Robust Transmission Design for IRS-Aided MISO Network with Reflection Coefficient Mismatch

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Abstract. Intelligent reflection surface (IRS) has been recognized as a revolutionary technology to achieve spectrum and energy efficient wireless communications due to its capability to reconfigure the propagation channels. However, due to the limited cost and space of each reflection element, it is difficult to accurately adjust the reflection coefficients of the passive elements. In this paper, we propose a worst-case robust reflection coefficient design for an IRS-aided single-user multiple-input single-output (SU-MISO) system where one IRS is deployed to enhance the received signal quality. Based on the fact of imperfect adjustment of reflection coefficients, our goal is to minimize the transmission power subject to the signal-noise ratio (SNR) constraint on the receiver end and the unit-modulus constraints on the reflection coefficients. The resulting optimization problem is non-convex and in general hard to solve. To tackle this problem, we adopt the linear approximation and alternating optimization (AO) methods to convert the original optimization problem into a sequence of convex subproblems that could be efficiently solved. We then extend our work to a practical situation where only limited phase shifts at each element are available. Numerical results demonstrate the robustness of the transmission scheme and show that high resolution for phase shifts is not an essential condition to approach the ideal performance.

Keywords: Intelligent reflection surface (IRS) · Robust design · Convex optimization · Reflection coefficient error · Discrete phase shift

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1 Introduction

Although the 5th generation (5G) cellular systems are still under deployment globally, both academia and industry have begun to seek next-generation solutions that are faster, smarter, and greener. Thanks to the technological breakthrough in meta-surfaces, the intelligent reflection surface (IRS) has drawn wide attention recently as a promising technology to achieve smart and reconfigurable wireless environments in the physical layer [1–3]. Specifically, the IRS is a planar composed of a large number of passive elements, each of which could independently adjust the amplitude and phase of the incident signals, thus collaboratively improve the performance of communication systems by upgrading the propagation environments of wireless signals [4].

Note that the IRS is significantly different from the existing technologies such as the amplify-and-forward (AF) relaying networks and the conventional massive multiple-input multiple-output (MIMO) systems [5]. Firstly, different from AFs that actively amplify and forward the received signals, the IRS is generally passive, which incurs a negligible power consumption. Secondly, the IRS is also different from the active antennas in massive MIMO due to their different architectures (passive versus active) and operating mechanisms (reflect versus transmit), resulting in a relatively low cost. Besides, the IRS is able to create a virtual line-of-sight (LoS) channel to bypass obstacles, which is meaningful for applications of millimeter wave (mmWave).

Due to the aforementioned advantages, the IRS has been considered as a key enabler for future wireless communications. The acquisition of channel state information (CSI) of auxiliary links [6–9], and the transmission/beamforming vector design [10–13] are two major design issues about IRS-aided communications. However, the above-mentioned works are all based on the perfect adjustment of reflection coefficients, which is impractical due to the complexity and cost constraints on reflection elements. In addition, prior works mainly consider continuous phase shifts at the IRS, which is also difficult to realize.

We note that there are several works focusing on the discrete phase optimization [14,15], but very complex processing methods are required. To address this problem, we investigate the worst-case robust transmission design problem based on the inaccurate reflection coefficients, which could also be interpreted as phase mismatch due to instability of meta-surfaces. More specifically, our goal is to minimize the total transmission power subject to the signal-noise ratio (SNR) constraint on the receiver end and the unit-modulus constraints on the reflection coefficients while ensuring no outage happens under all possible reflection coefficient error realizations. Numerical simulations were conducted which verified the robustness of the proposed transmission scheme with the obtained reflection phases being set to the nearest discrete values to evaluate the practical robust performance of the IRS-aided communication. Our main contributions can be summarized as follows:

- The original optimization problem is non-convex and generally hard to solve. By employing the linear approximation and alternating optimization (AO)

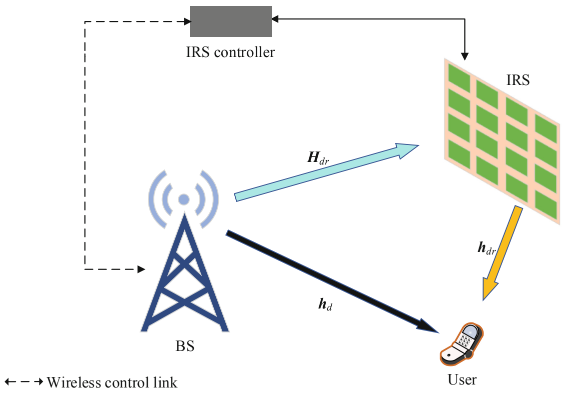

Fig. 1. IRS-aided communication system.

methods, we successfully convert the original problem into a sequence of convex subproblems that could be efficiently solved by using the CVX toolbox.
- We showed that a very high resolution for phase shifts is not necessary to achieve the ideal performance.

