Gradient-Based UAV Positioning Algorithm for Throughput Optimization in UAV Relay Networks

Xiangyu Li(✉), Tao Peng, and Xiaoyang Li

Wireless Signal Processing and Networks Laboratory (WSPN),
Key Laboratory of Universal Wireless Communications, Ministry of Education,
Beijing University of Posts and Telecommunications, Beijing, China
{lixymiracle,pengtao,lixiaoyang111}@bupt.edu.cn

Abstract. Under natural disaster or other emergency situations, the fixed communication infrastructures are unavailable, which brings great inconvenience to information interaction among people. In this paper, we design a UAV relay network, using a small-scale UAV fleet serves as communication relays of a team of ground users performing collaborate tasks. Aiming at the user's requirement for high communication capacity for multi service transmission, we present a distributed gradient-based algorithm of finding the optimal positions of UAV in UAV relay network to improve the network average end-to-end throughput in real-time. The system optimization objective is formulated by using Shannon-Hartley Theorem and received signal-to-noise ratio (SNR) that incorporates with UAV positions and ground user positions. Due to the non-smoothness of the objective function, we use generalized gradient instead. Each UAV moves along the generalized gradient direction of objective function to optimize the target locally, and finally, all UAV convergence to stable positions of optimizing the network throughput. Simulation results show the effectiveness of our method in improving the network average end-to-end throughput.

Keywords: UAV relay network · Throughput optimization · Gradient-based positioning

1 Introduction

Recently, the application of unmanned aerial vehicles (UAVs) has attracted increasing research interest. Small-scale UAV fleet is a practical choice for commercial applications due to their ease of deployment, low cost and hovering ability. Such aerial vehicles are suitable for natural disaster rescue, environmental

This work is supported in part by the National Natural Science Foundation of China (No. 61631004) and the National Science and Technology Major Project of China under Grant 2016ZX03001017.

H. Gao et al. (Eds.): ChinaCom 2019, LNICST 313, pp. 278–291, 2020.
https://doi.org/10.1007/978-3-030-41117-6_23

monitoring, delivery of goods and outdoor tourism. The using of single or multiple UAVs as communication relays or aerial base stations for ground users' communications in emergency situations and for public safety has been of particular interest because of their mobility and large coverage capabilities. Meanwhile, the development of wireless and mobile communication technologies has changed all aspects of our lives. The demand for high-capacity, high-reliability, low-latency wireless communication is increasing. In a UAV relay network, the locations of UAV affect various network performance metrics, including throughput, connectivity and coverage. The positions of ground nodes can usually be obtained by GPS or other navigation system, one practical way to optimize above metrics is positioning UAVs based on the nodes positions and network topology.

Many existing works have studied UAV relay network and its optimization. Rasario et al. [1] proposed a mechanism for the placement of UAV relays to support the transmission of high-quality live videos. [2] proposed an algorithm for the performance optimization of the ground-to-relay link of a UAV relay network by controlling the UAV heading angle. Zeng et al. [3] addressed the throughput maximization problem in a UAV relaying systems by optimizing the source/relay transmit power along with the relay trajectory. The paper [4] investigated a particle swarm optimization (PSO) based method of finding the optimal positions of unmanned aerial vehicles functioning as a communication relays to improve the network connectivity and communication performance of a team of ground nodes. Dixon et al. [5,6] presented a decentralized mobility control algorithm for optimizing the end-to-end communication capacity of a UAV relay chain system. The chaining controller drives the location of a virtual control point, using estimates of the communication objective function gradient calculated by stochastic approximation techniques, to locations of improving the relay performance. In [7], a centralized heuristic algorithm was proposed for positioning UAV to maximize the throughput of a software-defined disaster area UAV communication network. [8] developed a distributed controller to position a team of aerial vehicles in a configuration that optimizes communication link quality, to support a team of ground vehicles performing a collaborative task. They presented a gradient-based control approach where the agent locally minimizes a physically motivated cost function.

UAV deployment algorithm can be divided into three categories: heuristic algorithm [4,7], gradient based search [5,6,8] and others [2,3]. Heuristic algorithm has strong global search ability, which can obtain good UAV positions in given iterations. Gradient based search has continuous search trajectory, which is attractive in real-time optimization system. Other algorithm, such as UAV heading angle control, is applicable to specific UAV network.

There are some limitations in above works. The work [1,3,5,6] only consider one end-to-end transmission link, which is impractical for scenarios of multi-team collaboration, while paper [2] did not consider the actual available end-to-end communication capacity for ground users. The methods in [4] and [7] are centralized and time-consuming, causing poor network invulnerability, and not suitable for real-time optimization system. In [8], the objective function lacks physical meanings.

