




Dynamic Beam Optimization and Interference Mitigation Methods for Multi-beam Satellite Systems

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Abstract. In multi-beam satellite system, dynamically adjusting the beams to meet the communication demands of ground users, adapt to changes in user distribution, and consider satellite hardware limitations is a technological challenge. To end this, this paper proposes an adaptive beamwidth design scheme. The scheme utilizes an enhanced K-means (called eK-means) clustering algorithm for user grouping and assigns suitable beams to each group, ensuring that all users within the same group are positioned within the primary radiation region of the beam. Also, a greedy frequency allocation strategy is introduced to mitigate inter-group interference. Given its capacity to dynamically tailor the number and coverage of beams based on user distribution and demands of communication, the proposed scheme greatly bolsters the flexibility of multi-beam satellite systems. Simulation results showcase its exceptional performance.

Keywords: Multi-beam satellite · User grouping · Beamwidth optimization

1 Introduction

With the advancement of future wireless networks, there is an increasing demand for broad coverage, high throughput, and ubiquitous large-scale connectivity. Under these circumstances, satellite communication has emerged as an ideal complement and extension to terrestrial communication networks, as it offers broader coverage and cost-effective internet access services to a large number of terminal devices [1]. Multi-beam satellite communication technology, as a crucial area of study, significantly enhances the flexibility and adaptability of satellite communication by employing spot beams to directly serve ground users [2].

To enhance communication capacity, multi-beam satellite communication systems implement frequency reuse techniques, including full frequency reuse

(FFR) and four color frequency reuse (FcFR) [3]. FFR technology allows for the reuse of frequencies within the same area, thereby maximizing the utilization of limited spectrum resources [4]. Nevertheless, this approach requires complex signal processing techniques to reduce the mutual interference between different beams. In contrast, FcFR technology effectively reduces the interference between beams by using different frequencies or polarizations in adjacent beams, thereby enhancing system capacity [5]. To implement these reuse technologies, satellites are typically equipped with multi-beam antennas such as phased array antennas and digital active antennas [6]. These antennas can generate multiple beams, each with its specific mainlobe and sidelobes. Users situated in the mainlobe encounter consistent service, whereas the sidelobes might introduce interference or diminish signal quality. Phased array antennas can adjust the direction and width of the beams. The wider the beam, the more users can be served, but the received power decreases. Therefore, a trade-off is needed between the number of users being served and the signal-to-interference-plus-noise ratio (SINR) of the beams, while also considering the gain characteristics of the mainlobe and sidelobes, to ensure communication performance.

To address the challenge of trade-off in multi-beam satellite communication systems, user grouping strategies and beam optimization techniques are utilized. Xu et al. proposed a method that utilizes deep reinforcement learning algorithms to optimize beam hopping scheduling and coverage control, in order to address the challenge of matching ground cell communication demands with dynamically beam transmission capabilities [7]. Liu et al. performed user grouping based on location information and optimized the direction and width of beams using time-division technology to maximize the average data rate of satellites [8]. In [9], the authors address the heterogeneous service requirements under satellite systems by employing two different sizes of beams, which are adjusted according to the users' demands dynamically. Honnaiah et al. proposed an adaptive multi-beam strategy for high-throughput satellite systems to address the uneven distribution of users and fluctuating service demands [10]. The aforementioned works did not comprehensively consider the balance between beamwidth, user requirements, and interference factors.