2 The System Model

2.1 Transmission Model

We consider an IRS-aided single-user multiple-input single-output (SU-MISO) system where the base station (BS) and the IRS are equipped with N antennas and M reflection elements, respectively, as depicted in Fig. 1. The channel coefficients from the BS to the user, from the IRS to the user, and from the BS to the IRS are denoted by $\boldsymbol{h}_d \in \mathbb{C}^{N\times 1}$, $\boldsymbol{h}_{dr} \in \mathbb{C}^{M\times 1}$, and $\boldsymbol{H}_{dr} \in \mathbb{C}^{M\times N}$, respectively. Let $\boldsymbol{\theta} = [e^{j\theta_1}, \ldots, e^{j\theta_M}]$ denote the phase shifts and a diagonal matrix $\boldsymbol{\Theta} = \mathrm{diag}(\beta_1 e^{j\theta_1}, \ldots, \beta_M e^{j\theta_M})$ denote the corresponding reflection coefficient matrix. The reflection amplitude and phase shift of the nth element at the IRS are given by $\beta_n \in [0, 1]$ and $\theta_n \in [0, 2\pi)$, respectively. In order to maximize the signal reflection power, we set $\beta_n = 1, n \in \mathcal{M}$, where $\mathcal{M} = [1, \ldots, M]$, and then investigate the phase shifts optimization. Practically, it is difficult to realize continuous phase shifts due to cost and hardware limitations. Motivated by this fact, we first focus on the optimization of continuous phase shifts and then extend our work to the practical cases where each element at the IRS only has finite levels for phase shifts. In this paper, we consider the linear precoding at the BS, with $\boldsymbol{f} \in \mathbb{C}^{N\times 1}$ denoting the precoding vector. The transmission power is denoted by $\|\boldsymbol{f}\|_2^2$. Then, the received signal at the user is given by

$$y = (\boldsymbol{h}_d^{\mathrm{H}} + \boldsymbol{h}_{dr}^{\mathrm{H}} \boldsymbol{\Theta} \boldsymbol{H}_{dr}) \boldsymbol{f} s + n_g \quad (1)$$

where s is the transmitted information carrying symbol with zero mean and unit variance, i.e., $s \sim \mathcal{CN}(0, 1)$, and n_g denotes additive white Gaussian noise (AWGN) at the receiver with zero mean and variance σ^2, i.e., $n_g \sim \mathcal{CN}(0, \sigma^2)$. Note that $\boldsymbol{h}_{dr}^{\mathrm{H}}\boldsymbol{\Theta}\boldsymbol{H}_{dr} = \boldsymbol{v}^{\mathrm{H}}\boldsymbol{G}$, where the cascaded channel $\boldsymbol{G} = \mathrm{diag}(\boldsymbol{h}_{dr}^{\mathrm{H}})\boldsymbol{H}_{dr}$, the passive beamforming $\boldsymbol{v} = [v_1, \ldots, v_M]^{\mathrm{H}}$, the nth entry $v_n = e^{j\theta_n}, n \in \mathcal{M}$. Accordingly, the SNR of the received signal is given by

$$\mathrm{SNR} = \frac{|(\boldsymbol{h}_d^{\mathrm{H}} + \boldsymbol{v}^{\mathrm{H}}\boldsymbol{G})\boldsymbol{f}|^2}{\sigma^2}. \tag{2}$$