In view of the shortcomings of above literature, in this paper, we design a distributed UAV relay network according to the search and rescue scenario, supporting multi-users end-to-end communication. We propose a novel decentralized UAV positioning algorithm to optimize the average end-to-end throughput of the network in real-time. We consider the flow-pipe (FP) end-to-end communication capacity model [5], and generate the system optimization target by Shannon-Hartley Theorem based on the received signal-to-noise ratio (SNR). We design a gradient-based controller, driving the UAV moves along the gradient flow to maximize the target. Due to the non-smoothness of our objective function, the generalized gradient theory is introduced. We utilize the stochastic approximation to estimate the generalized gradient, and the convergence of the objective function to local maxima is proved by using the non-smooth stability analysis literature [9]. The optimization effectiveness is validated by simulation experiments. By contrast with PSO, this optimization result is global maxima.

The remainder of this paper is organized as follows. Section 2 describes the system model and optimization objective. Section 3 provides the detailed description of the proposed positioning algorithm. Section 4 shows the result of simulation experiments. In Sect. 5, we conclude this work.

2 System Model and Problem Formulation

In disaster or other emergency situations. The field is usually partitioned in several isolated regions and the fixed communication infrastructures are unavailable. The team members performing collaborate tasks in different regions cannot communicate directly with each other. The use of multiple UAV as communication relays for this scenario is feasible because of their mobility and large coverage capabilities. In this paper, we consider a distributed UAV mesh network.

2.1 UAV Relay Network Model

As shown in Fig. 1, a small-scale UAV fleet is employed as aerial base stations to serve a group of ground users. The network is designed as decentralized structure to reduce the dependency to control center which has high risk of being destroyed in complicated environment. All UAV play the same role and construct a top-layer network, their motions are self-controlled and in the same fixed altitude. Further, each UAV forms a sub-network with certain number of users and provides communication relay service for them. The motion of ground users is uncontrolled in their regions. For this model and optimization problem, we make following assumptions:

- The users are distributed in isolated areas, cannot communicate directly with other users without relays. The UAV communication range can cover the deployment area, and UAV can communicate with each other directly.
- UAV equips with multi-radios which occupy different channels, while ground users have one. Users in the same sub-network share a communication channel by time division multiple access (TDMA), while the air-to-air links using independent radio. No communication interference is considered in this work.

– Ground users can report their positions obtained by GPS to UAV in real-time. UAV shares these information with each other through interactions that occupy seldom communication bandwidth.
– The ground user has the willingness to communicate continuously with all other users in the network.

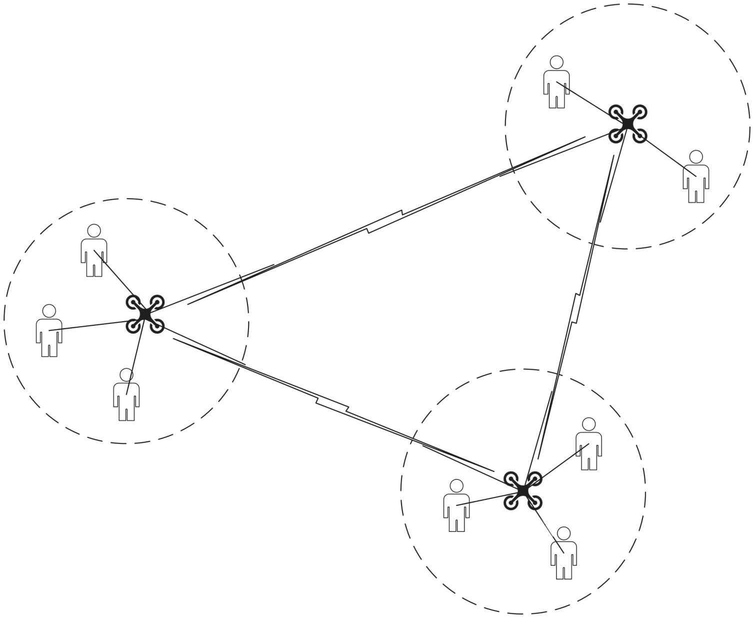


Fig. 1. System model of UAV relay network.

On a given area, a group of N UAV flying at height H are deployed as communication relays to serve K users. Let $\mathcal{N} = \{n_1, n_2, ..., n_N\}$ with $|\mathcal{N}| = N$ be the set of sub-networks of the system, the corresponding identifier of UAV is represented as $\mathcal{A} = \{a_1, a_2, ..., a_N\}$ with horizontal coordinates $\mathcal{P}_a = \{p_1, p_2, ..., p_N\}$, $p_i \in \mathbb{R}^2$. Let $\mathcal{K} = \{k_1, k_2, ..., k_N\}$ with $|\mathcal{K}| = K$ presents the number of ground users in each sub-network. For sub-network n_i, the corresponding users set can be expressed as $\mathcal{U}_i = \{u_{i1}, u_{i2}, ..., u_{ik_i}\}$ with horizontal positions $\mathcal{P}_{iu} = \{p_{i1}, p_{i2}, ..., p_{ik_i}\}$ where $p_{ij} \in \mathbb{R}^2$, the horizontal distance between these users and the host UAV is $\mathcal{D}_i = \{d_{i1}, d_{i2}, ..., d_{ik_i}\}$. The user u_{ij} in n_i is willing to communicate continually with all other users in the network.

