



# Modulation Pattern Recognition Based on Wavelet Approximate Coefficient Entropy

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**Abstract.** Aiming at the modulation pattern recognition of multiple signals in complex electromagnetic environments, a modulation pattern recognition method based on wavelet approximate coefficient entropy is proposed. Based on the traditional wavelet entropy, an improved wavelet entropy, wavelet approximate coefficient entropy, is proposed, which has strong ability to represent the modulation signal characteristics and has good noise suppression effect. The simulation results verify the correctness of the theoretical analysis, and show that the proposed method can effectively realize the modulation pattern recognition of multiple signals at low signal to noise ratio.

**Keywords:** Modulation pattern recognition · Wavelet approximate coefficient entropy · Signal to noise ratio · Recognition rate

## 1 Introduction

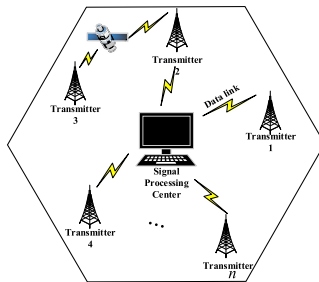
How to acquire and master the parameters of the complex electromagnetic environment in time is an important and challenging problem in information warfare. In reference [1], J. Mitola proposed the cognitive radio technology, which uses the learning ability of cognitive radio to autonomously sense the surrounding spectrum environment and respond to the actual electromagnetic situation in real time. In electronic countermeasures, the modulation recognition technology is applied to the spectrum sensing of receiver equipment, which can provide more necessary information for battlefield command and decision-making. For example, after intercepting the enemy's information, the first thing is to identify, demodulate and decrypt the enemy's signal, so as to obtain the enemy's confidential intelligence. At the same time, after the enemy's modulation system is proved, the effective communication can be purposefully interfered and the battlefield initiative can be obtained. Wavelet analysis is a local transform of time and frequency, which can effectively extract feature information from the signal, and is conducive to the perception of the surrounding electromagnetic environment. Many cognitive radio technologies are devoted to modulation recognition of communication signals by using cyclic spectrum [2], characteristic parameters and their

statistics [3, 4], time-frequency transform [5] and high-order cumulants [6]. These methods are difficult to achieve multi-resolution analysis of modulated signals, which increases the difficulty of obtaining effective information, and the real-time performance of signal analysis and processing is not good. In order to quickly and accurately grasp the current radio spectrum situation, wavelet analysis is applied to the modulation recognition of communication signals. In reference [7], the wavelet coefficients of speech signals after discrete wavelet analysis are used to realize effective recognition of Arabic speech numbers. In reference [8], a recognition algorithm based on wavelet variation coefficient difference and similarity feature is proposed to classify and recognize common digital modulation signals. In reference [9], continuous wavelet transforms (CWT) and multi-layer wavelet decomposition (MLWD) are used to extract the features of signals, and different classification features are adopted for different modulated signals. The algorithm does not need symbol period estimation and synchronization time estimation, which improves the operation speed and recognition rate of modulation signal recognition.

In order to improve the performance of modulation recognition at low signal-to-noise ratio, a modulation pattern recognition method based on wavelet approximate coefficient entropy (WACE) is proposed in this paper. Different from the traditional method, it can effectively extract the relevant features of the signal in the complex electromagnetic environment where the signal-to-noise ratio is low and the accurate modulation parameters are not easy to obtain, so as to realize the modulation pattern recognition of multiple signals.

The following contents are arranged as follows. In Sect. 1, the system model of modulation recognition is given. In the second section, the theory of wavelet analysis is introduced. In Sect. 3, the mathematical definition of WACE is given, and the entropy vector of wavelet approximate coefficient of each communication signal is calculated. In Sect. 4, the simulation analysis of the modulation recognition algorithm proposed in this paper through experiments shows that the algorithm is effective for modulation type recognition, and its performance in noise environment is analyzed. Finally, the conclusions are given.

## 2 System Model



**Fig. 1.** Model of communication signal modulation recognition system.

In this section, a complete communication signal modulation recognition system model will be established to simulate the communication situation in complex electromagnetic environment, and six representative and widely used communication signal modulation methods will be considered.