2.2 Problem Formulation

As mentioned above, many of the current IRS-aided systems concentrate on the transmission design and the acquisition of CSI corresponding to the cascaded links. However, most of them are based on the assumption of perfect adjustment of reflection coefficients, which is too idealistic due to the limited space and cost of the passive elements. The performance of the IRS is highly affected by the accuracy of the reflection coefficient adjustment and thus this problem should be addressed. In this paper, we adopt the bounded error model to investigate the worst-case robust transmission design of the IRS-aided communication, i.e., the practical passive beamforming could be rewritten as

$$\hat{\boldsymbol{v}} = \boldsymbol{v} + \triangle\boldsymbol{v}$$

where $\|\triangle\boldsymbol{v}\|_2 \leq \epsilon$. Then, the worst-case optimization problem can be formulated as

$$\min_{\boldsymbol{f},\boldsymbol{v}} \quad \|\boldsymbol{f}\|_2^2 \tag{3a}$$

$$\text{s.t.} \quad \frac{|(\boldsymbol{h}_d^{\mathrm{H}} + \hat{\boldsymbol{v}}^{\mathrm{H}}\boldsymbol{G})\boldsymbol{f}|^2}{\sigma^2} \geq \gamma, \tag{3b}$$

$$\|\triangle\boldsymbol{v}\|_2 \leq \epsilon, \tag{3c}$$

$$|v_m|^2 = 1, \forall m \in \mathcal{M}. \tag{3d}$$

The constraint (3b) is the minimum SNR target for the user; (3c) is the bounded model for coefficient error; (3d) corresponds to the unit-modulus requirements for the reflection coefficients.

The above problem is non-convex due to the following reasons: *(a)* The constraint (3b) is a quadratic infinite inequality, the solution set of which is non-convex; *(b)* The precoding $\boldsymbol{f}$ and the passive beamforming $\boldsymbol{v}$ are coupled, which is non-convex and hard to optimize simultaneously; *(c)* The unit-modulus constraints (3d) are strongly non-convex and intractable.

3 Robust Optimization Solutions

In this section, we aim to propose the solutions to Problem (3) which is non-convex due to the reasons mentioned in Sect. 2. Following the non-convexity analysis, we first deal with the quadratic inequality in (3b) by linear approximation and then uncouple the variables by the alternating optimization (AO) method, thus convert the primal problem into a sequence of convex subproblems that could be efficiently solved.

3.1 Problem Transformation

We first handle the quadratic inequality in (3b) by using the linear approximation method.

Lemma 1. *Letting $\boldsymbol{f}^{(n)}$ and $\boldsymbol{v}^{(n)}$ be the optimal solution obtained at the nth iteration, the linear lower bound of $|(\boldsymbol{h}_d^{\mathrm{H}}+\hat{\boldsymbol{v}}^{\mathrm{H}}\boldsymbol{G})\boldsymbol{f}|^2$ at $(\boldsymbol{f}^{(\mathrm{n})},\boldsymbol{v}^{(\mathrm{n})})$ could be denoted by*

$$\triangle\boldsymbol{v}^{\mathrm{H}}\boldsymbol{X}\triangle\boldsymbol{v}+2\,\mathrm{Re}\{\boldsymbol{x}^{\mathrm{H}}\triangle\boldsymbol{v}\}+c \tag{4}$$

where

$$\begin{aligned}
\boldsymbol{X} =& \boldsymbol{G}\boldsymbol{f}\boldsymbol{f}^{\mathrm{H},(\mathrm{n})}\boldsymbol{G}^{\mathrm{H}}+\boldsymbol{G}\boldsymbol{f}^{(\mathrm{n})}\boldsymbol{f}^{\mathrm{H}}\boldsymbol{G}^{\mathrm{H}}-\boldsymbol{G}\boldsymbol{f}^{(\mathrm{n})}\boldsymbol{f}^{\mathrm{H},(\mathrm{n})}\boldsymbol{G}^{\mathrm{H}},\\
\boldsymbol{x} =& -\boldsymbol{G}\boldsymbol{f}^{(n)}\boldsymbol{f}^{\mathrm{H},(n)}\boldsymbol{h}_d-\boldsymbol{G}\boldsymbol{f}^{(n)}\boldsymbol{f}^{\mathrm{H},(\mathrm{n})}\boldsymbol{G}^{\mathrm{H}}\boldsymbol{v}^{(n)}\\
&+\boldsymbol{G}\boldsymbol{f}^{(n)}\boldsymbol{f}^{\mathrm{H}}\boldsymbol{h}_d+\boldsymbol{G}\boldsymbol{f}^{(n)}\boldsymbol{f}^{\mathrm{H}}\boldsymbol{G}^{\mathrm{H}}\boldsymbol{v}+\boldsymbol{G}\boldsymbol{f}\boldsymbol{f}^{\mathrm{H},(n)}\boldsymbol{h}_d\\
&+\boldsymbol{G}\boldsymbol{f}\boldsymbol{f}^{\mathrm{H},(n)}\boldsymbol{G}^{\mathrm{H}}\boldsymbol{v}^{(n)},\\
c =& \,2\,\mathrm{Re}\{\boldsymbol{f}^{\mathrm{H}}(\boldsymbol{h}_d\boldsymbol{h}_d^{\mathrm{H}}+\boldsymbol{G}^{\mathrm{H}}\boldsymbol{v}\boldsymbol{h}_d^{\mathrm{H}}+\boldsymbol{h}_d\boldsymbol{v}^{\mathrm{H},(n)}\boldsymbol{G}\\
&+\boldsymbol{G}^{\mathrm{H}}\boldsymbol{v}\boldsymbol{v}^{\mathrm{H},(n)}\boldsymbol{G})\boldsymbol{f}^{(n)}\}-\boldsymbol{f}^{\mathrm{H},(\mathrm{n})}(\boldsymbol{h}_d\boldsymbol{h}_d^{\mathrm{H}}+\boldsymbol{G}^{\mathrm{H}}\boldsymbol{v}^{(n)}\boldsymbol{h}_d^{\mathrm{H}}\\
&+\boldsymbol{h}_d\boldsymbol{v}^{\mathrm{H},(n)}\boldsymbol{G}+\boldsymbol{G}^{\mathrm{H}}\boldsymbol{v}^{(\mathrm{n})}\boldsymbol{v}^{\mathrm{H},(\mathrm{n})}\boldsymbol{G})\boldsymbol{f}^{(n)}.
\end{aligned}$$

