsive differential equation, we can have $x(t) \geq \widetilde{x_2}(t) \rightarrow 0$ as $t \rightarrow +\infty$. So, the globally asymptotically stable periodic solution $(0, \widetilde{c_o}(t), \widetilde{c_{e1}}(t), \widetilde{c_{e2}}(t))$ exists.

ii) From condition (C_2) , (22), Theorem 2, we can have

$$x(t) \leq x_1(t) \leq \widetilde{x_1(t)} - \varepsilon \leq \frac{d_1 x^* e^{-d_1 l \tau}}{d_1 + c_1 x^* (1 - e^{-d_1 l \tau})} + \frac{(d_2 + E) x^{**} e^{-(d_2 + E)(1-l)\tau}}{(d_2 + E) + c_2 x^{**} (1 - e^{-(d_2 + E)(1-l)\tau})} - \varepsilon \triangleq m_1.$$

From condition (C_4) , (23), Theorem 6, we can easily obtain

$$x(t) \geq x_2(t) \geq \widetilde{x_2(t)} - \varepsilon \geq \frac{d_1 x^* e^{-d_1 l \tau}}{(d_1 + \beta_1 M_o) + c_1 x^* (1 - e^{-(d_1 + \beta_1 M_o) l \tau})} + \frac{(d_2 + E + \beta_2 M_o) x^{**} e^{-(d_2 + E + \beta_2 M_o)(1-l)\tau}}{(d_2 + E + \beta_2 M_o) + c_2 x^{**} (1 - e^{-(d_2 + E + \beta_2 M_o)(1-l)\tau})} - \varepsilon \triangleq m_2.$$

From above discussion and Remark 5, we can have $m_1 \leq x(t) \leq m_2$, $m_o \leq c_o(t) \leq M_o$, $m_{e1} \leq c_{e1}(t) \leq M_{e1}$, $m_{e2} \leq c_{e2}(t) \leq M_{e2}$. This completes the proof.

4 Discussion

In this work, we consider we construct a single population model with non-transient/transient impulsive harvesting and birth pulse in a polluted environment. From the conditions of Theorem 7, we can deduce that the transient impulsive harvesting amount μ has a threshold μ^* . If $\mu > \mu^*$, the globally asymptotically stable $(0, \widetilde{c_o(t)}, \widetilde{c_{e1}(t)}, \widetilde{c_{e2}(t)})$ exists. If $\mu < \mu^*$, system permanence. We can also obtain the threshold l^* for non-transient impulsive harvesting intervals. Our results present a biological management researching basis in a polluted environment.

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