Figure 1 shows the communication signal modulation recognition system model, including an integrated signal processing center and n potential modulated signal transmitters. In modern electronic warfare, the signal processing center is the core of the whole combat system. It is responsible for receiving and processing signals of various modulation types from all directions, including our communication signals and the enemy communication signals. At the same time, there are also jamming signals that the enemy intentionally transmits. Whether it is to accurately obtain the information transmitted by our side, or to intercept the enemy signal to obtain intelligence, or to interfere with the effective communication of the enemy, it is inseparable from the modulation recognition technology. In order to better reflect the diversity of communication signal modulation methods in complex electromagnetic environment, six typical modulation methods are selected in this paper, which are Frequency Shift Keying (FSK), Minimum Shift Keying (MSK), Quadrature Phase Shift Keying (QPSK) and 16 Quadrature Amplitude Modulation (16QAM), Offset-QPSK (OQPSK) and Binary Phase Shift Keying (BPSK).

### 3 Wavelet Theory

Wavelet analysis is a time-frequency analysis method. For any function  $f(t) \in L^2(R)$ , the continuous wavelet transform is as follows:

$$\begin{aligned}
 W_f(a, b) &= \langle f, \psi_{a,b} \rangle \\
 &= |a|^{-1/2} \int_{\mathbf{R}} f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt
 \end{aligned}
 \tag{1}$$

Where  $a$  is the scale factor,  $b$  is the shift factor,  $a, b \in \mathbf{R}; a \neq 0$ ,  $\psi_{a,b}$  is the wavelet sequence of basic wavelet  $\psi(t)$  after stretching and translation.

In practical application, continuous wavelet must be discretized. Discrete wavelet transform discretizes continuous parameters  $a, b$  into  $m, n$ . The basic wavelet functions of discrete wavelet transform are as follows:

$$\psi_{m,n}(t) = 2^{-\frac{m}{2}} \psi(2^{-m}t - n)
 \tag{2}$$

The discrete wavelet transform of any function  $f(t)$  is:

$$WT_f(m, n) = \int_{\mathbf{R}} f(t) \cdot \overline{\psi_{m,n}(t)} dt
 \tag{3}$$

After the discrete wavelet transform of the signal  $s(n)$ , under the  $j$ -th decomposition scale, the coefficient of high-frequency component at  $k$ -time is  $cD_j(k)$ , and the

coefficient of low frequency component is  $c A_j(k)$ . The signal components obtained by single reconstruction are  $D_j(k), A_j(k)$ . The original signal  $s(n)$  can be expressed as the sum of the components [10].

$$\begin{aligned} s(n) &= D_1(n) + A_1(n) \\ &= D_1(n) + D_2(n) + A_2(n) \\ &= \dots = \sum_{j=1}^m D_j(n) + A_m(n) \end{aligned} \quad (4)$$

In most application scenarios, discrete wavelet transform with multiple scales can reflect the time-frequency distribution of signals. The limit case may be considered. If all the low-frequency components are taken and the high-frequency components are discarded, the anti-noise performance will be greatly improved. However, some key information of the signal will be lost. This paper makes up for this problem by two means. On the one hand, if the original signal is added to the multi-resolution analysis of the modulated signal to form the wavelet domain features together with other scale analysis results, no information of the original signal will be lost. On the other hand, if the appropriate wavelet function is selected to make the energy of each scale more concentrated on the low-frequency components, the denoising effect is better.

db $N$  wavelet ( $N$  denotes the order of wavelet function) is a wavelet function constructed by I. Daubechies, a famous scholar of wavelet analysis in the world. It performs well in the field of signal denoising. Therefore, this paper adopts the wavelet, in which the selection of order  $n$  is considered in the following two aspects. Firstly, the  $N$  in db $N$  wavelet corresponds to the vanishing moment of wavelet function. The larger the vanishing moment, the smaller the high frequency coefficient, the more concentrated the signal energy, and the better the noise removal effect. Secondly, with the increase of vanishing moment  $N$ , too much noise will be concentrated in the low frequency components, which will affect the denoising effect. At the same time, the support length of wavelet function will be lengthened and the computational complexity will be increased obviously. In this paper, db5 wavelet function is selected to concentrate the signal energy to obtain the best denoising effect.