■

Proof: Let a be a complex scalar variable. Then, the first-order Taylor expansion is expressed as

$$|a|^2\geq a^{(n),*}a+a^*a^{(n)}-a^{(\mathrm{n}),*}a^{(n)}.$$

The linear approximation inequality holds for any fixed $a^{(n)}$, where $a^{(n)}$ is a constant scalar. Then we replace a and $a^{(n)}$ with $(\boldsymbol{h}_d^{\mathrm{H}}+(\boldsymbol{v}^{\mathrm{H}}+\triangle\boldsymbol{v}^{\mathrm{H}})\boldsymbol{G})\boldsymbol{f}$ and $(\boldsymbol{h}_d^{\mathrm{H}}+(\boldsymbol{v}^{\mathrm{H},(n)}+\triangle\boldsymbol{v}^{\mathrm{H}})\boldsymbol{G})\boldsymbol{f}^{(n)}$, respectively. The proof is completed. □

With the lower bound (4) of $|(\boldsymbol{h}_d^{\mathrm{H}}+\hat{\boldsymbol{v}}^{\mathrm{H}}\boldsymbol{G})\boldsymbol{f}|^2$, we have

$$\triangle\boldsymbol{v}^{\mathrm{H}}\boldsymbol{X}\triangle\boldsymbol{v}+2\mathrm{Re}\{\boldsymbol{x}^{\mathrm{H}}\triangle\boldsymbol{v}\}+c-\gamma\sigma^2\geq 0. \tag{5}$$

Note that (5) contains random variable $\triangle\boldsymbol{v}$ and it is not known to be tractable. A possible way is to derive a bound for the error constraint for any $\|\triangle\boldsymbol{v}\|_2\leq\epsilon$ and then perform optimizations based on the bound. Here, similar to [16], we directly derive the exact equivalent condition from (5) by employing the S-lemma [17]. Thus, we have the equivalent constraint of (5) formulated as

$$\begin{bmatrix}\boldsymbol{X}+w\boldsymbol{I} & \boldsymbol{x}\\ \mathbf{x}^{\mathrm{H}} & c-\gamma\sigma_u^2-w\epsilon^2\end{bmatrix}\succeq 0, w\geq 0 \tag{6}$$

where $\boldsymbol{I}$ is an identity matrix and w is a slack variable. Thus we have the reformulated optimization problem

$$\min_{\boldsymbol{f},\boldsymbol{v},w} \quad \|\boldsymbol{f}\|_2^2 \tag{7a}$$

$$\text{s.t.} \quad \begin{bmatrix} \boldsymbol{X}+w\boldsymbol{I} & \boldsymbol{x} \\ \boldsymbol{x}^{\mathrm{H}} & c-\gamma\sigma^2-w\epsilon^2 \end{bmatrix} \succeq 0, \tag{7b}$$

$$|v_m|^2 = 1, \forall m \in \mathcal{M}, \tag{7c}$$

$$w \geq 0. \tag{7d}$$

According to the non-convexity analysis, Problem (7) is still non-convex due to the coupled precoding at the BS and passive beamforming at the IRS, which is hard to optimize simultaneously. Thus, in the next section, we propose to use the AO method to handle the coupling optimization problem by iteratively updating precoding with the passive beamforming fixed and optimizing passive beamforming with the precoding fixed, respectively.