2.2 SNR Field and Communication Capacity

Communication quality can be characterized by received signal-to-noise ratio (SNR). Denote the transmit power of node as P, and the received additive white

Gaussian noise (AWGN) power is σ^2, the available SNR between user u_{ij} and UAV a_i is given as

$$SNR_{ij} = \frac{PG_{ij}}{\sigma^2} \tag{1}$$

where G_{ij} is the channel power gain. We assume that the channel power gain from the UAV to users follows the free-space path loss model, which can be expressed as

$$G_{ij} = \frac{\rho_0}{d_{ij}^{\alpha}} \tag{2}$$

where ρ_0 denotes the channel power gain at the reference distance $d = 1m$, α represents the path-loss exponent and $d_{ij} = (H^2 + \|p_i - p_{ij}\|^2)^{\frac{1}{2}}$ is the distance between the user u_{ij} and UAV a_i. Further, the SNR between these two nodes can be written as

$$SNR_{ij} = \frac{\gamma_0}{d_{ij}^{\alpha}} \tag{3}$$

where $\gamma_0 = P\rho_0/\sigma^2$ denotes the reference received signal-to-noise ratio (SNR) at $d = 1m$. The capacity of communication link between any two nodes is a function of the channel bandwidth and the SNR of received signal. We use the Shannon-Hartley Theorem, the unit bandwidth maximum achievable rate in bps/Hz can be expressed as

$$\begin{aligned} r_{ij} &= \log_2(1 + SNR_{ij}) \\ &= \log_2(1 + \frac{\gamma_0}{d_{ij}^{\alpha}}) \end{aligned} \tag{4}$$

2.3 UAV Relay Network Average End-to-End Throughput

We assume the capacity of a single link is divided equally by communications carried by the link. The flow-pipe (FP) model [5] is implemented in end-to-end throughput calculation, it equals to the capacity of the worst link of all links in an end-to-end communication.

For an air-to-ground link, the theoretical capacity between user u_{ij} and UAV a_i is r_{ij}. The rate is firstly divided because the channel is shared by TDMA within the sub-network, and each user can get $1/k_i$ time slot in a unit time. We get the real link rate

$$r'_{ij} = \frac{r_{ij}}{k_i} \tag{5}$$

The link carries the number of $2(K-1)$ communications, including up-link and down-link transmissions. And for each communication, the available average rate is

$$\bar{r}_{ij} = \frac{r_{ij}}{2k_i(K-1)} \tag{6}$$

With regard to air-to-air links, the link capacity between UAV a_m and UAV a_n is R_{mn}, the link undertakes the cross sub-networks transmission with the number of $2k_mk_n$ and for each communication, the available rate is

$$\bar{R}_{mn} = \frac{R_{mn}}{2k_mk_n} \tag{7}$$

For a communication between u_{ni} and u_{nj} in sub-network n, the end-to-end throughput can be expressed as

$$\min(\bar{r}_{ni}, \bar{r}_{nj}) \tag{8}$$

Respectively, the sum of end-to-end throughput and communication amounts in sub-networks are

$$\sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{U}_n} \sum_{j \in \mathcal{U}_n, j \neq i} \min(\bar{r}_{ni}, \bar{r}_{nj}), \sum_{n \in \mathcal{N}} k_n(k_n - 1) \tag{9}$$

For a communication between $u_{n_1 i}$ in sub-network n_1 and $u_{n_2 j}$ in sub-network n_2, the throughput can be expressed as

$$\min(\bar{r}_{n_1 i}, \bar{R}_{n_1 n_2}, \bar{r}_{n_2 j}) \tag{10}$$

Respectively, the sum of end-to-end throughput and amounts of cross sub-network communication are

$$\sum_{n_1 \in \mathcal{N}} \sum_{n_2 \in \mathcal{N}, n_2 \neq n_1} \sum_{i \in \mathcal{U}_{n_1}} \sum_{j \in \mathcal{U}_{n_2}} \min(\bar{r}_{n_1 i}, \bar{R}_{n_1 n_2}, \bar{r}_{n_2 j}), \sum_{n_1 \in \mathcal{N}} \sum_{n_2 \in \mathcal{N}, n_2 \neq n_1} k_{n_1} k_{n_2} \tag{11}$$