In this paper, we propose an adaptive beamwidth design scheme. The scheme groups users based on users' location information rather than their channel state information. Optimization of beam direction and beamwidth is conducted for each group to enhance system throughput and guarantee user Quality of Service (QoS). By employing FcFR, each group of users share the same time-frequency resources. We formulate an optimization problem that considers the trade-off among system throughput, user received SINR and satellite hardware constraints, which is known to be an NP-hard problem. To address this, we develop an algorithm that combines enhanced K-means (eK-means) clustering and greedy frequency allocation. The algorithm decomposes the problem into two stages: In the initial stage, due to the difficulties in directly acquiring inter-beam interference (IBI), we employ an iterative process with the objective of maximizing total received power. This process helps determine the optimal user

grouping and beam radius for different group number. In the second stage, we utilize a graph-based greedy frequency allocation method to find the optimal frequency coloring solution for various group numbers. Finally, we compare and evaluate different designs to determine the optimal beamwidth that satisfies the given constraints. The effectiveness of the proposed scheme is validated and demonstrated through simulation results.

2 System Model and Problem Formulation

2.1 System Setup

This paper focuses on a downlink satellite communication system that comprises a satellite catering to K ground users indexed by $\mathcal{K} = \{1, 2, \dots, K\}$ as shown in Fig. 1. Assuming the satellite is equipped with phased array antennas, it can provide flexible beam direction and beamwidth. The satellite payload generates a total of N beams on the ground. Here, N is not a fixed constant, but a variable, and $1 \leq N \leq K$. All ground users are situated within a common horizontal plane, and their positions are known. Denote \mathbf{u}_k and \mathbf{z}_k as the coordinates of the k th user and the coordinates of the beam center, respectively.

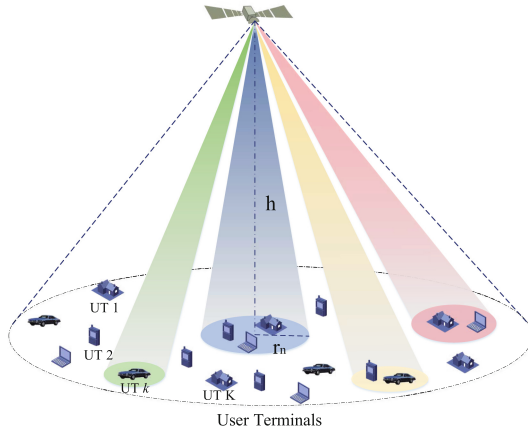


Fig. 1. System model.

Due to the inability of a single beam's mainlobe to cover all users simultaneously, it is necessary to group users and utilize multiple beams to provide services to the users. In this paper, the number of beams is equal to the number of user groups. We define a set of indicator variables, denoted as $b_{k,n}$, to describe the grouping of each user. Specifically, if the k th user belongs to the n th group, the value of $b_{k,n}$ is set to 1; otherwise, $b_{k,n}$ is set to 0. Each user can only be served by one beam, i.e.,

$$\sum_{n=1}^N b_{k,n} = 1, \quad \forall k. \quad (1)$$

Given the assumption that the beam coverage area has a circular shape, it is required that all users within the same group are positioned within the mainlobe of the beam. Then, we have

$$\sum_{n=1}^N b_{k,n} \|\mathbf{u}_k - \mathbf{z}_n\| \leq \sum_{n=1}^N b_{k,n} r_n, \quad \forall k, \quad (2)$$

where r_n is the coverage radius of n th beam.

In order to improve the communication throughput, this paper employs FcFR. Due to the reuse of frequencies in the downlink, the implementation of the multi-beam technique can result in IBI caused by the overlapping sidelobes in the radiation pattern.

Ground users employ omnidirectional antennas, and each user has the same gain of received antenna which is denoted as G^r . When the user is expected to locate at the center of the target beam, the received signal power of the expected user in the n th beam is

$$P_n^r = \frac{1}{2} \alpha G_{\text{tx}}^m(\phi_n) G^r P_n^t \left(\frac{\lambda}{4\pi l} \right)^2, \quad (3)$$

where α is the power attenuation, G_{tx}^m is transmitter antenna gain, ϕ_n is the angle of the expected user from the antenna axis of the beam, P_n^t is the transmission power of the n th beam, λ is the wavelength, and l is the path length between the user and the satellite. The relationship between the offaxis angle θ and transmit antenna gain is denoted as [11].

