# An Efficient Compression and Reconstruction Framework for Electromagnetic Spectrum Data

Dong Xiao, Jiangzhi Fu, Lu Sun, and Yun Lin(✉)

College of Information and Communication, Harbin Engineering University, Harbin, People's Republic of China
linyun@hrbeu.edu.cn

**Abstract.** Spectrum monitoring often demands a great quantity of spectrum data, and the massive characteristics of spectrum data make it consume a lot of resources in the process of transmission and storage. At the same time, the compressed acquisition system of spectrum data often has the problem of low reconstruction accuracy of the original data, and the reconstruction accuracy and compression performance cannot be achieved simultaneously. This paper studies the factors affecting the reconstruction error in the process of electromagnetic spectrum data utilization. In this paper, an electromagnetic spectrum compression and reconstruction framework called QRK-SVD is proposed. Aiming at the problems of slow dictionary convergence and low accuracy in dictionary learning, QRK-SVD purposely uses k-means clustering to construct the initial dictionary, which effectively improves the compression accuracy and system robustness. QRK-SVD increases the minimum singular value of sensing matrix through QR decomposition to optimize the problem of low accuracy of random observation matrix in compressed system. We designed a set of spectrum data acquisition and compression system based on QRK-SVD. It can adapt to various collection scenarios, greatly reduce the amount of data transmitted and stored, and has high reconstruction accuracy. The measured data proves that the performance of QRK-SVD is better than the traditional K-SVD framework in different data compression situations.

**Keywords:** Dictionary learning · QRK-SVD · Electromagnetic spectrum · Data compression

## 1 Introduce

With the increasing update of information science, wireless network devices can be seen everywhere in daily life, which also leads to the increasing shortage of spectrum resources. The spectrum resources in the electromagnetic space are distributed unevenly, scheduling is uneven, and the electromagnetic environment is facing deterioration. The utilization rate of a large number of spectrum resources is low, and only 15% to 85% of spectrum resources are utilized [1]. Spectrum resources are limited, and the available spectrum resources are difficult to meet the needs of various communication services. Exponential growth of radio services, repeated prohibitions of illegal radio equipment,


Y. Chenggang et al. (Eds.): MobiMedia 2022, LNICST 451, pp. 67–80, 2022.
https://doi.org/10.1007/978-3-031-23902-1_6

ecological deterioration of environmental noise and other factors make it imperative to effectively and reasonably monitor and manage spectrum resources [2]. The management of spectrum utilization inevitably requires a great quantity of spectrum data. Currently, with the development of networking radar, electromagnetic space network, wireless communication and other technologies, a large amount of electromagnetic spectrum data has been collected in various regions. Electromagnetic spectrum data is a kind of electromagnetic big data, which has the characteristics of high dimension, many types, and wide sources. The amount of electromagnetic spectrum data collected for a long time is extremely large, which makes it extremely inconvenient to transmit, store, and use spectrum data in real time [3, 4].

In order to solve the "dimension disaster" caused by massive data, Candes et al. proposed Compressed Sensing (CS) [5]. Compressed sensing is a technology that can effectively reduce the data sampling rate and compress data redundancy. However, most signals do not have sparseness, and the theory of signal sparse representation came into being. Its purpose is to linearly represent a given non-sparse signal using some coefficients. The sparse representation of the signal meets the requirements of compressed sensing technology and can be compressed, transmitted and stored.

For the problem of dictionary learning, authors in [6] proposed the Sparsenet dictionary learning algorithm [7], which uses the sparse coding of images to emergence the receptive field of a single cell. The optimization effect of the algorithm in the literature [6] is not ideal. To further optimize the above algorithm, a method of updating the dictionary by alternating optimization is proposed, namely MOD (Method of optimal directions) [8]. However, MOD requires matrix inversion, which leads to high algorithm complexity. K-SVD [9] is proposed on the basis of MOD by updating dictionary atoms one by one, which does not need to calculate matrix inverse and is more efficient. Dictionary learning under various constraints has been proposed one after another, and online dictionary learning with batch dynamic updates is proposed [10]. In this paper, we study the sparse representation and compression and reconstruction process of electromagnetic spectrum data. Aiming at the acquisition, compression, transmission, storage and reconstruction of electromagnetic spectrum data, the QRK-SVD framework is proposed to promote the compression and reconstruction capability of spectrum data.

