FRI Sampling for Ultra-wideband Gaussian Pulses Based on Non-ideal LPF

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Abstract. In the field of radar signal processing, the ultra-high sampling frequency has always limited the development of radar technology. Finite rate of innovation (FRI) sampling theory can effectively reduce the sampling frequency of radar pulses in recent years. But the existing radar pulses sampling systems based on FRI have not considered the non-ideal effects caused by non-ideal filters in hardware implement, which affects the accuracy of system reconstruction. In this paper, we proposed a FRI sampling scheme for ultra-wideband gaussian pulses based on non-ideal LPF, which can achieve high-precision reconstruction under non-ideal physical component environment. The proposed system has two identical channels based on non-ideal LPF. The two channels samples the ultra-wideband gaussian pulses and the basis signal with a sub-Nyquist sampling frequency after filtered by non-ideal LPF, then we can obatin Fourier coefficients with non-ideal effects. Then we propose a new estimation algorithm to reconstruct the ultra-wideband gaussian pulses, which can eliminate non-ideal effects and improve the reconstruction performance. Finally, simulation results have verified the effectiveness of the proposed scheme.

Keywords: Finite rate of innovation (FRI) · Ultra-wideband Gaussian pulses · Non-ideal LPF

1 Introduction

The ultra-wideband gaussian pulses means ultra-high bandwidth. According to the traditional Nyqiust sampling theorem [1], the sampling rate must be greater than twice the highest frequency of the signal, so that the original signal can be reconstructed without distortion. So sampling an ultra-wideband signal requires a very high sampling frequency, which is difficult to achieve. In order to effectively reduce the sampling frequency of signal and reconstruct the original signal, many experts have carried out a series of research on the sub-Nyquist sampling method [2,3]. Finite rate of innovation (FRI) sampling theory is a feasible solution to reduce the sampling frequency of ultra-wideband gaussian pulses, whose base pulse is known.

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Since finite rate of innovation firstly proposed by Vetterli [4], and many experts have been researching on it. Then the following conclusions are obtained: as long as it is a function that satisfies the fixed constraint conditions, the function of the exponential spline curve, and any function with the rational Fourier transform can be used as the sampling kernel [5]. Therefore, Vetterli and Dragotti proposed tightly supported sampling kernels in the time domain, which are mainly divided into three categories: exponential regenerative kernels, rational number sampling nuclei and polynomial regenerative kernels [6]. The above-mentioned time-domain tightly supported sampling kernels all use the moment of the signal to recover the signal. The aforementioned sampling kernels are often oriented towards a small number of FRI signals [7–9], so their versatility is extremely poor. Therefore, Eldar and Tur designed a sampling structure based on SoS sampling kernel for the sampling and reconstruction of arbitrary shape pulse signals [10].

The above FRI sampling structure is mostly single-channel, in order to further reduce the signal sampling rate. The multi-channel FRI sampling structure came into being. Eldar and Gedalyahu [11] designed a filter bank multi-channel sampling structure suitable for half-period signals (the amplitude changes in each cycle while the delay is unchanged). In this structure, the input signal is first sent to P channels at the same time, the signal is filtered in each channel, and then the discrete sampling value is obtained through low-speed sampling, and then a correction filter is used to filter the discrete sampling value. After the data is passed into the joint sub-empty Harbin Institute of Technology Master's degree thesis-4 to perform calculations, the free parameters of the original signal can be restored [12]. Tur and Gedalyahu [13] designed a parallel multi-channel mixing sampling structure. Its characteristic is to use the known single frequency signal to mix and integrate the input signal under test in each channel, and then obtain the necessary information for reconstruction through subsequent processing. Xiaoyao Wei [14] gradually extended the traditional FRI sampling structure framework to an approximate FRI framework that works with arbitrary sampling cores, and realized the reconstruction of the input signal of the parameterized model.

The estimation algorithm of free parameters is another core research problem in FRI sampling theory. Therefore, Dragotti and Blu et al. [15] made appropriate improvements on the basis of the null filter method and proposed a comprehensive least squares method. Crespo and Erdozain et al. conducted in-depth research on subspace invariance, and proposed a state space method for this purpose [16]. Its reconstruction effect is far superior to the former. According to the idea of parameter optimization, Yonina and Michaeli [17] designed a method to reconstruct the signal by solving an unconstrained optimization problem. This method is applicable to a wide range of conditions, and the reconstruction effect is good. It has low requirements on the sampling core, strong versatility, and complex calculations in the reconstruction process as its main features. Goyal and Tan [18] designed a simple, efficient, and high-precision signal parameter reconstruction method based on statistical ideas, but this method has strict requirements on the type of sampling kernel, and its versatility is poor.