$$G_{\text{tx}}(\theta) = \begin{cases} G_m, & \theta < \theta_b \\ G_m - 3(\theta/\theta_b)^2, & \theta_b \leq \theta \leq a\theta_b \\ G_m + L_s, & a\theta_b \leq \theta \leq b\theta_b \\ \max\{\delta, 0\}, & \text{else,} \end{cases} \quad (4)$$

where $a = 2.88$, $b = 6.32$, $L_s = -25$ dB, θ_b denotes the angle corresponding to the 3dB beamwidth and $\delta = G_m + L_s + 20 - 25 \log_{10} \left(\frac{\theta}{\theta_b} \right)$. G_m represents the maximum gain of the satellite's transmit antenna, and its form is given by [12]

$$G_m = 10 \log_{10} \left[4.93 \left(\frac{70}{\theta_b} \right)^2 \right]. \quad (5)$$

The beam coverage radius is calculated by

$$r = h \tan \theta_b, \quad (6)$$

where h is the height of the satellite. The interference signal power affected by IBI of the n th beam is

$$P_n^I = \sum_{m=1}^M \frac{1}{2} \alpha G_{\text{tx}}^m(\phi_m) G^r P_n^t \left(\frac{\lambda}{4\pi l} \right)^2, \quad (7)$$

where M denotes the total number of beams of the same color and ϕ_m represents the angle at which the expected user deviates from the axis of the n th beam of the same color. Then, the SINR of the n th beam is expressed as

$$\gamma_n = \frac{P_n^r}{N_0 W_n + P_n^I}, \quad (8)$$

where N_0 is the noise power spectral density and W_n represents the bandwidth allocated by the target beam n . So, the data rate of the n th beam can be expressed as

$$R_n = W_n \log_2(1 + \gamma_n). \quad (9)$$

2.2 Problem Formulation

Our goal is to comprehensively consider the limitations imposed by satellite hardware, dynamically adjust beams to meet the communication demands of ground users, adapt to changes in user distribution, and ultimately maximize system throughput. The problem is formulated as

$$\begin{aligned} \mathcal{P}_1 : & \max_{b,z,r} \sum_{n=1}^N R_n \\ \text{s.t. } & C_1: \sum_{n=1}^N b_{k,n} \|\mathbf{u}_k - \mathbf{z}_n\| \leq \sum_{n=1}^N b_{k,n} r_n, \quad \forall k \\ & C_2: \sum_{n=1}^N b_{k,n} = 1, \quad \forall k \\ & C_3: b_{k,n} \in \{0, 1\}, \quad \forall k, n \\ & C_4: r_n \geq 0.443 \frac{\lambda}{D} h \\ & C_5: R_n \geq R_{th} \\ & C_6: \sum_{n=1}^N P_n^t \leq P_{total} \end{aligned} \quad (10)$$

where D represents the diameter of the satellite phased array antenna, R_{th} denotes the minimum data rates and P_{total} is the total power of the satellite. The constraint C_4 means that the radius of the beam cannot be smaller than the beam resolution. Constraint C_5 indicates that the limitations for ensuring the minimum data rate for user QoS. C_6 represents that the total power allocated to all beams must not exceed the overall system power. The proposed problem \mathcal{P}_1 is a non-convex mixed integer programming problem, and its structure is similar to the problem discussed in [8], falling under the category of NP-hard.

3 Joint User Grouping and Dynamic Beam Resource Allocation Scheme

To solve problem \mathcal{P}_1 , we adopt a staged strategy. Firstly, we perform a user grouping and determine the optimal beam coverage radius for each group to maximize the total received power. Then, we proposed a frequency allocation scheme based on graph theory, aiming to reduce IBIs. Through this approach, we aim to enhance the system throughput while ensuring the communication quality.