## 4 Modulation Recognition Technology Based on Wavelet Approximate Coefficient Entropy

In this section, we mainly analyze two kinds of traditional wavelet entropy, which are wavelet energy entropy (WEE) and adaptive wavelet entropy (AWE).

Combining multi-resolution wavelet transform with information entropy, the definition and calculation method of wavelet energy entropy of signal can be obtained [10].

Suppose that any digital signal  $s(n)$  with  $n$  sampling points is decomposed on  $M$  scales, and on a given decomposition scale  $m$ , the wavelet coefficient vector is  $\mathbf{A}_m = (a_{m,1}, a_{m,2}, \dots, a_{m,n})$ ,  $m = 1, 2, \dots, M$ . The wavelet coefficient vectors  $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_M$  of each decomposition scale can form a vector sequence  $\{\mathbf{A}\}$ . The vector norm of wavelet coefficients is used to describe the closeness of wavelet coefficients at

different scales and the energy on scale  $m$  can be defined as  $E_m = \|\mathbf{A}_m\|^2 = \sum_{i=1}^n |a_{m_i}|^2$ . The

normalized energy  $p_m = E_m / \sum_{j=1}^M E_j$  of each scale wavelet coefficient is taken as the distribution of energy sequence instead of the probability distribution of signal. Thus, the entropy obtained based on energy distribution is called WEE, which is defined as  $H_{we} = H(p_1, p_2, \dots, p_M) = \sum_{j=1}^M p_j \log_2 p_j$ .

The concept of AWE is based on information entropy. In reference [11], the definition of AWE is given by combining the theory of information entropy with discrete wavelet transform.

$$E(S) = \frac{\sum_m |S_m|^P}{N} \tag{5}$$

Among them, the AWE  $E$  is a real number.  $S$  is the signal after the original signal  $s(n)$  is decomposed by discrete wavelet.  $P$  is an exponential weight, and its value range is  $1 \leq P < 2$ .  $S_m$  is the  $m$ -layer signal of the original signal after discrete wavelet transform and  $N$  is the length of  $S_m$ .

The two kinds of wavelet entropy mentioned above, including WEE and AWE, have achieved good results in their respective fields. However, if it is used in cognitive radio modulation recognition, especially when the signal-to-noise ratio (SNR) of the modulation signal to be recognized is low, the two kinds of wavelet entropy are difficult to achieve good recognition results. For example, in reference [12], the AWE is used for multi signal modulation recognition. When combined with BP neural network, the average recognition rate is about 95%. However, when the SNR is low, the recognition performance of this method for some modulated signals will decline rapidly.

Based on this, a new improved wavelet entropy, WACE, is proposed in this paper. It is the entropy value calculated from all wavelet approximate coefficients of the signal, and the wavelet approximate coefficient vector can be expressed as

$$\mathbf{W}_m = (w_{m,1}, w_{m,2}, \dots, w_{m,n}) \tag{6}$$

Where, the subscript  $m$  represents the decomposition scale parameter, and its value range is  $1, 2, \dots, M$ . Vector element  $w_{m,i} (i = 1, 2, \dots, n)$  is wavelet approximation coefficient. If the original signal is regarded as  $\mathbf{W}_0$ , then a new vector sequence  $\{\mathbf{W}\}$  can be formed from  $\mathbf{W}_0, \mathbf{W}_1, \mathbf{W}_2, \dots, \mathbf{W}_M$ . Each subsequence in the sequence  $\{\mathbf{W}\}$  is weighted by 2-norm, and the exponential term in the 2-norm is treated as the weight, and the weighted 2-norm of each scale wavelet approximation coefficient vector is calculated as

$$\|\mathbf{W}_m\| = \sqrt{\left( \sum_{i=1}^n |w_{m,i}|^\gamma \right)} \tag{7}$$

Where,  $\gamma$  is the index weight item. After this step, the vector sequences  $\{\mathbf{W}\}$  of wavelet approximation coefficients in different scales are transformed into 2-norm weighted sequences  $\{\|\mathbf{W}\|\}$ . After 2-norm weighting, the original signal is added to the vector sequence to ensure that the information of the original signal is not lost in the feature extraction of wavelet domain.