## 2 Dictionary Learning and QRK-SVD

### 2.1 Dictionary Learning

As shown in Fig. 1, let $X$ be a discrete signal. $X$ satisfies $X = \Psi\theta$ under a specific set of basis (dictionary) $\Psi$, where $\theta$ is sparse, signal $X$ is said to be sparse under basis $\Psi$, where $\Psi$ is called the sparse basis of signal $X$, and $\theta$ is marked as the sparse representation of signal $X$ under sparse basis $\Psi$. Different sparse bases are generally not universal. Sparse bases are usually for a specific signal. For example, common natural image signals are sparse under discrete cosine transform basis, discrete wavelet transform basis or discrete fourier transform basis.

Moreover, the performance of general sparse basis can not meet our requirements. Dictionary learning can design a dictionary with less reconstruction error according to the signal. One of the widely used algorithms is K-SVD. It mainly includes two

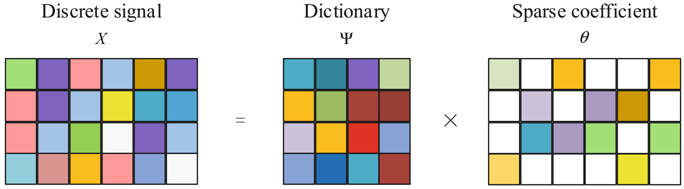


**Fig. 1.** Signal sparse representation

parts: sparse decomposition and dictionary updating. First, $D$ is generated as the initial dictionary, and the corresponding optimization problem is:

$$\min_{D,\theta}\left\{\|X - D\theta\|_F^2\right\} \quad \text{s.t. } \|\theta_i\|_0 \leq s, \forall i. \tag{1}$$

where $X$ is the discrete signal and $\theta$ is the sparse coefficient matrix. Then fix dictionary $D^{(t-1)}$ and use OMP algorithm to solve the following optimization problems:

$$\min_{\theta_i}\left\{\left\|x_i - D^{(t-1)}\theta_i\right\|_2^2\right\} \quad \text{s.t. } \|\theta_i\|_0 \leq s, \forall i. \tag{2}$$

When K-SVD updates the dictionary, each atom is updated. The error matrix of the signal corresponding to the $k$-th atom removed is:

$$E_k = X - \sum_{j \neq k} d_j \theta_{\mathrm{T}}^j. \tag{3}$$

where $\theta_{\mathrm{T}}^j$ is the $j$-th row vector of $\theta$. Then perform singular value decomposition on $E_k^R$, which is a matrix determined by the nonzero-valued indices of $\theta_{\mathrm{T}}^j$:

$$E_k^R = U\Sigma V^{\mathrm{T}}. \tag{4}$$

Update the dictionary $\hat{d}_k = u_1$ and the coefficient $\hat{\theta}_k^R = \Sigma[1, 1]v_1$ at the same time.

## 2.2 QRK-SVD

We often need to reconstruct a high-dimensional original sparse matrix from a low-dimensional observation matrix, as shown in Eqs. (5) and (6). The most commonly used method is orthogonal matching pursuit. We can linearly transform the signal $X$ to get $Y$. When reconstructing $X$ from $Y$, there will be errors in both the matching algorithm and dictionary learning. We propose to choose a more accurate initial dictionary in dictionary learning and improve the sensing matrix to make observations contain more information.

$$Y = \Phi X = \Phi D\theta \tag{5}$$

$$\theta_i^* = \arg\min_{\theta_i^* \in R^m} \|\theta_i\|_0, \; s.t \; \Phi D\theta_i = Y. \tag{6}$$

In dictionary learning, the selection of the initial matrix is very important. For real-time systems, the efficiency of data compression is very important. It is worth thinking about how to achieve a high reconstruction SNR with fewer iterations. In K-SVD, the initial dictionary selection method is to randomly select several columns of $X$, which is random and has poor performance. In QRK-SVD, each column of $X$ is clustered by k-means, and each cluster center $\mu_i$ is selected as the initial dictionary, see Eq. (7). $x_i$ that is not in the initial dictionary can be well represented by its own class center $\mu_{ji}$, as shown in Eq. (8). The overall error of the initial dictionary of QRK-SVD will be smaller than that of K-SVD, see Eq. (9).

$$D_1 = (\mu_1, \mu_2, \ldots, \mu_m) \tag{7}$$

$$x_i \approx D_1\theta_i^* = (\mu_{1i} \ldots \mu_{ji} \ldots \mu_{mi}) \begin{pmatrix} \theta_{1i}^* \\ \vdots \\ \theta_{ji}^* \\ \vdots \\ \theta_{mi}^* \end{pmatrix} \tag{8}$$

$$\left\| Y - D_1^*\theta^* \right\|_2 \leq \left\| Y - D_1\theta^* \right\|_2. \tag{9}$$

where $D_1^*$ represents initial dictionary for QRK-SVD, $D_1$ represents initial dictionary for K-SVD.