3.1 User Grouping Based on eK-Means Algorithm

Due to the inability to obtain the IBI, we adopted an alternative strategy that maximizes the total received power of all beams to improve system throughput. To achieve this, we fixed the total number of beams N in problem \mathcal{P}_1 and proceeded with the problem-solving process. Since $1 \leq N \leq K$, we perform a one dimensional exhaustive search over the range $[1, K]$ to find the optimal solution. Therefore, problem \mathcal{P}_1 can be transformed into K subproblems. Specifically, when N is fixed, we need to solve the following subproblem

$$\begin{aligned} \mathcal{P}_2 : \max \sum_{n=1}^N P_n^r \\ \text{s.t. constrains } C_1, C_2, C_3, C_4. \end{aligned} \quad (11)$$

According to Eqs. (3)–(6), the strength of beam received power is closely related to the beam coverage radius. Therefore, a key aspect in solving problem \mathcal{P}_2 is to perform appropriate user grouping and select suitable beam coverage radius for each group.

The K-means algorithm has emerged as a highly favored data processing technique in recent years, known for its straightforward implementation and low computational complexity. However, the performance of this algorithm can be significantly influenced by the initial selection of cluster centers, which may lead to a local optimum instead of a global optimum [13]. The K-means++ algorithm enhances clustering quality by refining the selection of initial centroids based on the K-means algorithm. Nevertheless, this algorithm has higher complexity and its effectiveness is limited on certain datasets [14]. Based on these limitations, this paper proposes eK-means algorithm for user grouping.

Specifically, let $\mathcal{U} = \{\mathbf{u}_k\}, k = 1, \dots, K$ be the set of user coordinates and the distance between \mathbf{u}_k and \mathcal{C} is defined as $d(\mathbf{u}_k, \mathcal{C}) = \min_{\mathbf{c}_y \in \mathcal{C}} \|\mathbf{u}_k - \mathbf{c}_y\|$, where $\mathcal{C} = \{\mathbf{c}_1, \dots, \mathbf{c}_Y\}$ is a set of points and $\|\cdot\|$ denotes the Euclidean distance. Then, we define the cost of \mathcal{U} relative to \mathcal{C} as

$$\Phi_{\mathcal{U}}(\mathcal{C}) = \sum_{\mathbf{u}_k \in \mathcal{U}} d^2(\mathbf{u}_k, \mathcal{C}) = \sum_{\mathbf{u}_k \in \mathcal{U}} \min_{y \in \{1, \dots, Y\}} \|\mathbf{u}_k - \mathbf{c}_y\|^2 \quad (12)$$

Then, for the proposed algorithm, we first randomly select a user from \mathcal{U} as the initial center and add to the user center set \mathcal{C} . Subsequently, we calculate the

initial clustering cost $\Phi_{\mathcal{U}}(\mathcal{C})$ according to (12), and perform $T = O(\log(\Phi_{\mathcal{U}}(\mathcal{C})))$ iterations. In each iteration, we sample each user point with the probability $p_{\mathbf{u}_k}$, which is given by

$$p_{\mathbf{u}_k} = \min \left\{ \frac{l \cdot d^2(\mathbf{u}_k, \mathcal{C})}{\Phi_{\mathcal{U}}(\mathcal{C})}, 1 \right\} \quad (13)$$

where l is an oversampling factor. Subsequently, we add the sampled points to \mathcal{C} , updating the value of $\Phi_{\mathcal{U}}(\mathcal{C})$. The final number of user centers is $|\mathcal{C}| = l \times T$, where $|\cdot|$ denotes the cardinality of the set. Then, we calculate the number of points in \mathcal{U} closer to \mathbf{c}_y than any other point in \mathcal{C} , denoted by ω_y . According to $\omega_y, y \in \{1, \dots, Y\}$, we obtain the set of initial centroids from \mathcal{C} . Finally, we perform standard K-means algorithm for user grouping.