Assuming that the signal is decomposed on  $M$  scales, the approximate coefficient vector of wavelet on scale  $m$  is  $\mathbf{W}_m = (w_{m,1}, w_{m,2}, \dots, w_{m,n})$ . The energy on scale  $m$  is defined as

$$E_m = \|\text{varvec}W_m\|^2 = \sum_{i=1}^n |w_{m,i}|^\gamma, \quad m = 0, 1, \dots, M \tag{8}$$

In order to increase the number of wavelet entropy features of the signal to be identified, the WACE is given by the following expression according to the concept of AWE.

$$E_m = \frac{E_m}{L_m} = \frac{\|\mathbf{W}_m\|^2}{L_m} = \frac{\sum_{i=1}^n |w_{m,i}|^\gamma}{L_m} \tag{9}$$

$$\boldsymbol{\gamma}_{approx} = (\gamma_{0-approx}, \gamma_{1-approx}, \dots, \gamma_{M-approx})^T \tag{10}$$

Where  $E_m$  represents the entropy of wavelet approximation coefficients at the  $m$ -th level of discrete wavelet decomposition,  $L_m$  is the length of the  $m$ -th wavelet approximation coefficient, and  $\boldsymbol{\gamma}_{approx}$  is the exponential weight vector. In this way, the meaning of WACE is the average energy of wavelet approximate coefficient per length of signal in a certain scale, or the average energy of wavelet approximate coefficient of digital signal at each sampling point. Because this improved wavelet entropy represents the average energy of each wavelet approximate coefficient length in any signal, and reflects the uncertainty of signal at different decomposition scales, so it is called WACE.

For different signals, the WACE at a certain scale can reflect the characteristics of the signal at that scale. When a signal is decomposed in  $M$ -level by discrete wavelet transform, the entropy of  $M + 1$  wavelet approximate coefficients can be calculated according to formula (11), in which the approximate coefficient entropy of each layer represents certain wavelet domain characteristics of the signal. In order to make them represent the signal together, the entropy vector is composed of the approximate coefficient entropy of wavelet in each layer and is expressed as

$$\mathbf{E}_{approx} = (E_{0-approx}, E_{1-approx}, \dots, E_{M-approx})^T \tag{11}$$

Where  $\mathbf{E}_{approx}$  is the entropy column vector of wavelet approximate coefficient when the decomposition scale is  $M$ .

Compared with WEE and AWE, WACE has many advantages in modulation recognition. On the one hand, by discarding the high-frequency coefficients after

discrete wavelet decomposition and using db5 wavelet with larger vanishing moment, the extracted WACE vector has stronger anti-noise ability; on the other hand, by selecting different weight vectors  $\gamma_{approx}$ , the proportion of low-frequency components is increased, and the high-frequency noise interference is suppressed. Under the same noise environment, the computational complexity can be reduced and the recognition speed can be faster.

In this paper, the unit column vector whose weight vector matrix is 1.5 times is selected. On the one hand, after adding the index term of 1.5, the residual noise in low-frequency coefficients of each scale can be further weakened, and the key information which is conducive to feature extraction can be amplified. On the other hand, if the exponential weight changes in the same direction with the number of decomposition levels, the key information in the lower scale coefficients will be obliterated and the feature extraction of modulation signal will be disturbed, which will lead to the decrease of recognition rate or recognition speed. Otherwise, if the exponential weight vector changes in the opposite direction with the number of decomposition levels, part of the noise in the small-scale coefficients will be amplified, so that the useful features of the modulated signal may not be extracted, which will also lead to the decrease of recognition rate. Of course, according to the different problems to be solved, different exponential weights can be applied to make the WACE achieve better analysis and processing effect, that is, the improved wavelet entropy has good portability in other fields.