3.2 The Alternating Optimization Method

We first fix the passive beamforming coefficient and optimize the precoding vector. With reflection coefficients fixed, the unit-modulus constraints (7c) could be ignored. We then have the subproblem of Problem (7) formulated as

$$\min_{\boldsymbol{f},w} \quad \|\boldsymbol{f}\|_2^2 \tag{8a}$$

$$\text{s.t.} \quad \begin{bmatrix} \boldsymbol{X}+w\boldsymbol{I} & \boldsymbol{x} \\ \boldsymbol{x}^{\mathrm{H}} & c-\gamma\sigma^2-w\epsilon^2 \end{bmatrix} \succeq 0, \tag{8b}$$

$$w \geq 0. \tag{8c}$$

Problem (8) is a semidefinite program (SDP) problem and could be solved by employing the CVX toolbox [18,19]. Note that in the above problem reflection coefficients are constant numbers, which satisfy $\boldsymbol{v} = \boldsymbol{v}^{(n)}$.

We then consider the subproblem about the passive beamforming. With the precoding fixed, Problem (7) is reduced to a feasibility-check problem. We introduce a slack variable $\eta \geq 0$ to improve the convergence speed, which could be interpreted as 'SNR residual'; see [11] for details. Thus (5) could be rewritten as

$$\triangle\boldsymbol{v}^{\mathrm{H}}\boldsymbol{X}\triangle\boldsymbol{v} + 2\mathrm{Re}\{\boldsymbol{x}^{\mathrm{H}}\triangle\boldsymbol{v}\} + c - \gamma\sigma^2 - \eta \geq 0. \tag{9}$$

The subproblem about the passive beamforming of Problem (7) could be formulated by

$$\max_{\eta,\boldsymbol{v},w} \quad \eta \tag{10a}$$

$$\text{s.t.} \quad \begin{bmatrix} \boldsymbol{X}+w\boldsymbol{I} & \boldsymbol{x} \\ \boldsymbol{x}^{\mathrm{H}} & c-\gamma\sigma^2-w\epsilon^2-\eta \end{bmatrix} \succeq 0, \tag{10b}$$

$$w \geq 0, \eta \geq 0, \tag{10c}$$

$$|v_m|^2 = 1, \forall m \in \mathcal{M}. \tag{10d}$$

Algorithm 1. Penalty convex-concave procedure optimization for IRS

```
Initialize: set i = 0, and initialize v^[0], p > 1.
repeat
    if i < I_max then
        Update v^[i+1] from Problem (11);
        λ^[i+1] = min{pλ^[i], λ_max};
        i = i + 1;
    else
        Initialize with a new random v^[0], and set i = 0.
    end if
until ||b||_2 ≤ χ and ||v^[i] − v^[i−1]||_2 ≤ z
Output v^(n+1) = v^[i].
```

It is worth noting that Problem (10) is still non-convex due to the unit-modulus constraints (10d). Here we adopt penalty convex-concave procedure (CCP) to handle the non-convex part [10]. Following the penalty framework, our first step is to relax the unit-modulus constraints and then implement penalty on the target function according to the degree of violation of the constraints. The intensity of penalty increases as the number of iterations goes on. The modulus of each reflection coefficient is compelled to be 1 when the penalty is sufficiently large.

Specifically, $|v_m|^2 = 1, \forall m \in \mathcal{M}$ could be rewritten as $1 \leq |v_m|^2 \leq 1, \forall m \in \mathcal{M}$. Note that the constraints $|v_m|^2 \geq 1, \forall m \in \mathcal{M}$ are non-convex, which are again handled by linear approximation, i.e., $|v_m^{[i]}|^2 - 2\mathrm{Re}(v_m^* v_m^{[i]}) \leq -1, \forall m \in \mathcal{M}$, where $v_m^{[i]}$ is the optimal solution obtained at the ith iteration. Here, we let the modulus of $v_m, \forall m \in \mathcal{M}$ violate the unit-modulus constraints and impose slack variables $\boldsymbol{b} = [b_1, \ldots, b_{2M}]^{\mathrm{T}} \geq 0$ over the relaxed constraints, i.e., $|v_m^{[i]}|^2 - 2\mathrm{Re}(v_m^* v_m^{[i]}) \leq b_m - 1, |v_m|^2 \leq 1 + b_{M+m}, \forall m \in \mathcal{M}$. The slack variables could be interpreted as the penalty term for optimization variables that violate the unit-modulus constraints. Thus, we have the reformulated problem as follows