After user grouping, it is necessary to determine the radius of the beam serving per group. Considering that the beam received power decreases as the beam radius increases, we maximize the beam received power by selecting the minimum circle covering [15] that covers all users in the user group, satisfying the beam resolution. The section details are summarized in **Stage 1** of Algorithm 1.

3.2 Graph-Based Frequency Allocation Method

In order to effectively reduce IBI and enhance system throughput, we adopt a conventional FcFR scheme. In this scheme, the SINR of the n th beam is influenced not only by its own signal strength but also constrained by the sidelobe interference from beams of the same color. The level of sidelobe interference is closely related to the relative angle between beams. We define \mathbf{G} as the adjacency matrix, where $[\mathbf{G}]_{n,m}$ represents the interference generated by the m th beam on the n th beam, i.e., $\frac{1}{2}\alpha G_{\text{tx}}^m(\phi_m) G^r P_n^t (\frac{\lambda}{4\pi l})^2$. It is worth noting that the gains of each beam vary depending on the angular difference of θ_b , resulting in the asymmetry of the adjacency matrix.

By grouping users and utilizing N beams to serve user groups, we consider a graph \mathcal{G} with N vertices, defined the vertex set as $\mathcal{V} = \{v_1, \dots, v_N\}$. In this graph, $[\mathbf{G}]_{n,m}$ represents the weight from vertex n to vertex m , constituting a directed weighted graph. We define $\bar{\mathbf{G}} = \mathbf{G} + \mathbf{G}^T$, converting the initial directed graph into an undirected graph for easier mathematical processing and analysis. This transformation facilitates subsequent mathematical processing and analysis while maintaining the basic structure.

In order to effectively utilize the FcFR technology, we first randomly select four vertices as initial points and assign them to four independent clusters. Next, for the remaining $N - 4$ vertices, we evaluate the sum of edge weights between each point and all points in the four clusters. Utilizing these sums, we assign every unassigned point to the cluster with the minimum sum of edge weights. After iteration, this allocation method reduces IBI of beams of the same color, thereby improving the overall communication performance. Define the i th cluster as \mathcal{V}_i , where $\sum_i |\mathcal{V}_i| = |\mathcal{V}|$. The section details are summarized in **Stage 2** of Algorithm 1.

Algorithm 1: Adaptive beamwidth design algorithm**Input:** The user coordinates $\mathcal{U} = \{\mathbf{u}_k\}, k = 1, \dots, K$ **Output:** The user group indicator $\{b_{k,n}\}$, the center coordinates $\{\mathbf{z}_n\}$, the beam coverage radius $\{r_n\}$ and the coloring solution \mathcal{Y}_N

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1 for  $N = 1 : K$  do
2   Stage1 :
3   Choose a point at random from  $\mathcal{U}$  and add to  $\mathcal{C}$ ;
4   Calculate  $\Phi_{\mathcal{U}}(\mathcal{C})$ ;
5   for  $t = 1 : O(\log(\Phi_{\mathcal{U}}(\mathcal{C})))$  do
6     Calculate the chosen probability  $p_{\mathbf{u}_k}$ ;
7     Sample each point from  $\mathcal{U}$  independently with  $p_{\mathbf{u}_k}$  and add to  $\bar{\mathcal{C}}$ ;
8     Update  $\mathcal{C} \leftarrow \mathcal{C} \cup \bar{\mathcal{C}}$ ;
9   Calculate  $\omega_y$  and obtain the set of initial centroids from  $\mathcal{C}$ ;
10  Perform standard K-means algorithm to divide users into  $N$  groups;
11  Calculate center coordinate  $\mathbf{z}_n$ , coverage radius  $r_n$  and received power  $P_n^r$ ;
12  Stage2 :
13  Calculate  $\bar{\mathbf{G}}$  and let  $\mathcal{V} = \{v_n\}, n = 1, \dots, N$ ;
14  Choose four vertices randomly from  $\mathcal{V}$  and assign them to four separate
    groups ;
15  Let  $\tilde{\mathcal{V}} = \mathcal{V} - \bigcup_i \mathcal{V}_i$ ;
16  for  $iter = 1 : N - 4$  do
17    Randomly choose  $\tilde{v}_j \in \tilde{\mathcal{V}}$ ;
18     $\mathcal{V}_i = \operatorname{argmin}_{p \in \mathcal{V}_i} [\bar{\mathbf{G}}]_{\tilde{v}_j, p}$ ;
19     $\mathcal{V}_i = \mathcal{V}_i \cup \{\tilde{v}_j\}$ ;
20     $\tilde{\mathcal{V}} = \tilde{\mathcal{V}} - \{\tilde{v}_j\}$ ;
21  Obtain coloring solution  $\mathcal{Y}_N$ ;
22  Calculate  $P_n^I$ ;
23  if  $R_n < R_{th}$  then
24     $\sum_{n=1}^N R_n = 0$ ;
25  else
26    Calculate  $\sum_{n=1}^N R_n$ ;
27 Evaluate different designs and record the best  $\{b_{k,n}\}, \{\mathbf{z}_n\}, \{r_n\}, \mathcal{Y}_N$ ;