## 5 Simulation Results and Analysis

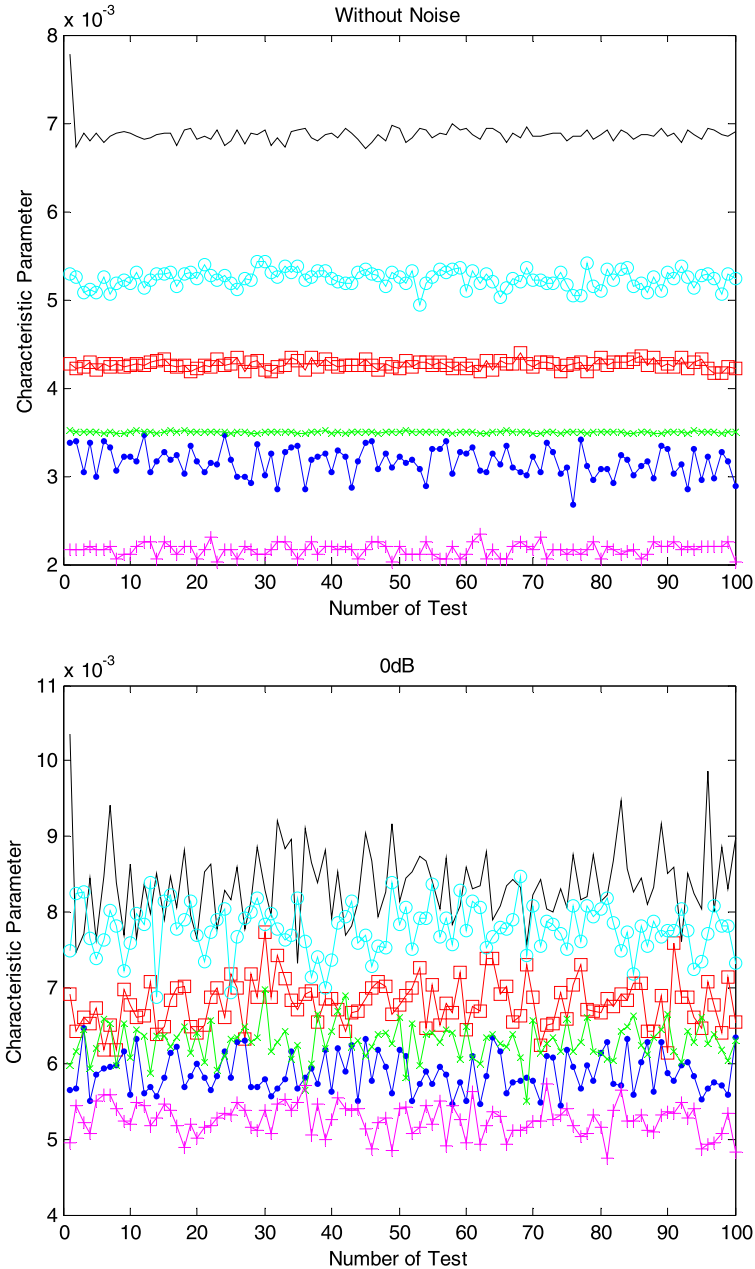
In this section, six kinds of modulation signals are simulated by using the model proposed in this paper, and the modulation recognition conditions under different SNR are compared. The performance comparison with other literature schemes is carried out to prove the effectiveness of the proposed method.

For AWE, because it is a function of signal, its average value is used as the characteristic parameter of signal recognition; for WEE, the wavelet energy entropy of signal is directly used as the feature parameter of signal recognition; for WACE, each element in entropy vector is used for linear weighting to obtain the characteristic parameter of signal recognition (Table 1).