Some of the above FRI sampling systems did not propose specific hardware implementation schemes, and some did not consider the impact of non-ideal effects in hardware implementation. In actual design, LPF is usually used as the sampling kernel function. There is a non-ideal effect between the ideal LPF and the non-ideal LPF, which is bound to have a bad influence on the reconstruction accuracy. So we proposed a new FRI sampling for ultra-wideband gaussian pulses based on non-ideal LPF. In the proposed system, we introduced a sampling channel of the base signal to obtain sampling samples of the base signal containing non-ideal effects. Then through the processing of the algorithm, we can eliminate the non-ideal effects. Finally we can reconstructed signal by these samples.

2 Problem Formulatiom

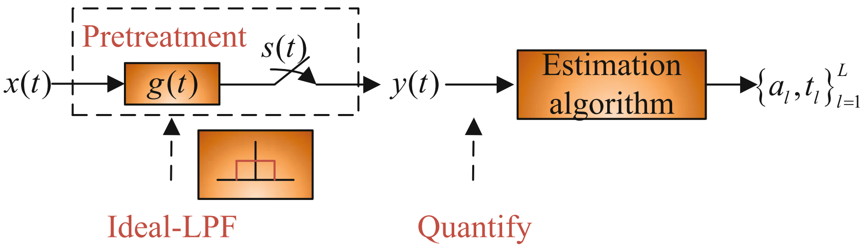

Fig. 1. Classic FRI sampling system

With the classic FRI sampling system, which show in Fig. 1. Taking the basic sinc sampling kernel as an example, we modeled the process of FRI sampling and reconstruction of the ground penetrating radar ultra-wideband gaussian pulses. In a radar system, the superposition of the received echo can be used to express the complete echo signal, and without noise and other interference, the radar echo signal can be expressed as

$$x(t) = \sum_{l=1}^{L} a_l h(t - nT - t_l), \tag{1}$$

where $h(t)$ is a known gaussian pulse waveform, $\{a_l, t_l\}_{l=1}^{L}$ correspond to the amplitude parameter and time delay parameter of the detected pulse respectively and T is radar pulse repetition interval. Then $x(t)$ is filtered by ideal LPF, The signal after filtered can be expressed as

$$y(t) = x(t) * g(t) = \int_{-\infty}^{\infty} x(t)g(t-\tau)d\tau, \tag{2}$$

where $g(t)$ is the frequency response of ideal filter. After sampled with sub-Nyquist sampling frequency, The samples can be expressed as

$$y[n] = y(t)_{t=nT_s}, \tag{3}$$

where T_s is the sampling interval.

According to (3), the samples contain part of the fourier coefficients of the gaussian pulses. After simplification, a typical parameter estimation problem can be obtained. By solving this problem, the amplitude and delay parameters of the pulses can be obtained. However in the hardware implementation process, it will bring non-ideal effects and affect the reconstruction accuracy because the filter cannot achieve the same effect of simulation. In order to eliminate non-ideal effects caused by non-ideal LPF, we proposed a new FRI sampling for ultra-wideband gaussian pulses which is based on non-ideal LPF.

3 FRI Sampling for Ultra-Wideband Gaussian Pulses Based on Non-ideal LPF

3.1 Sampling Framework

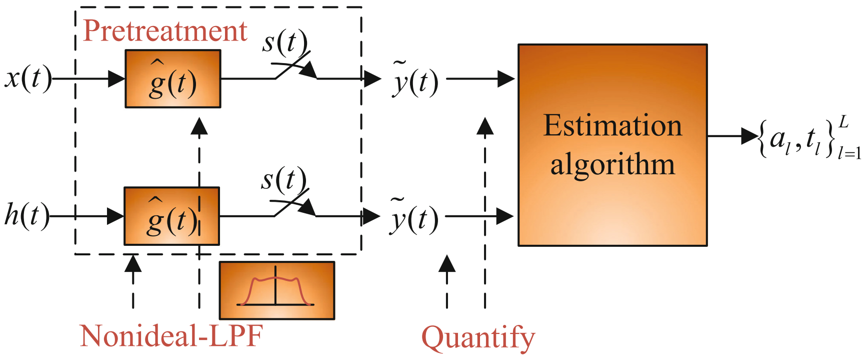

Fig. 2. Proposed FRI sampling system

The proposed FRI sampling framework has been showed in Fig. 2, and it has two parallel channels. In both channels we use a continuous pulses signal to sampling gaussian pulses with a sub-Nyquist sampling frequency, and the continuous pulses signal can be expressed as