The system average end-to-end throughput of the UAV relay network can be defined as

$$J = \frac{\sum\limits_{n \in \mathcal{N}} \sum\limits_{i \in \mathcal{U}_n} \sum\limits_{j \in \mathcal{U}_n, j \neq i} \min(\bar{r}_{ni}, \bar{r}_{nj}) + \sum\limits_{n_1 \in \mathcal{N}} \sum\limits_{n_2 \in \mathcal{N}, n_2 \neq n_1} \sum\limits_{i \in \mathcal{U}_{n_1}} \sum\limits_{j \in \mathcal{U}_{n_2}} \min(\bar{r}_{n_1 i}, \bar{R}_{n_1 n_2}, \bar{r}_{n_2 j})}{\sum\limits_{n \in \mathcal{N}} k_n(k_n - 1) + \sum\limits_{n_1 \in \mathcal{N}} \sum\limits_{n_2 \in \mathcal{N}, n_2 \neq n_1} k_{n_1} k_{n_2}} \tag{12}$$

where J is a function of all nodes positions p due to the definition of link rate r and R.

2.4 Problem Formulation

As can be seen above, the system average end-to-end throughput associates with the positions of all nodes. Ground users are moving in corresponding areas and uncontrollable while UAV are self-controlled and can adjust their positions dynamically. Our goal is real-timely maximizing (12) by distributed adaptive deployment of UAV. Namely, to determine

$$p^* = arg \max_{p \in \mathbb{R}^{2 \cdot N}} (J) \tag{13}$$

where p^* presents a group of positions of UAV.

3 Gradient-Based UAV Positioning Algorithm

This section describes a distributed gradient-based controlling algorithm to optimize the system average end-to-end throughput. We design a gradient controller for each UAV, driving the UAV moves along the gradient direction of global objective function. The target is optimized locally in these movement process. We lead into the generalized gradient due to the locally non-differentiable of the objective function. Furthermore, we prove the stability of the controller by non-smooth analysis.

3.1 Generalized Gradient Controller

Gradient ascent (decent) is a general method in solving various engineering optimization problems. Consider the problem of maximizing a continuous differentiable function $J(p)$ where $p \in \mathbb{R}^d$, which is equivalent to find p^* that maximizing $J(p)$ such that the gradient at p^*

$$g(p^*) = \nabla J(p^*) = \frac{\partial J}{\partial p}(p^*) = 0 \tag{14}$$

using gradient ascent, the iteration optimization process can be expressed as

$$p_{k+1} = p_k + \lambda g(p_k) \tag{15}$$

where p_k is the variable at the k^{th} iteration, $\lambda > 0$ is the iteration movement step and $g(p_k)$ indicates the normalized gradient of the objective function at p_k. Following the direction of gradient flow in each iteration, the objective function will converge to local maxima finally.

Owing to the existence of minimum functions in (12) lead to locally non-differentiable at points where the elements in minimum function are equal, gradient ascent method has theoretical defects. We must instead use the generalized gradient.

The generalized gradient and its properties are studied in discontinuous dynamic systems [9]. For a locally Lipschitz function J, $\mathbb{R}^d \to \mathbb{R}$, and $\Omega_J \in \mathbb{R}^d$ denotes the set of points where J fails to be differentiable. The generalized gradient ∂J at a non-differentiability point p, can be expressed as the convex hull of all possible limits of the gradient at neighboring points where the function J is differentiable [9], written as

$$\partial J(p) = co\left\{\lim_{i\to\infty} \nabla J(p_i) : p_i \to p, p_i \notin S \cup \Omega_J\right\} \tag{16}$$

where co denotes convex hull. If function J is continuously differentiable at p, then the generalized gradient $\partial J(p) = \nabla J(p)$. Moreover, the generalized gradient vector field [9] $Ln\partial(J)$, $\mathbb{R}^d \to \mathbb{R}$, where $Ln : \mathcal{B}(\mathbb{R}^d) \to \mathcal{B}(\mathbb{R}^d)$ is a set-valued map that associates to each subset S of $\mathbb{R}^d$ the set of least-norm elements of its closure $\overline{S}$. Furthermore, $Ln(\partial J/\partial p)$ is a direction of ascent of J at $p \in \mathbb{R}^d$.

We then design a generalized gradient controller. For UAV i, the controller can be expressed as

$$\dot{p}_i = Ln(\partial J)(p_i) \tag{17}$$

where $Ln(\partial J)$ is the generalized gradient vector field of J, and p_i is the position of UAV i.

Calculating the generalized gradient of (12) is a difficult task. The method of stochastic approximation for estimating the generalized gradient of non-smooth function and the appropriate conditions for convergence has been studied in [10]. In this paper, we apply a stochastic approximation approach named least squares gradient estimation (LSGE) mentioned in [5] (Fig. 2).