**Table 1.** Simulation parameters

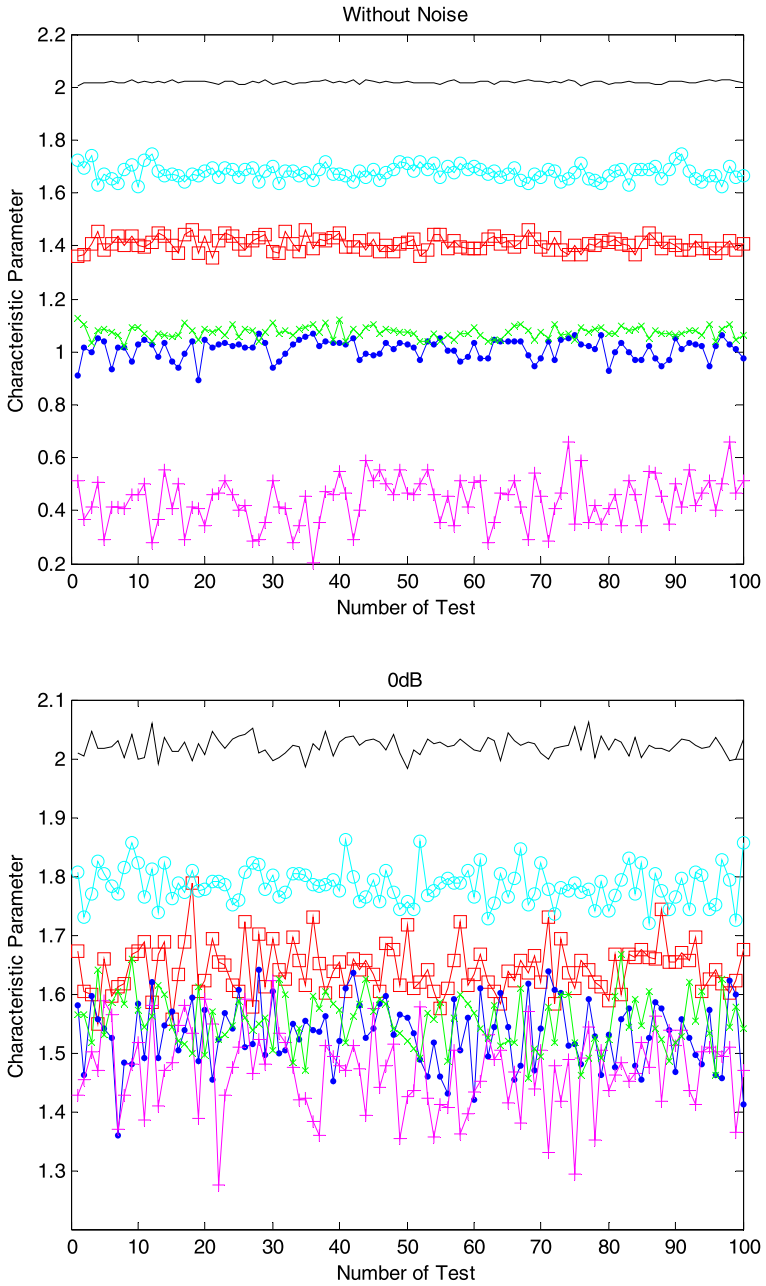
Sampling rate	10 MHz	Chip rate	0.5 MHz
Carrier frequency	0–6 MHz	Noise	AWGN
SNR	–5–10 dB	Wavelet basis	db5
Wavelet decomposition level	5	Number of tests	100

It can be seen from Fig. 2 that in the ideal environment without noise, the three kinds of wavelet entropy can distinguish six kinds of modulation signals, and the recognition is completely correct. However, with the decrease of SNR, the recognition performance of AWE and WEE method for six kinds of modulation signals decreases