$$s(t) = \sum_{n=0}^{\infty} \delta(t - nT_s), \tag{4}$$

where T_s is the sampling interval. Its CTFT can be expressed as

$$S(\Omega) = \Omega_s \sum_{n=0}^{\infty} \delta(\Omega - n\Omega_s). \tag{5}$$

We model $x(t)$ as linear addition of base gaussian pulse, and $x(t)$ can be written as

$$x(t) = \sum_{l=1}^{L} a_l h(t - t_l), \tag{6}$$

where $h(t)$ is a known gaussian pulse waveform, and $h(t)$ can be written as

$$h(t) = \exp(-\frac{t^2}{2\sigma^2}) \tag{7}$$

In the one channel, the gaussian pulses $x(t)$ is sampled by $s(t)$ after filter by non-ideal LPF, whose frequency response can be written as $\widehat{g}(t)$. The signal after sampled can be showed as

$$\widetilde{y}(t) = [x(t) * \widehat{g}(t)] \cdot s(t). \tag{8}$$

For subsequent processing, we need to quantify the sample $\widetilde{y}(t)$, and we can obtain the samples

$$\widetilde{y}[n] = \widetilde{y}(t)_{t=nT_s} (n = 0, 1, \cdots, N-1, N = \frac{T}{T_s}). \tag{9}$$

In the other channel, the base gaussian pulse $h(t)$ is sampled by $s(t)$ after filter by non-ideal LPF, whose frequency response can be written as $\widehat{g}(t)$. The signal after sampled can be showed as

$$\widetilde{h}(t) = [h(t) * \widehat{g}(t)] \cdot s(t). \tag{10}$$

And after quantization, we can obtain the samples

$$\widetilde{h}[n] = \widetilde{h}(t)_{t=nT_s} (n = 0, 1, \cdots, N-1, N = \frac{T}{T_s}). \tag{11}$$

From (9) and (11) we can obatin the samples which contain part of the fourier coefficients of the gaussian pulses. But those samples $\widetilde{y}[n]$ and $\widetilde{h}[n]$ contain non-ideal effects caused by non-ideal LPF, which can lead to low reconstruction accuracy. We design a new estimation algorithm to eliminate non-ideal effects by these samples.

3.2 Estimation Algorithm

The new estimation algorithm can be summarized as Theorem 1, and the proof process is given.

Theorem 1. *Consider a gaussian pulses x(t) in (6), where the base function h(t) is known, and h(t) in (7). Assume that these two pulses are input the proposed system in Fig. 2, then we can obatin that the samples are given by* $\widetilde{y}[n]$ *and* $\widetilde{h}[n]$*, respectively. If the sampling frequency satisfies* $f_s \geq 2f$*, where* f *is the cutoff frequency of non-ideal LPF. Finally we can estimate the* $\{a_l, t_l\}_{l=1}^{L}$ *from those samples uniquely under high precision*

Proof. According to (8) and convolution theorem, the CTFT of (8) can be written as

$$\widetilde{Y}(\Omega) = \frac{\Omega_s}{(2\pi)^2} \sum_{n=0}^{\infty} (X(\Omega - n\Omega_s)\widehat{G}(\Omega - n\Omega_s)) \tag{12}$$

It is obvious that $\widetilde{Y}(\Omega)$ is a period signals and the period is $\Omega_s = 2\pi/T_s$. When the sampling frequency satisfies $f_s \geq 2f$, where f is the cutoff frequency of non-ideal LPF, according to the Shannon-Nyquist sampling theorem we can avoid frequency aliasing. and its frequency domain information can use a period to complete, so (12) can be simplified to

$$\widetilde{Y}(\Omega) = \frac{1}{2\pi T_s} X(\Omega)\widehat{G}(\Omega). \tag{13}$$

According to (10) and convolution theorem, the CTFT of (10) can be written as

$$\widetilde{H}(\Omega) = \frac{\Omega_s}{(2\pi)^2} \sum_{n=0}^{\infty} (H(\Omega - n\Omega_s)\widehat{G}(\Omega - n\Omega_s)), \tag{14}$$

and its frequency domain information can use a period to complete, so (14) can be simplified to

$$\widetilde{H}(\Omega) = \frac{1}{2\pi T_s} H(\Omega)\widehat{G}(\Omega). \tag{15}$$