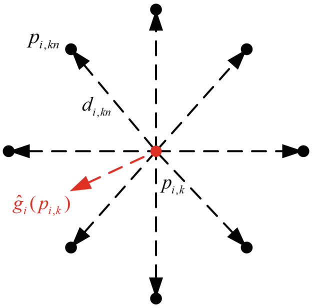


Fig. 2. An example of sample points and sample vectors around a control point.

The LSGE algorithm uses the least square method to fit the objective function and provides the least-square estimate of the gradient of the target point. We take n sample points $\{p_{i,k_1}, p_{i,k_2}, ..., p_{i,k_n}\}$ around the position of UAV i at $p_{i,k}$ (defined as control point) of time k with sample angle $2\pi/n$ and define the sample vector

$$\boldsymbol{d}_{i,k_n} = p_{i,k_n} - p_{i,k} \tag{18}$$

where $\boldsymbol{d}_{i,k_n} = h\boldsymbol{v}_{i,k}$, $\boldsymbol{v}_{i,k}$ with $\|\boldsymbol{v}_{i,k}\| = 1$ is the vector direction and $h > 0$ is the length of sample vector. Define the sample matrix $H_{i,k}$ used by the LSGE algorithm [5]

$$H_{i,k} = \begin{bmatrix} 1 & \boldsymbol{d}_{i,k_1} \\ \vdots & \vdots \\ 1 & \boldsymbol{d}_{i,k_n} \end{bmatrix} \tag{19}$$

then the LSGE method is given by

$$\begin{bmatrix} \hat{J}(p_{i,k}) \\ \hat{g}_i(p_{i,k}) \end{bmatrix} = (H_{i,k}^T H_{i,k})^{-1} H_{i,k}^T \begin{bmatrix} J(p_{i,k_1}) \\ \vdots \\ J(p_{i,k_n}) \end{bmatrix} \tag{20}$$

where $\hat{J}(p_{i,k})$ is an estimate value of the objective function at $p_{i,k}$, $J(p_{i,k_n})$ denotes the measurement of objective function at sample point p_{i,k_n}, which can be precisely calculated as the positions of all other nodes can be obtained in real time. $\hat{g}_i(p_{i,k})$ is the estimation of generalized gradient of UAV i at $p_{i,k}$. The error and variance analysis of this method in estimating the generalized gradient is presented in [11].

As the generalized gradient provide the movement direction, the position iteration process for UAV i can be expressed as

$$p_{i,k+1} = p_{i,k} + \lambda \hat{g}_i(p_{i,k}) \tag{21}$$

3.2 Non-smooth Analysis of the Controller

In this section, we present the stability analysis of generalized gradient controller. We achieve this by non-smooth analysis in discontinuous dynamic system [9].

The existence of generalized gradient vector field of the objective function depends on the fact that the function is locally Lipschitz and regular. A function $f : \mathbb{R}^d \to \mathbb{R}^m$ is locally Lipschitz at $x \in \mathbb{R}^d$ if there exist a L_x, $\varepsilon \in (0, \infty)$ such that

$$\|f(y) - f(y')\|_2 \leq L_x \|y - y'\|_2 \tag{22}$$

for all y, $y' \in B(x, \varepsilon)$ where $B(x, \varepsilon)$ is a ball centered at x of radius ε.

A function is said to be regular when its right directional derivative $f'(x; v)$ is equal to its generalized directional derivative $f^0(x; v)$ [9], where

$$f'(x; v) = \lim_{h \to 0^+} \frac{f(x + hv) - f(x)}{h} \tag{23}$$

$$f^0(x; v) = \lim_{h \to 0^+} \sup_{y \to x} \frac{f(y + hv) - f(y)}{h} \tag{24}$$

With regard to the locally Lipschitz and regular function, various results are available to facilitate the computation of generalized gradient [9]. Followings are the two properties for conserving the locally Lipschitz and regular property of our objective function.

Sum Rule: If f_1, f_2: $\mathbb{R}^d \to \mathbb{R}$, are locally Lipschitz and regular at $x \in \mathbb{R}^d$, s_1, $s_2 \in \mathbb{R}$, then the function $s_1 f_1 + s_2 f_2$ is locally Lipschitz and regular at x and the generalized gradient $\partial(s_1 f_1 + s_2 f_2)(x) = s_1 \partial f_1 + s_2 \partial f_2$.

The Minimum (Maximum) of A Finite Set of Continuous Differentiable Functions is Locally Lipschitz and Regular: For $k \in \{1, 2, ..., m\}$, let $f_k : \mathbb{R}^d \to \mathbb{R}$ be locally Lipschitz at $x \in \mathbb{R}^d$, and define the functions f_{max}, f_{min}: $\mathbb{R}^d \to \mathbb{R}$ by

$$f_{max} = max\{f_k(x) : k \in \{1, 2, ..., m\}\} \tag{25}$$

$$f_{min} = min\{f_k(x) : k \in \{1, 2, ..., m\}\} \tag{26}$$

f_{max}, f_{min} are locally Lipschitz and regular.