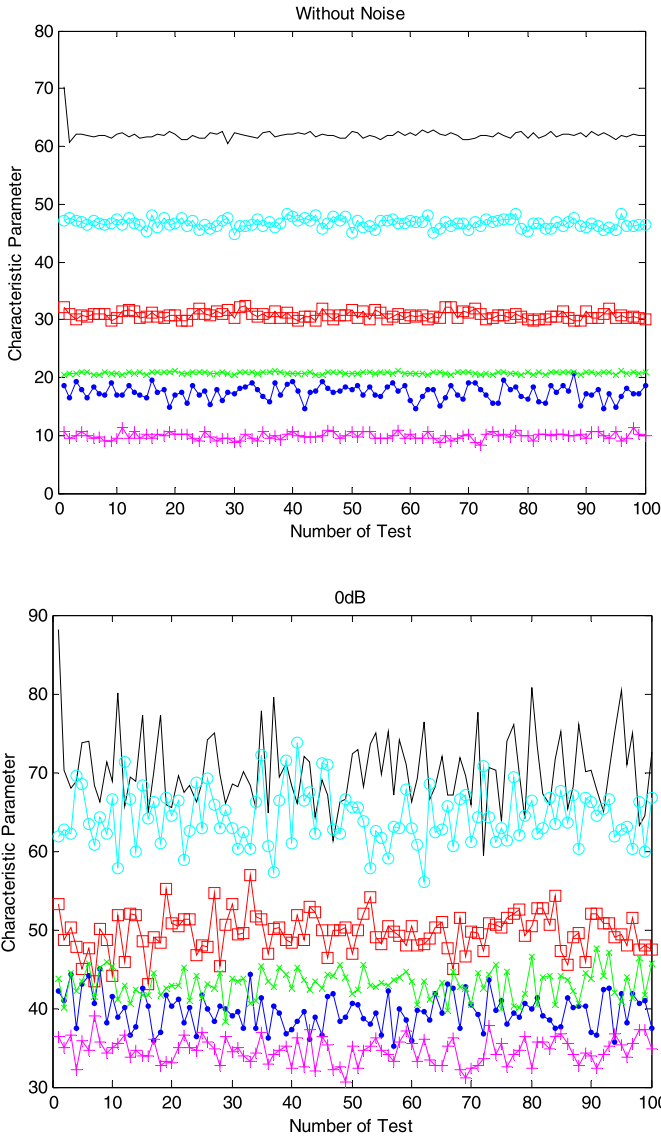
(a) AWE (Without Noise& SNR of 0dB)

**Fig. 2.** Comparison of characteristic parameters of different wavelet entropy for different modulated signals.



(b) WEE (Without Noise& SNR of 0dB)

**Fig. 2.** (continued)



(c) WACE (Without Noise& SNR of 0dB) Linear weight:[8,16,32,16,8,4]

**Fig. 2.** (continued)

obviously, and the range of characteristic parameters of each signal has significant cross aliasing. In comparison, the recognition performance of WACE is not much reduced. With SNR of 0dB, the probability of correct recognition of all six signals is still above 70%, which is much higher than the other two wavelet entropy recognition methods.

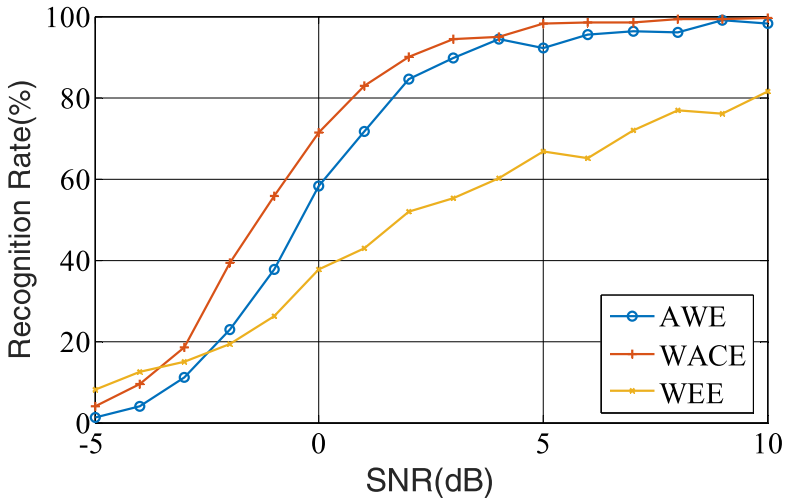


Fig. 3. Comparison curve of recognition rate of different wavelet entropy.