In coefficient reconstruction, the selection of the observation matrix is very important, and the observation matrix is generally selected as a Gaussian random matrix. However, it is not easy to generate random numbers in hardware, and the precision of random matrices is not high. Literature research shows that so as to enhance the independence of matrix column vector, it is essential to make its minimum singular value larger. The QR decomposition can increase the minimum singular value of the matrix without changing the original properties of the matrix, so we use QR decomposition to increase the reconstruction accuracy of the sensing matrix. The QRK-SVD data compression and reconstruction framework is shown in Fig. 2.

QR decomposition decomposes the transpose of the sensing matrix $\Psi^T$ into $Q$ matrix and $R$ matrix, as shown in Eq. (10). Perform Then set all elements in $R$ except on the diagonal to zero to obtain matrix $\frown{R}$, as shown in Eq. (11), matrix $I$ is a unit matrix. The optimized matrix $\frown{\Psi}$ is obtained by multiplying the matrix $Q$ and $\frown{R}$, as shown in Eq. (12).

$$\Psi^T = (\Phi D)^T = QR \tag{10}$$

$$\overset{\frown}{R} = IR \tag{11}$$

$$\overset{\frown}{\Psi} = (Q\overset{\frown}{R})^T \tag{12}$$

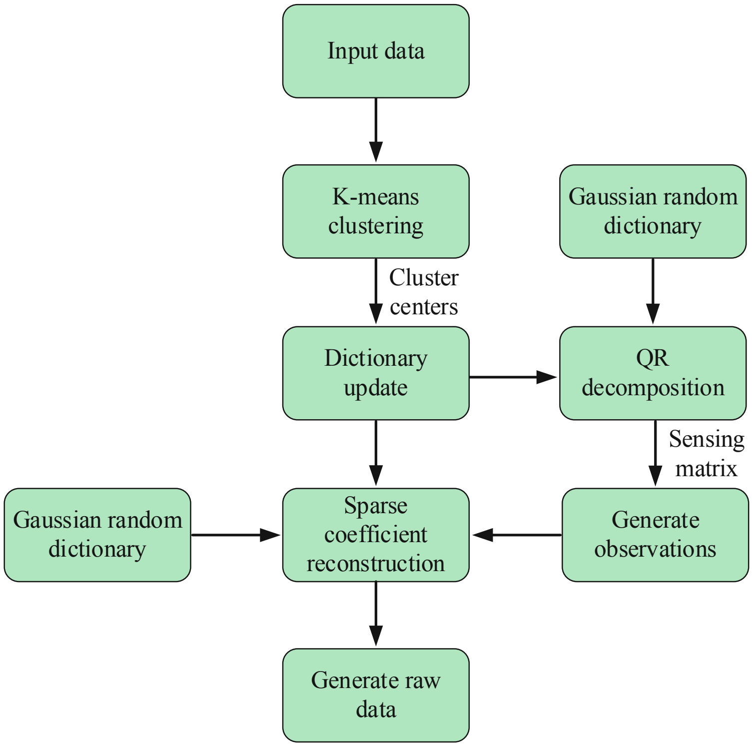


**Fig. 2.** The QRK-SVD data compression and reconstruction framework

## 3 Data Set and System Model

### 3.1 Electromagnetic Spectrum Data Set

The experimental dataset is an open-source measured spectrum dataset from the Broadband Spectrum Observatory of the Wireless Networks and Communications (WiNCom) Research Center of the Illinois Institute of Technology (IIT). The spectrum dataset was collected at a fixed location in Turku, Finland, and included a continuous 8-day spectrum data from January 20 to January 27, 2015 to 2018. The total dataset size is over 900 GB, and the acquisition frequency band is from 30 MHz to 6 GHz. See Table 1 for details.

In most cases, what we want to analyze and study is often a certain frequency band that may have key signals, rather than all frequency bands, so in the experiment, we consider the compression of data in one frequency band. The experimental data set selects the spectrum data of the first frequency band in the data collected in 2018, and the frequency band of the data is 17.93–22.14 MHz. Figure 3 shows the original time-frequency graph of the data from 8:00 am to 9:00 am in the second day.