We know the relationship between $x(t)$ and $h(t)$, so we can obtain the frequency relationship between $x(t)$ and $h(t)$ and it can be expressed as

$$X(\Omega) = \sum_{l=1}^{L} a_l H(\Omega) e^{-j\Omega t_l}. \tag{16}$$

Take k samples, equivalent to $\Omega = 2\pi k/T$, we can obatin

$$X(\frac{2\pi k}{T}) = \sum_{l=1}^{L} a_l H(\frac{2\pi k}{T}) e^{-j\frac{2\pi k}{T} t_l} \tag{17}$$

By using (13) and (15), we can obtain the samples

$$Y(\Omega) = \frac{\widetilde{Y}(\Omega)}{\widetilde{H}(\Omega)} = \frac{\frac{1}{2\pi T_s} X(\Omega)\widehat{G}(\Omega)}{\frac{1}{2\pi T_s} H(\Omega)\widehat{G}(\Omega)} = \frac{X(\Omega)}{H(\Omega)}, \tag{18}$$

Take k samples and after simplification, the samples can be written as

$$Y[k] = \sum_{l=1}^{L} a_l e^{-jkwt_l} = \sum_{l=1}^{L} a_l e^{-jk\frac{2\pi}{T}t_l}, \tag{19}$$

which does not contain non-ideal effects, and (19) is a typical parameter estimation problem. We can solve this problem by use annihilation filter algorithm or others algorithm.

For better understanding of Theorem 1, we conclude the process in Algorithm 1.

Algorithm 1. Estimation algorithm

Input: Time length T; Number of components L; Bandwidth B; Sampling period T_s; Samples obtained by the proposed system $\tilde{y}[n]$ and $\tilde{h}[n]$, where $n = 0, 1, \cdots, N-1$ and $N = \lfloor T/T_s \rfloor$.

Output: Estimated parameters $\{\hat{a}_l, \hat{t}_l\}_{l=1}^{L}$.

1: Computer the CTFT of the samples of the one channel $\widetilde{Y}(\Omega) = \frac{1}{2\pi T_s} X(\Omega)\widehat{G}(\Omega)$;

2: Computer the CTFT of the samples of the other channel $\widetilde{H}(\Omega) = \frac{1}{2\pi T_s} H(\Omega)\widehat{G}(\Omega)$;

3: Eliminate non-ideal effects by $Y(\Omega) = \frac{\widetilde{Y}(\Omega)}{\widetilde{H}(\Omega)} = \frac{\frac{1}{2\pi T_s} X(\Omega)\widehat{G}(\Omega)}{\frac{1}{2\pi T_s} H(\Omega)\widehat{G}(\Omega)} = \frac{X(\Omega)}{H(\Omega)}$;

4: **for** $k = -\lfloor BT \rfloor$ to $\lfloor BT \rfloor$ **do**

5: $\quad Y[k] = \tilde{Y}\left(\frac{2\pi}{T}k\right) / \tilde{R}\left(\frac{2\pi}{T}k\right)$;

6: **end for**

7: Estimate $\{\hat{a}_l, \hat{t}_l\}_{l=1}^{L}$ by solving (19) under annihilation filter algorithm [19].

8: **return** $\{\hat{a}_l, \hat{t}_l\}_{l=1}^{L}$.

3.3 Anti-noise Methods

Because in the hardware implementation process, noise is inevitable. We propose two methods to improve the noise robustness of the system.

Method 1: Use reconstruction algorithm which has strong noise robustness. ESPRIT and MUSIC algorithms [20,21] has better anti-noise performance than annihilation filter algorithm, when estimating parameters with a noise environment.

Method 2: Increase Properly the sampling frequency which should be greater than two times the filter cutoff frequency, and we can use more samples to estimated parameters.