According to the decision tree, the modulation signals are identified in Fig. 3. The recognition rate of the three wavelet entropy methods is very high under high SNR, so we mainly consider the recognition under the condition of low SNR. It can be seen from the figure that compared with AWE, the recognition rate of WACE method is improved by about 15% at low SNR. When the SNR is higher than 2 dB, the correct recognition rate of WACE is higher than 90%. And when the SNR is higher than 5 dB, the recognition rate is stable above 98%. Compared with WEE method, except the recognition rate at  $-5$  dB, the recognition rate of WACE is much higher than that of WEE method under other SNR. Simulation results show that compared with the existing methods, the proposed WACE has better performance in feature extraction, stronger anti-interference ability, and the computational complexity is almost the same as that of the original method. At the same time, it can optimize the selection of exponential weight vector and linear weight vector to achieve better recognition performance.

## 6 Conclusions

Based on the traditional wavelet energy entropy and adaptive wavelet entropy, a new improved wavelet entropy, wavelet approximate coefficient entropy, is proposed. It can extract the correlation features from the modulated signal better and has better anti-noise performance. In this paper, the system model of modulation recognition is established and six typical modulation modes of communication signals are selected. The simulation results show that the modulation recognition method based on wavelet approximate coefficient entropy can improve the performance of modulation recognition in low signal-to-noise ratio, and the effectiveness of the modulation recognition method is proved.

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## References

1. Mitola, J., Maguire, G.Q.: Cognitive radio: making software radios more personal. *IEEE Pers. Commun.* **6**(4), 13–18 (1999)
2. Dong, S., Li, Z., Zhao, L.: A modulation recognition algorithm based on cyclic spectrum and SVM classification. In: 2020 IEEE 4th Information Technology, Networking, Electronic and Automation Control Conference (ITNEC), Chongqing, China, pp. 2123–2127 (2020)
3. Yang, Y., Zhang, X.: A modified method for digital modulation recognition based on instantaneous signal features. In: 2019 3rd International Conference on Electronic Information Technology and Computer Engineering (EITCE), Xiamen, China, pp. 1351–1354 (2019)
4. Huang, Y., Jin, W., Li, B., et al.: Automatic modulation recognition of radar signals based on Manhattan distance-based Features. *IEEE Access* **7**, 41193–41204 (2019)
5. Bai, J., Gao, L., Gao, J., et al.: A new radar signal modulation recognition algorithm based on time-frequency transform. In: 2019 IEEE 4th International Conference on Signal and Image Processing (ICSIP), Wuxi, China, pp. 21–25 (2019)
6. Zhao, Y., Xu, Y., Jiang, H., et al.: Recognition of digital modulation signals based on high-order cumulants. In: 2015 International Conference on Wireless Communications & Signal Processing (WCSP), Nanjing, pp. 1–5 (2015)
7. Mohammed, E., Abdwahad, A., Ali, G.: Spoken arabic digits recognition using discrete wavelet. In: 2014 UKSim-AMSS 16th International Conference on Computer Modelling and Simulation, Cambridge, pp. 275–279 (2014)
8. Wang, L., Guo, S., Jia, C.: The digital modulation recognition technique based on the wavelet envelope difference. *Appl. Electron. Tech.* **43**(2), 95–98 (2019)
9. Chen, J., Kuo, Y., Li, J., et al.: Modulation identification of digital signals with wavelet transform. *J. Electron. Inf. Technol.* **28**(11), 2026–2019 (2016)
10. Chen, L., Qu, W.: Detection of aero engine instability based on wavelet entropy theory. *Microcomput. Appl.* **27**(6), 54–57 (2011)
11. Avci, E., Avci, D.: The performance comparison of discrete wavelet neural network and discrete wavelet adaptive network based fuzzy inference system for digital modulation recognition. *Exp. Syst. Appl.* **35**(1–2), 90–101 (2008)
12. Zhang, C., Yang, L., Wang, X.: Discrete wavelet neural network group system for digital modulation recognition. In: 2011 IEEE 3rd International Conference on Communication Software and Networks, Xi'an, pp. 603–606 (2011)