### 3.2 System Model

The electromagnetic spectrum data compression storage system is shown in Fig. 4. On the local end, spectrum data collection is mainly implemented, which is divided into nyquist

**Table 1.** Dataset information

| Data information | |
|---|---|
| Data volume | Over 900 GB |
| Collecting time | In the four years from 2015 to 2018, there are 32 days from January 20 to January 27 |
| Collecting frequency band | 30 MHz~6 GHz |
| Time resolution | 10 s for 30 MHz~130 MHz and<br>3 s for 130 MHz~6 GHz |
| Frequency resolution | 78.125 kHz for 30 MHz~130 MHz and 3 GHz~6 GHz,<br>39.0625 kHz for 130 MHz~3 GHz |
| Data form | Spectrum power value and GPS data |
| Data format | Mat file format and excel table |

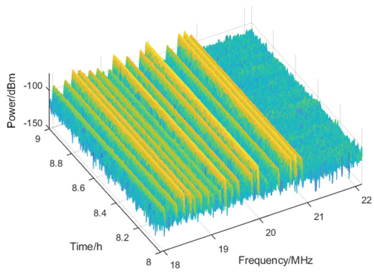


**Fig. 3.** Time-frequency graph of the second day spectrum

sampling and undersampling. Nyquist sampled data is compressed by CS module and QR decomposition. The compressed value is transmitted over the network to the cloud, where the compressed value is stored in the cloud database. Then the compressed value can generate initial dictionary, and dictionary learning is performed, and the dictionary is fed back to the local end. The sparse coefficients are reconstructed in the cloud using the dictionary and compressed data, and raw spectral data is generated for analysis. The following describes the detailed work on the local side and the cloud.

**Local Slide**

At the local slide, we use RF spectrum analyzers, acquisition antennas, power supply equipment, transportation equipment, etc. to sample the electromagnetic spectrum. The collection methods of electromagnetic spectrum data are generally divided into Nyquist sampling and Nyquist sampling. We compress the Nyquist sampled data through the CS module, and then go through QR decomposition to improve its reconstruction performance.

Under-nyquist sampling is a collection method based on compressive sensing theory, its process graph is shown in Fig. 5. The RF signal $x(t)$ is divided into $m$ signals, each