We combine the two rules above to valid the property of our objective function and utilize Proposition 11 in [9] to prove the stability of our controller (16) as Theorem 1.

Theorem 1. UAV follows the generalized gradient vector field of J such that $\dot{p}_i = Ln(\partial J / \partial p_i)$ will asymptotically converge to the critical points of J where the strongly stable critical points are local maxima of J.

Proof: On account of the continuous differentiability of functions (6) and (7), (8) and (10) are locally Lipschitz and regular using the second rule above. Then, consider the objective function J, which is an algebraic composition of a series of minimum functions, by applying the sum rule, we conclude that J is a locally Lipschitz and regular. According to Proposition 11 in [9], the strict maximizer of J are strongly equilibria of the non-smooth gradient flow of J. Further, we find a compact and strongly invariant set for this dynamic system follow the example of [8]. If the UAV flies out of the ground user field, it will fail to communicate with other nodes which leads to the generalized gradient $\partial J/\partial p_i$ for agent i goes to zero. So the ground user field is a strongly invariant set that with any initial conditions, the UAV will converge to the set of critical points of J. Then our distributed generalized gradient controller can be expressed as

$$\begin{aligned}\dot{p}_i &= Ln(\frac{\partial J}{\partial p_i}) \\ &= Ln(\frac{\partial(\sum_{n\in\mathcal{N}}\sum_{a\in\mathcal{U}_n}\sum_{b\in\mathcal{U}_n,a\neq b}\min(\bar{r}_{na},\bar{r}_{nb}) + \sum_{n_1\in\mathcal{N}}\sum_{n_2\in\mathcal{N},n_2\neq n_1}\sum_{a\in\mathcal{U}_{n_1}}\sum_{b\in\mathcal{U}_{n_2}}\min(\bar{r}_{n_1a},\bar{R}_{n_1n_2},\bar{r}_{n_2b}))}{\partial p_i\cdot(\sum_{n\in\mathcal{N}}k_n(k_n-1)+\sum_{n_1\in\mathcal{N}}\sum_{n_2\in\mathcal{N},n_2\neq n_1}k_{n_1}k_{n_2})} \\ &= \hat{g}_i(p_i)\end{aligned} \tag{27}$$

where $\hat{g}_i(p_i)$ is the generalized gradient estimation value.

4 Simulation Experiences

In this section, we show the performance of our algorithm in optimizing the UAV relay network average end-to-end throughput. We consider a group of users distribute in a $3000 * 3000\,\text{m}^2$ area, and a small-scale of UAV fleet provides communication relay service. Each UAV forms a sub-network with several users and their users are unchanged during the optimization process. We assume UAV can gain the positions of all nodes through information interaction.

The algorithm is implemented using MATLAB. The simulation parameters are provided in Table 1. The iteration step size λ can affect the stability of

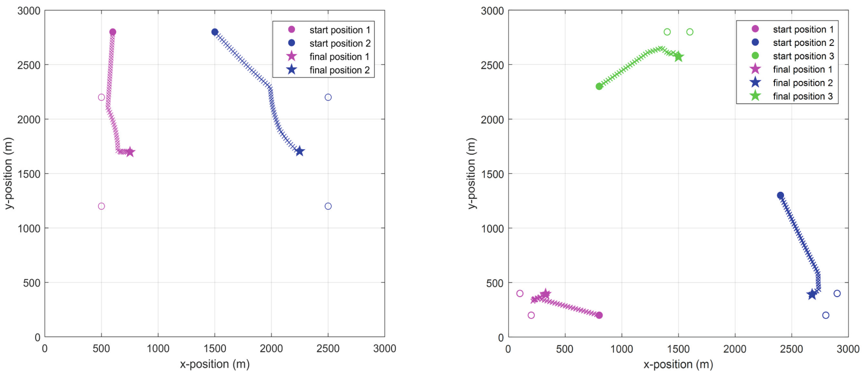
Table 1. Simulation parameters

Parameter term	Value	Unit
UAV flight height (H)	100	m
Node transmit power (P)	0.1	W
Channel power gain at reference distance = 1 m (ρ_0)	−50	dB
Received noise power (σ^2)	−110	dBm
Path-loss exponent (α)	2	n/a
Length of sample vector (h)	10	m
Iteration step size (λ)	8	m

controller around convergence positions of objective function, if the value is too big. We use 8 in following simulations.