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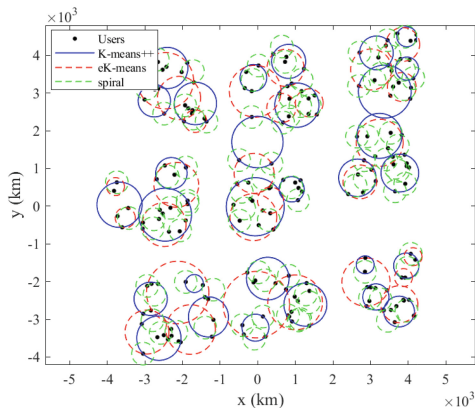
4 Simulation Results

In this section, the simulation results are presented to evaluate the performance of the proposed scheme. We assume that user coordinates within the satellite coverage area follow a continuous uniform distribution. The satellite power and bandwidth are evenly allocated to each beam. The values of the major simulation setup parameters are summarized in Table 1. To validate the effectiveness of the proposed method, we compare it with the spiral algorithm [16], the K-means algorithm and the K-means++ algorithm [14].

Figure 2 illustrates the user grouping results of the three algorithms. A total of 180 users are randomly generated within a plane area (represented by black

Table 1. Simulation Parameters

Parameter	Value
The height of satellite h	35786 km
The covered radius of satellite	5000 km
Bandwidth W	200 MHz
Satellite power P_{total}	20 dBW
Phased array antenna diameter D	1 m
Frequency band f	20 GHz
User antenna gain G^r	40 dBi
Minimum data rates limitation R_{th}	70 kbps
Noise power spectral density N_0	-174 dBm


Fig. 2. User grouping of three algorithms with 180 users.

dots), with the number of groups set to 30. The beamwidth is dynamically adjusted based on the number of users in each group. In this case, the system throughput achieved with the eK-means, K-means++ and spiral algorithms is 53.8 Mbps, 52.1 Mbps and 47.5 Mbps, respectively. Therefore, the proposed scheme achieves a higher throughput at the same cost.

Figure 3 shows the correlation between the system throughput of all algorithms and the number of users, with each point representing the average results over 300 random instances. It can be observed that proposed algorithm consistently outperforms spiral, K-means and K-means++ algorithms and this advantage does not significantly diminish as the number of users increases. Moreover, eK-means, K-means and K-means++ algorithms exhibit superior performance compared to spiral algorithm, revealing that adjusting the beamwidth can effectively enhance system throughput. The superior performance of K-means++ over K-means is evident as discussed in detail in Sect. 3. Although K-means++ offers attractive simplicity and speed compared to the standard algorithm, it