$$\max_{\eta, \boldsymbol{v}, w, \boldsymbol{b}} \quad \eta - \lambda^{[i]} \|\boldsymbol{b}\|_2 \tag{11a}$$

$$\text{s.t.} \quad \begin{bmatrix} \boldsymbol{X} + w\boldsymbol{I} & \boldsymbol{x} \\ \boldsymbol{x}^{\mathrm{H}} & c - \gamma\sigma^2 - w\epsilon^2 - \eta \end{bmatrix} \succeq 0, \tag{11b}$$

$$w \geq 0, \eta \geq 0, \boldsymbol{b} \geq 0, \tag{11c}$$

$$|v_m^{[i]}|^2 - 2\mathrm{Re}(v_m^* v_m^{[i]}) \leq b_m - 1, \forall m \in \mathcal{M}, \tag{11d}$$

$$|v_m|^2 \leq 1 + b_{M+m}, \forall m \in \mathcal{M}. \tag{11e}$$

The above problem is an SDP and could be solved by the CVX tool, and the algorithm is summarized in Algorithm 1.

Note that the feasible solution directly obtained from Problem (11) may not be the solution of Problem (10) due to the limited intensity of penalty and iterations. The feasibility is guaranteed by imposing a strong penalty over the target function and a large number of iterations. The variable $\lambda^{[i]}$ is the

Algorithm 2. Alternating optimization

1: **Initialize:** initialize $\boldsymbol{v}^{(0)}$ and $\boldsymbol{f}^{(0)}$, set $n = 0$
2: **repeat**
3: Update $\boldsymbol{f}^{(n+1)}$ from Problem (8);
4: Update $\boldsymbol{v}^{(n+1)}$ from Problem (10);
5: $n = n + 1$;
6: **until** The objective value (3) converges.

regularization factor to measure the influence of the penalty factor $\|\boldsymbol{b}\|_2$; $\|\boldsymbol{b}\|_2 \leq \chi$ and $\|\boldsymbol{v}^{[i]} - \boldsymbol{v}^{[i-1]}\|_2 \leq z$ control the convergence of Algorithm 1, see [10] for more details. Note that in Problems (10) and (11) the precoding vector is constant, which satisfies $\boldsymbol{f} = \boldsymbol{f}^{(n)}$. We iteratively solve Problem (8) and Problem (10), thus the reformulated Problem (7) could be efficiently solved. The alternating optimization algorithm is summarized in Algorithm 2. The convergence of AO is guaranteed because the feasible solution to Problem (10) is also feasible to Problem (8), thus the value of target function (3), i.e., the total transmission power is non-increasing. Therefore, AO is guaranteed to converge.

It is worth noting that only limited quantization bits are deployed into each element at the IRS practically. Hence, only a finite number of phase shift levels are available at the passive elements. Specifically, let q denote quantization bits for phase shifts at each element, and the discrete values are generated by uniformly quantizing the continuous phase shift set $[0, 2\pi)$. Thus, the set of discrete phase shift values is given by

$$\mathcal{F}_d = \{0, \triangle\theta, \ldots, (L-1)\triangle\theta\} \tag{12}$$

where $\triangle\theta = 2\pi/L, L = 2^q$. We then set the phases obtained from AO to the nearest discrete values to evaluate the practical performance of IRS-aided communication.

4 Simulation Results

In this section, numerical simulations are performed to verify the robustness of the proposed algorithm and evaluate the performance of the practical IRS-aided communication systems.