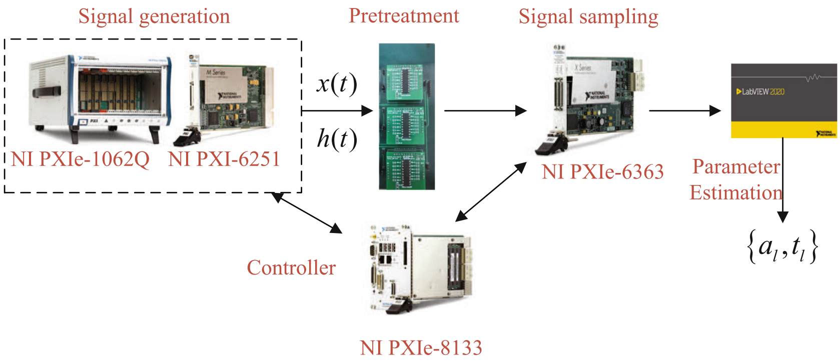

Fig. 3. Specific hardware implementation structure of the proposed FRI sampling system

4 Hardware Prototype

We propose a hardware implementation system based on this method, and the specific structure diagram of this hardware system is shown in the Fig. 3. It can be seen from the Fig. 3 that includes a pretreatment board, which consists of power supply module and filter module. In this system, we use MAX275 as a non-ideal sampling kernel function. MAX275 is a universal active filter produced by MAXIM Corporation in the United States. It contains two independent second-order active filter circuits, which can perform low-pass and band-pass filtering at the same time, and can also realize fourth-order active filtering through cascading. The center frequency/cutoff frequency can reach 300 kHz. MAX275 does not require a clock circuit, so compared with switched capacitor filters, its noise is lower and its dynamic characteristics are better. It can be widely used in various precision test equipment, communication equipment, medical instruments and data acquisition systems. Because the cutoff frequency of the filter can be adjusted by adjusting the size of the resistor. We also proposed an experimental platform based on the PXI instruments, which are used for the signal generation, sampling, and parameter estimation. The functions of these instruments are as follows:

NI PXIe-1062Q: PXI chassis, provide containers for other PXI devices. It has 4 PXI peripheral slots, 1 PXI Express slot with system timing function, and 2 PXI Express hybrid slots that can support PXI and PXI Express peripheral modules.

NI PXI-6251: Data acquisition card, use for signal generation $x(t)$ and $h(t)$ with NI PXIe-1062Q, then output them. This equipment provides low-cost reliability for various applications ranging from laboratory automation, research, design verification/testing to manufacturing testing.

NI PXIe-8133: Controller, control the operation of the entire system, equivalent to a host computer.

NI PXIe-6363: Data acquisition card, use for signal sampling. It has high-throughput PCI Express bus and multi-core optimized drivers and application software to provide high-performance features.

5 Simulation and Hardware Experiments

In order to verify the feasibility and advancement of the proposed system, we designed several experiments to prove it. The parameters in the experiment are set as follows: $x(t)$ is show in (6) and $h(t)$ is show in (7), $t_l = [0.2, 0.4, 0.6, 0.8]$ set as delay parameters, $a_l = [0.9, 0.6, 0.8, 0.5]$ set as amplitude parameters, $t_0 = 0.5$ is the initial delay of the base gaussian pulse, and $\sigma = 0.001$.

Simulation 1: In this simulation without noise, we verify the feasibility of the system and prove that it can reconstruct the unknown parameters of the signal through the samples. We compare the experimental results with the classic FRI sampling system, which show in Fig. 1. The experimental results with ideal-LPF can be show in Fig. 4 and the experimental results with nonideal-LPF can be show in Fig. 5. Figure 4 proved that this method can reconstruct unknown parameters. and it is obvious that with non-ideal LPF the reconstruction results of proposed system is closer to the original parameters than that of classic FRI system.

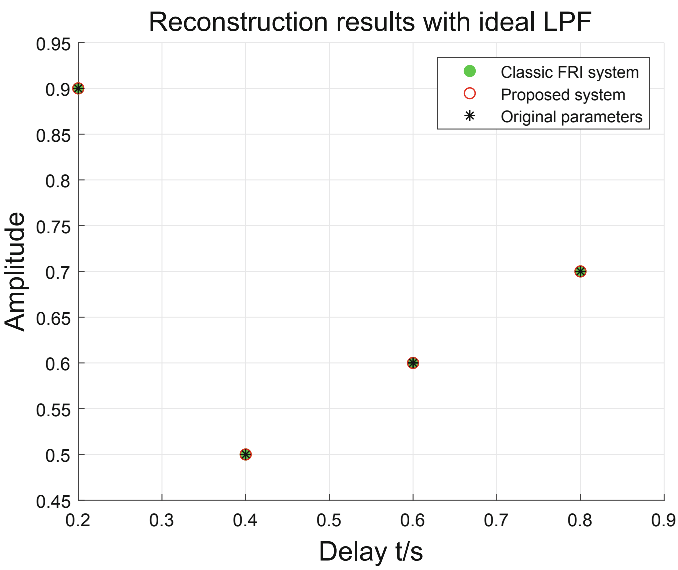

Fig. 4. Simulation results with ideal-LPF under non-noise.