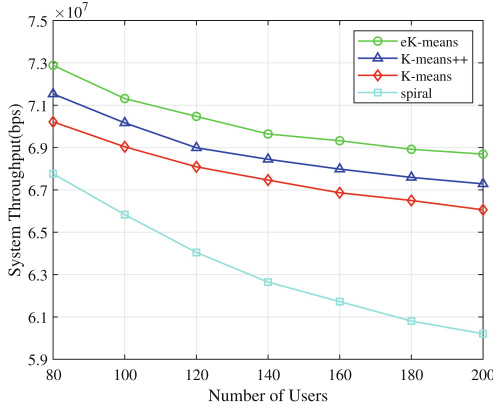


Fig. 3. Comparison of the system throughput under different number of users.

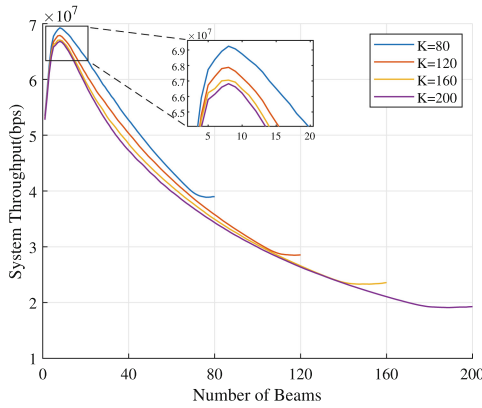


Fig. 4. System throughput verses number of beams with different number of users.

does not guarantee accuracy. The reasons why proposed algorithm outperforms K-means++ are as follows: (1) eK-means carefully selects centroids in both stages. In the first stage, it samples $l \times T$ centroids, which is more than N . In the second stage, K-means++ is applied to prune the centroids and select N centroids from the $l \times T$ centroids. (2) It may be due to the fact that eK-means updates the distribution function $\Phi_{\mathcal{U}}(\mathcal{C})$ only once per round, i.e., it recalculates $\Phi_{\mathcal{U}}(\mathcal{C})$ after selecting l centroids, whereas K-means++ recalculates $\Phi_{\mathcal{U}}(\mathcal{C})$ after selecting each centroid. It is observed that as the number of users increases, the system throughput tends to decrease. This is because the total transmit power of the system remains constant, and as the number of users increases, the distance between beam centers is more likely to be smaller than in the case of fewer users, resulting in increased IBI.

Furthermore, we conducted simulations to analyze the system throughput for different numbers of beams. Figures 4 and 5 present the results for different

numbers of users and different minimum coverage radius of beams, respectively. The results indicate that typically about 10 beams are needed to achieve the highest system throughput. This suggests that it is possible to search for the optimal number of groups N within a smaller range without iterating over all possible values, significantly reducing the complexity.

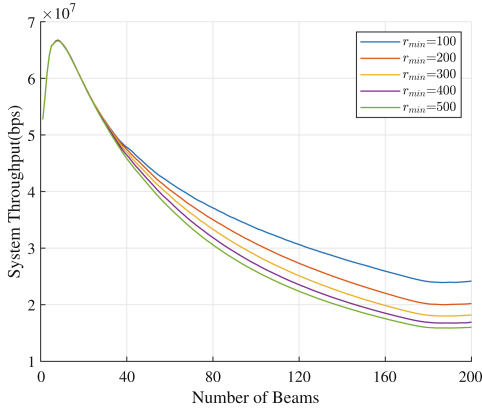


Fig. 5. System throughput verses number of beams with different minimum beam coverage radius.

5 Conclusion

This paper proposes an adaptive beamwidth design scheme that can adjust the number of beams and coverage range dynamically based on user distribution, communication requirements and satellite hardware constraints. This scheme enhances the flexibility of multi-beam satellite systems greatly. Compared to benchmark schemes, the proposed algorithm attains superior system throughput. In future research, we will explore integrating hybrid architectures and efficient user scheduling algorithms to further improve system throughput in large-scale user scenarios.

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