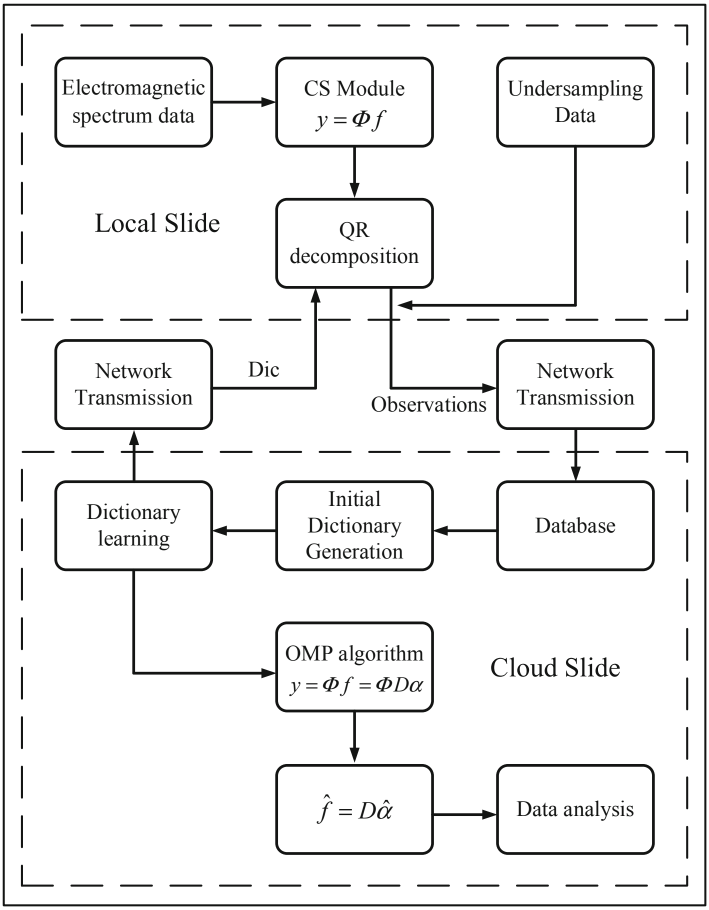


**Fig. 4.** Electromagnetic spectrum data compression system

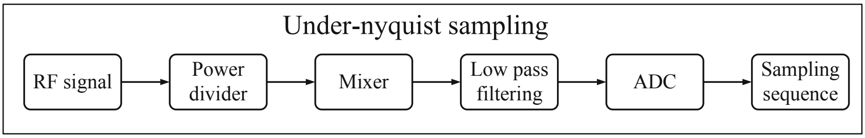


**Fig. 5.** Under-nyquist sampling process

signal is mixed with a pseudo-random code $p_i(t)$ with a period of $T_p$, and then passed through a low-pass filter $h(t)$ with a cutoff frequency of $1/(2T_s)$, and finally get the sequence $y[n]$ after sampling, and the discrete-time Fourier transform of $y[n]$ can be denoted by Eq. (13).

$$Y_i\left(e^{j2\pi fT_s}\right) = \sum_{n=-\infty}^{\infty} y_i[n]e^{-j2\pi f_n T_s} = \sum_{l=-L_0}^{L_0} c_{il}X\left(f - lf_p\right), \quad f \in \mathcal{F}_s \qquad (13)$$

where $X(f)$ is the spectrum of $x(t)$, $c_{il}$ is the fourier series coefficient of $p_i(t)$.

The collected electromagnetic spectrum data is generally temporarily stored in the local computer. The current local storage method consumes a lot of storage resources. Cloud storage, cloud computing and other methods are more suitable for the processing of big data. When we transmit the collected electromagnetic data to the cloud server, we also face the problem of a large amount of transmitted data, so we need to compress the collected electromagnetic data in the local computer first. This process uses a matrix with a smaller dimension. Multiplied by the electromagnetic data matrix, the model can be expressed as Eq. (14).

$$Y = \left[y_1, y_2, \ldots y_n\right] = \Phi F = \Phi\left[f_1, f_2, \ldots f_n\right]. \tag{14}$$

Similarly, we can also express Eq. (13) in this form.

$$\begin{aligned} Y &= \left[Y_i\left(e^{j2\pi fT_s}\right), Y_i\left(e^{j2\pi fT_s}\right), \ldots, Y_i\left(e^{j2\pi fT_s}\right)\right] \\ &= A\left[X\left(f - L_0 f_p\right), X\left(f - (L_0 + 1)f_p\right), \ldots, X\left(f + L_0 f_p\right)\right]. \end{aligned} \tag{15}$$

In the Eq. (14) and (15), the observation value matrix is denoted by $Y$, $\Phi$ and $A$ is the observation matrix.

**Cloud Side**

After the observations are uploaded to the cloud through the network, we can directly store the observations in the cloud server database. However, the observed values cannot be directly used for data analysis. Before analyzing the electromagnetic spectrum data, we need to obtain the original electromagnetic spectrum data. The known prior condition is the observation matrix $\Phi$, and the electromagnetic spectrum data itself is not sparse, so we cannot reconstruct the original electromagnetic spectrum data with only the observation matrix $\Phi$ and the observation value matrix $Y$.

In the cloud, we also need to train a complete or over-complete dictionary of electromagnetic spectrum data. The dictionary is the sparse domain of the data, and the data is represented by the dictionary in the form of multiplying the dictionary and the sparse coefficient, as shown in Eq. (16). The over complete dictionary includes more dictionary atoms than the signal dimension, while in the complete dictionary, the number of the two is equal. First, let the number of clusters equal the number of dictionary atoms, and minimize the error $E$, see Eq. (17), to get cluster center $(\mu_1, \mu_2, \ldots \mu_n)$. Let the initial dictionary $D_1 = (\mu_1, \mu_2, \ldots \mu_n)$.

$$f = D\alpha \tag{16}$$

$$E = \sum_{i=1}^{k} \sum_{f \in C_i} \|f - \mu_i\|_2^2 \tag{17}$$

where $f$ is the spectrum data, the dictionary is denoted by $D$, and the sparse coefficient is denoted by $\alpha$.