Figure 3(a) presents a two-uav four-user scenario, user1 (500, 1200), user2 (500, 2200), user3 (2500, 1200) and user4 (2500, 2200) are plotted with hollow circles, and uav1 starts at (600, 2800) serving user1-2 and uav2 starts at (1500, 2800) serving user3-4. The controller drives the UAV along the generalized gradient flow and stops at positions marked out with stars. The change of system average end-to-end throughput during the optimization process is shown is Fig. 4, increasing from 0.4363 bps/Hz to 0.6884 bps/Hz. We choose different start positions for UAV, the final convergence positions are same.

A scenario of three-uav and six-user is shown in Fig. 3(b). The positions of nodes, the sub-networks partition and the UAV optimization trajectory is plotted with different colors. From the blue line in Fig. 4, we see a significant increase



(a) two-uav four-users (scenario 1) (b) three-uav six-users (scenario 2)

Fig. 3. The optimization of two sample scenarios.

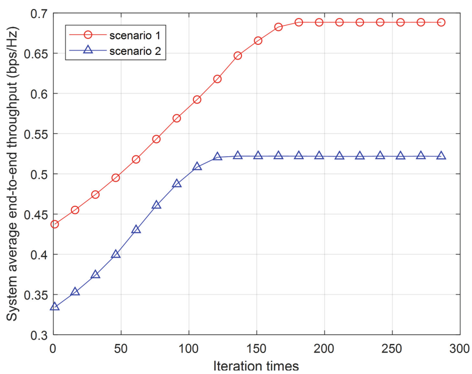


Fig. 4. Variation of throughput versus iteration times.

in system throughput, from 0.3330 bps/Hz to 0.5221 bps/Hz. The convergence positions of UAV instruct that the air-to-air link rates are the bottleneck of the end-to-end communications, causing UAV moves closer to get higher link rates. Different UAV start positions get the same optimization result.

Particle swarm optimization (PSO) [4] is a heuristic iteration algorithm, which can obtain global optimal solution by multiple particles joint searching solution space. As the searching process is random in each iteration, UAV can fly only if getting the final result. Let P_{num} be the particle number, N denotes the UAV number, I denotes the iteration times. For PSO, in each optimization, the total computation times of fitness function is $P_{num}I$. And in a distributed network, each UAV need to executes PSO algorithm to obtain its target position. Therefore, the total computation times is $NP_{num}I$ in each movement. In general, dozens of particles need be used in large-scale optimization problems and hundreds of iteration is necessary. This cause long computation time and is difficult to meet the real-time requirements if ground users are moving in their regions. While for our algorithm, the iteration process is continuous, UAV can fly in each iteration which has very little computation as described in Sect. 3. If users are moving, the UAV can adjust their flight direction in real-time.

Although PSO is not feasible to tackle this problem, it has great global search capability. We implement PSO with high iteration times and view the result of PSO as the optimal optimization result to validate the effectiveness of

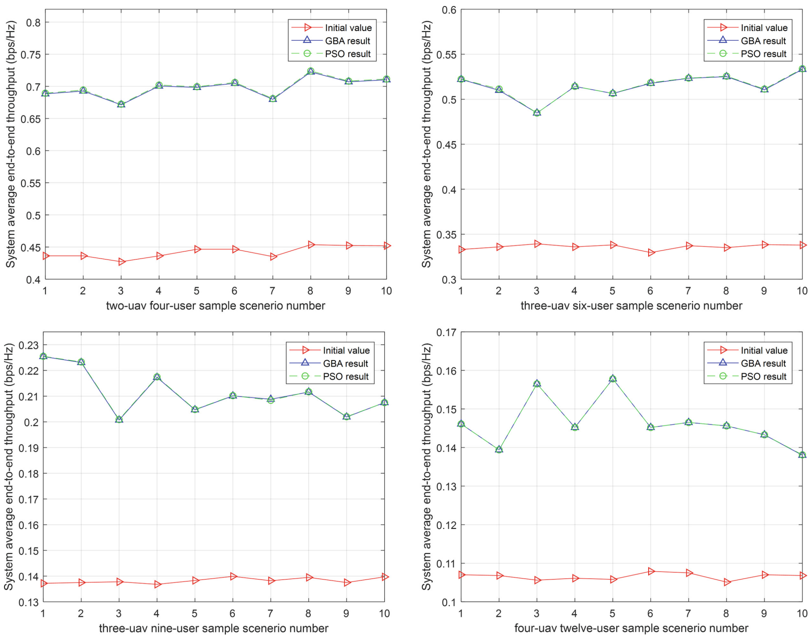


Fig. 5. Variation of throughput versus iteration times.

our method. We let the locations of a group of UAV as a particle, and using (12) as the fitness function.

We take 40 sample simulation scenarios and apply the gradient-based algorithm (GBA) and PSO to optimize the system average end-to-end throughput, respectively. The optimization result is presented in Fig. 5.