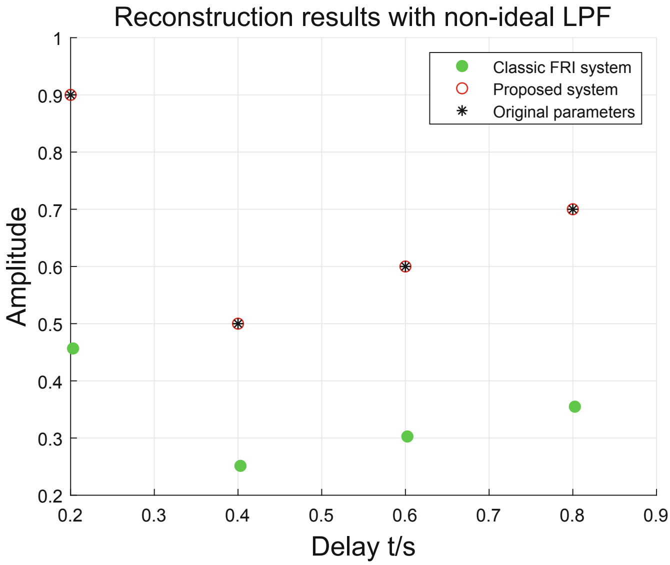

Fig. 5. Simulation results with nonideal-LPF under non-noise.

Simulation 2: In order to verify the strong robustness of the noise of the proposed system, simulation 2 is produced. In this simulation, we use the normalized mean square error (NMSE) to measure the accuracy of reconstruction and compare it with the classic FRI sampling system. Since the error of amplitudes is proportional to the corresponding time delays, we only use the NMSE of time delays to measure the efficiency of the proposed method. In order to obtain more data, annihilation filter algorithm and ESPRIT algorithm are used to estimate the unknown time delays and amplitudes parameters. The simulation results of the NMSE of estimated time delays are show in Fig. 6. From the figure, we can see the NMSE of proposed system always less than the NMSE of classic FRI system. When systems with high SNR 20 dB, the FRI system using de-noising ESPRIT algorithm closes to the proposed system, which used annihilating filter algorithm. When the signal-to-noise ratio of the system reaches 60 dB, the proposed system using annihilation filter algorithm keep it steady in −85 dB. And the proposed system using ESPRIT algorithm has the smallest NMSE.

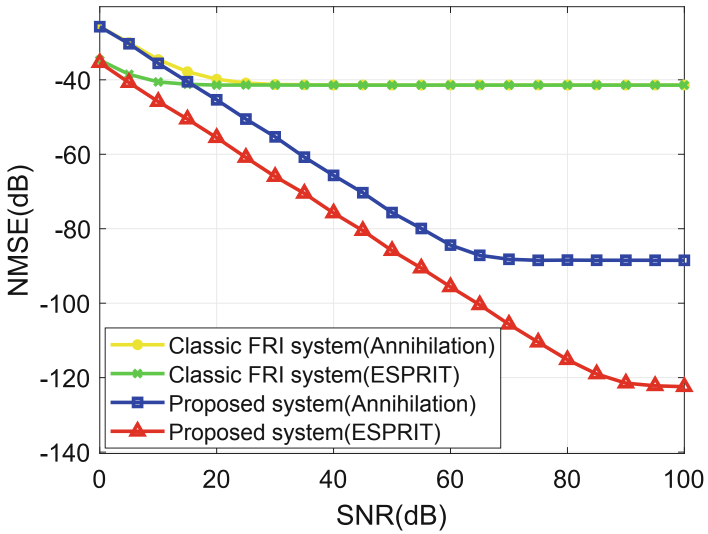

Fig. 6. Simulation results with noise

6 Conclusions

In this paper, we proposed a FRI sampling scheme for ultra-wideband gaussian pulses, which is based on non-ideal LPF and can eliminate the effects of non-ideal physical components and improve the reconstruction results. The proposed scheme is consist of two sampling channels, in which we can obtain a part of Fourier coefficients of both the import signal and the basis signal. Then we proposed a new estimation algorithm to estimate the unknown parameters from the obtained Fourier coefficients. We also design a hardware implementation system and several experiments to verify the feasibility of the proposed system. Simulation results have verified the effectiveness of the proposed scheme.

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