When the dictionary of a certain frequency band is trained in the cloud and the observed values are known, we can use various methods to reconstruct the sparse coefficients, such as greedy algorithm, convex optimization algorithm and so on. In this paper, the most commonly used OMP is selected to construct the sparse coefficients. The OMP ensures the accuracy and efficiency of reconstruction. Through the OMP, we can obtain the estimated value of the sparse coefficient reconstructed from the observation matrix and the dictionary. The estimated value of the signal can be obtained by multiplying the dictionary and the estimated value, and the estimated value of the signal can be sent to the data analysis model for data analysis and data mining.

There are bound to be errors in the entire system model, and there are three main types of errors. One is the acquisition error generated during acquisition, which can be improved by improving the accuracy of the acquisition device. The second type of error is the dictionary error generated when learning the dictionary. Because a dictionary cannot perfectly match all signal sets, there must be a dictionary error. The way to improve this error is to select appropriate system parameters and perform dictionary learning. The third error is the reconstruction error generated when using the OMP. The reason for this error is that the OMP cannot reconstruct a completely accurate sparse coefficient. The selection of observation matrix, sparsity and observation size will affect the reconstruction accuracy. The QRK-SVD framework can effectively reduce the latter two errors and improve the system performance.

# 4 Experiments

## 4.1 Evaluation Indicators

Through data acquisition, data compression, data transmission, data decompression and other processes, the estimated value of electromagnetic spectrum data is obtained in the cloud. We measure the accuracy of signal reconstruction by reconstruction SNR, which is defined as:

$$\mathrm{SNR} = 10\lg\frac{1}{\mathrm{RRMSE}^2} = 10\lg\frac{\|X\|_2^2}{\left\|X-\hat{X}\right\|_2^2}. \tag{18}$$

Among them, RRMSE is the relative root mean square error, $X$ is the original electromagnetic spectrum data, and the estimated value is denoted by $\hat{X}$.

We define the ratio of the size of the observations to the size of the original sample as the compression ratio (Cr), which is defined as Eq. (19).

$$Cr = \frac{row(Y)}{row(X)} \times 100\%. \tag{19}$$

## 4.2 Dictionary Error and Reconstruction Error

We select the data from 8:00 am to 9:00 am in the first frequency band on the second day, the fourth day, and the seventh day to study the relationship between the dictionary error and the reconstruction error. We performed 50 iterations with a dictionary sparsity of 0.05, a dictionary size of 380, and the Cr value of 25%. Figure 6 shows the relationship between dictionary error and reconstruction error.

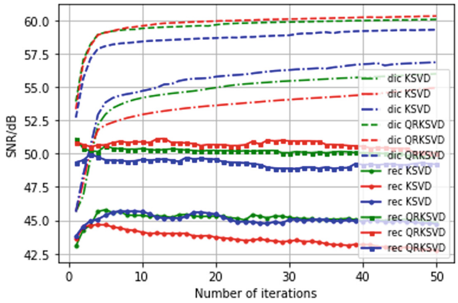


**Fig. 6.** The reconstruction SNR changes with the number of iterations

As can be seen from Fig. 6, no matter what day the data is, the reconstructed SNR of QRK-SVD is greatly improved than that of K-SVD. When the number of iterations is small, the dictionary error is not much different from the reconstruction error. When the number of iterations is large, the reconstruction performance of the K-SVD method decreases, while the QRK-SVD method still has good reconstruction performance. This is mainly because the selection strategy of the initial dictionary is better and because the QR decomposition reduces the correlation between the observation matrix and the dictionary, QRK-SVD achieves higher reconstruction accuracy.

### 4.3 Reconstruction Capability with Different Dictionary Sizes

We select the electromagnetic spectrum data from 8:00 to 9:00 in the first frequency band on the second day, the fourth day, and the seventh day to study the reconstruction ability of K-SVD under different dictionary sizes. We let the dictionary size vary from 360 to 400, the dictionary sparsity is 0.05, the number of iterations is 5, and the Cr value is shown in the figure. Figure 7 shows the effect of dictionary size on reconstruction performance under different Cr values.

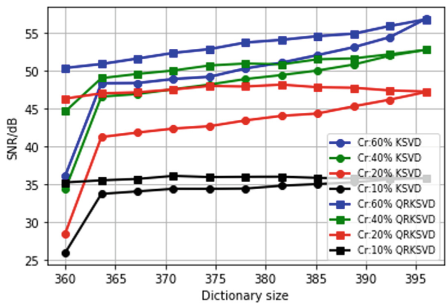


**Fig. 7.** The reconstruction SNR changes with the size of the dictionary

As can be seen from Fig. 7, no matter what day the data is, under any Cr value, the performance of QRK-SVD is better than that of K-SVD. And as the dictionary size increases, the reconstructed SNR also increases. Since the dictionary size increases, more atoms participate in the sparse representation, and the sparse representation power also increases.