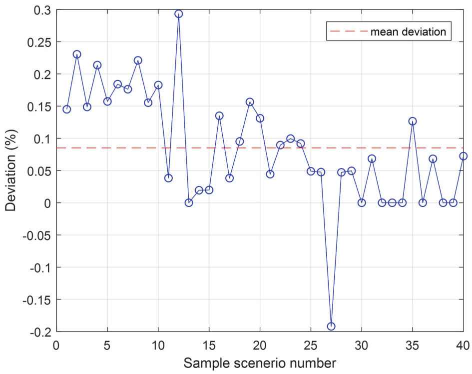


Fig. 6. Variation of throughput versus iteration times.

We can observe a great improvement in system average end-to-end throughput by using GBA. The final results of our method approximate the PSO result closely, illustrating that our algorithm can obtain optimal solution in theory. And the local maxima is global maxima in these scenarios of users distributing in isolated areas. In other scenarios, multi local maxima may exist, different UAV starting positions may converge to different local maxima. For this system, local maxima is also acceptable because the optimization is in real-time. The deviation (shown in Fig. 6) between our result and optimal value comes from gradient estimation.

In these 40 sample scenarios, the average deviation of network average end-to-end throughput of GBA with respect to optimal value is 0.085%.

5 Conclusions

In this paper, we designed a pragmatic UAV relay network to provide communication relays for ground users in emergency situations. The optimization of system average end-to-end throughput has been formulated and addressed to satisfy users' demand for communication capacity. To tackle this problem, we proposed a novel gradient-based UAV positioning algorithm. Using the generalized gradient controller, the UAV deploys adaptively to maximize the optimization target in real-time, making the network have high communication capacity at all times. We have proved the convergence and stability of this method by applying the relative theories of discontinuous dynamic system. The effectiveness of our method has been demonstrated by simulation experiments, whose results approximate closely to the optimal values obtained by PSO.

References

1. Rosário, D., Filho, J.A., Rosário, D., Santosy, A., Gerla, M.: A relay placement mechanism based on UAV mobility for satisfactory video transmissions. In: 2017 16th Annual Mediterranean Ad Hoc Networking Workshop (Med-Hoc-Net), Budva, pp. 1–8 (2017). https://doi.org/10.1109/MedHocNet.2017.8001638
2. Zhan, P., Yu, K., Swindlehurst, A.L.: Wireless relay communications with unmanned aerial vehicles: performance and optimization. IEEE Trans. Aerosp. Electron. Syst. **47**(3), 2068–2085 (2011). https://doi.org/10.1109/TAES.2011.5937283
3. Zeng, Y., Zhang, R., Lim, T.J.: Throughput maximization for UAV-enabled mobile relaying systems. IEEE Trans. Commun. **64**(12), 4983–4996 (2016). https://doi.org/10.1109/TCOMM.2016.2611512
4. Ladosz, P., Oh, H., Chen, W.: Optimal positioning of communication relay unmanned aerial vehicles in urban environments. In: 2016 International Conference on Unmanned Aircraft Systems (ICUAS), Arlington, VA, pp. 1140–1147 (2016). https://doi.org/10.1109/ICUAS.2016.7502562
5. Dixon, C., Frew, E.W.: Optimizing cascaded chains of unmanned aircraft acting as communication relays. IEEE J. Sel. Areas Commun. **30**(5), 883–898 (2012). https://doi.org/10.1109/JSAC.2012.120605
6. Dixon, C.: Controlled mobility of unmanned aircraft chains to optimize network capacity in realistic communication environments. Ph.D. dissertation, University of Colorado (2010)
7. ur Rahman, S., Kim, G., Cho, Y., Khan, A.: Positioning of UAVs for throughput maximization in software-defined disaster area UAV communication networks. J. Commun. Netw. **20**(5), 452–463 (2018). https://doi.org/10.1109/JCN.2018.000070
8. Gil, S., Schwager, M., Julian, B.J., Rus, D.: Optimizing communication in air-ground robot networks using decentralized control. In: 2010 IEEE International Conference on Robotics and Automation, Anchorage, AK, pp. 1964–1971 (2010). https://doi.org/10.1109/ROBOT.2010.5509622
9. Cortes, J.: Discontinuous dynamical systems. IEEE Control Syst. Mag. **28**(3), 36–73 (2008). https://doi.org/10.1109/MCS.2008.919306
10. Bartkutė, V., Sakalauskas, L.: Simultaneous perturbation stochastic approximation of nonsmooth functions. Eur. J. Oper. Res. **181**(3), 1174–1188 (2007). https://doi.org/10.1016/j.ejor.2005.09.052. ISSN 0377-2217
11. Brekelmans, R., Driessen, L., Hamers, H., Hertog, D.D.: Gradient estimation schemes for noisy functions. J. Optim. Theory Appl. **126**(3), 529–551 (2005). https://doi.org/10.1007/s10957-005-5496-2