We consider a uniform linear array (ULA) at the BS equipped with $N = 6$ antennas and a uniform rectangular array (URA) at the IRS equipped with $M = 16$ elements. A rectangular coordinate is adopted to locate the system, i.e., the BS is located at (0 m, 0 m), the IRS is located at (50 m, 10 m), and the user is located at (70 m, 0 m), respectively. The distance-dependent path loss model is given by $PL = -30 - 10\alpha \log_{10}(d)$ dB, where α is the path loss exponent and d is the distance in meters. We adopt Ricean channels to model $\boldsymbol{h}_d$, $\boldsymbol{h}_{dr}$, and $\boldsymbol{H}_{dr}$. All the Ricean factors are set to be 5. The LoS parts are given by the product of the steering vectors of the transmitter and receiver. The non-LoS parts follow Rayleigh distribution. The adjustment error bound of IRS reflection coefficients

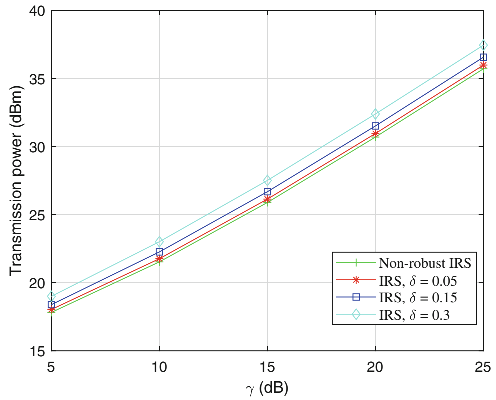

Fig. 2. Transmission power versus target SNR γ.

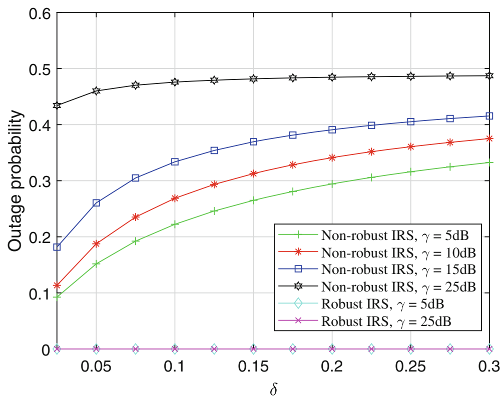

Fig. 3. Outage probability versus coefficient uncertainty δ.

is defined as $\epsilon = \delta\|\boldsymbol{v}\|_2$, where δ could be interpreted as the amount of error. The power of AWGN is set to be -80 dBm.

Figure 2 shows the transmission power versus the target SNR of the user. It is observed that the total power increases with the target SNR and the coefficient error δ. The transmission power of the robust design is higher than that of the non-robust design as expected. The additional power can be interpreted as the price to overcome the performance degradation caused by the mismatch. Note that even when the uncertainty δ in reflection coefficients is highly increased, the problem is still feasible at the cost of more energy consumption.

Figure 3 plots the outage probability of the non-robust transmission design. The outage probability is defined as the probability that the target SNR of the user is not satisfied. It is observed that the outage probability increases with the coefficient error δ and the target SNR γ when we ignore the error of reflection

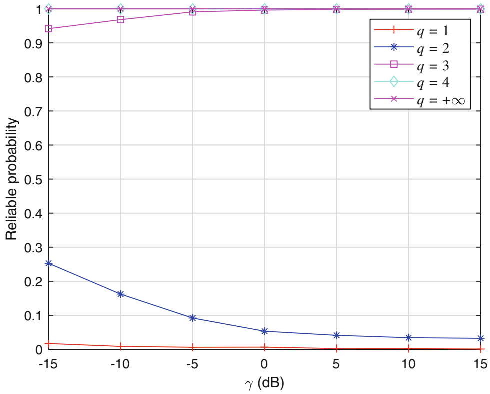

Fig. 4. Reliable probability versus target SNR γ under $\delta = 0.05$.

coefficients. However, when the robust transmission scheme is adopted, we can guarantee that no outage occurs.

In Fig. 4, we extend our work to the practical case where only finite phase levels are available. The reliable probability is defined as the probability that the SNR of the received signal is higher than γ. It is observed that when the number of quantization bits is small, i.e., 1-bit or 2-bit, the IRS-aided communication suffers severe performance degradation. However, the 3-bit IRS is arguably able to approach the ideal communication quality.

5 Conclusion

We have investigated the robust transmission design problem based on the inaccurate adjustment of the reflection coefficients at the IRS. The non-convex parts of the obtained optimization problem were successfully tackled by using the linear approximation and AO methods and thus could be efficiently solved. Numerical results verified the robustness of the transmission scheme and showed that high resolution for phase shifts is not mandatory to approach the ideal IRS performance.

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