### 4.4 Reconstruction Capability with Different Sample Sizes

We select the electromagnetic spectrum data from 8:00 to 9:00 in the first frequency band on the second day, the fourth day, and the seventh day to study the reconstruction ability of K-SVD under different sample sizes. The sample size is changed from 36 to 1800, the dictionary sparsity is 0.05, the dictionary size is 380, and the Cr value is 20%. Figure 8 shows the reconstruction SNR curve under the change of sample size.

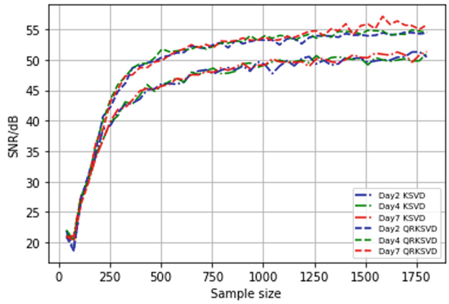


**Fig. 8.** The reconstruction SNR changes with the size of sample

As can be seen from Fig. 8, QRK-SVD outperforms K-SVD at any sample size. Because in the case of the same Cr value, the larger the spectrum sample size, the more the information content of the observed value. In this way, the reconstruction accuracy that can be obtained based on the orthogonal matching pursuit is also greater. However, it is not wise to choose a large sample size, as this will make the running time of the algorithm become longer and increase of the dictionary size, which reduces the system efficiency.

### 4.5 The Amount of Data at the Same Reconstruction Accuracy

We study the size of the transmitted data size of the two frameworks under the same reconstructed SNR. The dictionary size is 380, the dictionary sparsity is 0.05, and the number of iterations is 5. When the reconstruction SNR is 35 dB, 40 dB, and 45 dB, the amount of transmitted data of the two frameworks is shown in Fig. 9, 10 and 11.

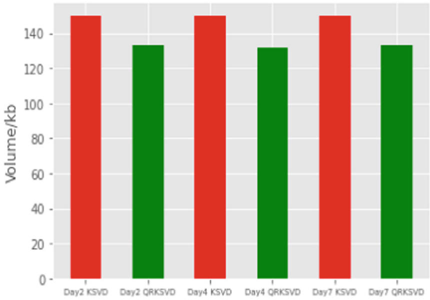


**Fig. 9.** Comparison of the data volume of the two frameworks at 35 dB

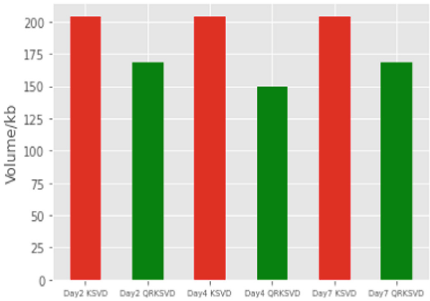


**Fig. 10.** Comparison of the data volume of the two frameworks at 40 dB

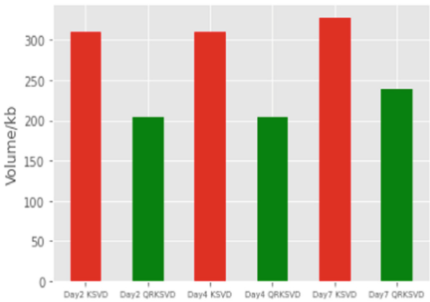


**Fig. 11.** Comparison of the data volume of the two frameworks at 45 dB

As can be seen from the figure, the amount of data in the QRK-SVD framework under any reconstruction SNR is smaller than that in the K-SVD framework. And when the reconstruction SNR is large, this advantage is more obvious.

## 5 Summarize

This paper studies the whole process of electromagnetic spectrum data acquisition, compression, transmission and utilization. It is proposed to use dictionary learning to improve the performance of spectral data compression and reduce the storage capacity. The reconstruction error of electromagnetic spectrum data is related to many factors, including the number of algorithm iterations, the sample size, and the size of the dictionary. This paper studies the influence of these factors on the reconstruction error. The electromagnetic spectrum data compression and reconstruction framework of QRK-SVD is proposed, and the reconstruction ability of QRK-SVD under different parameter conditions is analyzed. For different dictionary sizes, the reconstruction ability of the overcomplete dictionary is better than that of the complete dictionary, but the storage space required is also larger. The increase of dictionary size can effectively improve the reconstruction performance. When the sample size increases, the reconstruction ability of QRK-SVD continues to improve, indicating that QRK-SVD can adapt to samples of various sizes, which is very valuable in practical applications, because the size of each compressed sample cannot be guaranteed to be the same in actual spectrum data collection. In the follow-up research, we can think about how to ensure that there is no error in the transmission process and at the same time have a strong real-time transmission capability. Whether a more efficient dictionary can be designed and whether a stronger observation matrix can be designed to meet the requirement reconstruction needs further research and analysis.

## References


1. Zheng, X.: Design of spectrum sharing platform based on USRP for heterogeneous wireless network. In: 2021 Asia-Pacific Conference on Communications Technology and Computer Science (ACCTCS). IEEE (2021)
2. Lin, J., et al.: Spectrum resource trading and radio management data sharing based on blockchain. In: 2020 IEEE 3rd International Conference on Information Systems and Computer Aided Education (ICISCAE). IEEE (2020)
3. Sankaran, S., Ehsani, R.: Introduction to the electromagnetic spectrum. In: Manickavasagan, A., Jayasuriya, H. (eds.) Imaging with Electromagnetic Spectrum, pp. 1–15. Springer, Heidelberg (2014). https://doi.org/10.1007/978-3-642-54888-8_1
4. Guo, L., Wang, M., Lin, Y.: Electromagnetic environment portrait based on big data mining. Wirel. Commun. Mob. Comput. **2021**, 1–13 (2021)
5. Donoho, D.L.: Compressed sensing. IEEE Trans. Inf. Theory **52**(4), 1289–1306 (2006)
6. Olshausen, B.A., Field, D.J.: Emergence of simple-cell receptive field properties by learning a sparse code for natural images. Nature **381**(6583), 607–609 (1996)
7. Tošić, I., Frossard, P.: Dictionary learning. IEEE Signal Process. Mag. **28**(2), 27–38 (2011)
8. Engan, K., Aase, S.O., Husoy, J.H.: Method of optimal directions for frame design. In: 1999 IEEE International Conference on Acoustics, Speech, and Signal Processing. Proceedings, ICASSP99 (Cat. No. 99CH36258), vol. 5. IEEE (1999)
9. Aharon, M., Elad, M., Bruckstein, A.: K-SVD: an algorithm for designing overcomplete dictionaries for sparse representation. IEEE Trans. Signal Process. **54**(11), 4311–4322 (2006)
10. Mairal, J., et al.: Online learning for matrix factorization and sparse coding. J. Mach. Learn. Res. **11**(1), 19–60 (2010)
11. Papyan, V., et al.: Theoretical foundations of deep learning via sparse representations: a multilayer sparse model and its connection to convolutional neural networks. IEEE Signal Process. Mag. **35**(4), 72–89 (2018)
12. Lin, Y., et al.: Multisensor fault diagnosis modeling based on the evidence theory. IEEE Trans. Reliab. **67**(2), 513–521 (2018)

13. Lu, Y., Wang, Y.: A physics-constrained dictionary learning approach for compression of vibration signals. Mech. Syst. Signal Process. **153**, 107434 (2021)
14. Wang, M., et al.: Transfer learning promotes 6G wireless communications: recent advances and future challenges. IEEE Trans. Reliab. **70**(2), 790–807 (2021)
15. Duan, L., Yang, X., Li, A.: WSN data compression model based on K-SVD dictionary and compressed sensing. In: Zeng, J., Qin, P., Jing, W., Song, X., Lu, Z. (eds.) ICPCSEE 2021. CCIS, vol. 1451, pp. 429–442. Springer, Singapore (2021). https://doi.org/10.1007/978-981-16-5940-9_33
16. Shen, F., et al.: 3D compressed spectrum mapping with sampling locations optimization in spectrum-heterogeneous environment. IEEE Trans. Wirel. Commun. **21**(1), 326–338 (2021)
17. Liu, J.: Research on power quality signals reconstruction method based on K-SVD dictionary learning. In: 2020 39th Chinese Control Conference (CCC). IEEE (2020)
18. Wang, L., et al.: IK-SVD: dictionary learning for spatial big data via incremental atom update. Comput. Sci. Eng. **16**(4), 41–52 (2014)