






Performance Optimization and Power Allocation of Amplify-and-Forward System with Multi-source

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Abstract. In this paper, a performance optimization scheme based on the maximum signal-to-noise ratio (SNR) at destination of amplify-and-forward (AF) system with multi-source nodes and multi-relay nodes is proposed. For the underlying AF systems, the signal is transmitted from several identical source nodes to several parallel relay nodes via the wireless channel, and then transmitted by the relay nodes to the destination node. In order to recover the original signal, the SNR under limited system power constraint is modeled as a constrained optimization problem. Then, we transform the constrained optimization problem to a convex function with respect to the power allocation, and obtain the expression of maximum SNR at destination and corresponding power allocation. Simulation results show that the SNR at the destination of the proposed scheme significantly outperforms the one in which the limited power is equally allocated among all source nodes and all relay nodes.

Keywords: AF system · Multi-source · Parallel relay networks · Power allocation · Performance optimization · Constrained optimization problem

1 Introduction

AF is an important relaying strategy, in which one or more relays amplify the source signal to complete the communication between the source node and the destination node. It can extend wireless network coverage or improve communication quality and is widely used in wireless communication system, wireless sensor system and so on. Especially, in the communication networks of military, government and financial institution, where the relay node must be regarded as eavesdropper node, AF is safer than decode-and-forward (DF).

Due to the widely application, AF strategy have gotten numerous researches in recently years. Many researches focus on outage probability (OP) [1–5] of opportunistic AF relaying. In addition, Rodriguez and Tran [6] had studied the achievable rates over Rayleigh fading channels. Singh and Gupta [7], Liu and Song [8] and Islam [9] studied the quality of service (QoS), symbol error rate (SER) and achievable rate of the AF system, respectively. While Nagendra and Vimal [10] derived further closed-expressions of the outage probability and the average channel capacity. Recently, Wang and Wang [11] proposed a scheme, in which they studied how to guarantee SNR at destination. Based on AF system, there are more researches, such as Su and Chen [12] and Chen and Fang [13], they design a robust transceiver for Multiple Input Multiple Output (MIMO) system, and so on.

As we know, power allocation of wireless communication system plays a vitally important role, so it is always a field that attracts much more attentions. Laneman and Wornell [14] studied an AF scheme with cooperative diversity. They equally divided up the total available power among one source node and all relay nodes. However, it is found that the performance of this scheme is sub-optimal. Zhao and Adve [15] studied the AF cooperative diversity system and proposed a selection scheme to choose the “best” relay node to assist the communication. The scheme can get a better outage behavior and throughput than all-participate scheme. After that, much more power allocation scheme was optimized from selecting relay node [16–20].

However, to the best of our knowledge, for an AF system composed of several identical source nodes and several parallel relay nodes in which the source node cannot directly communicate with the destination node, when the total system power is limited and must be allocated among several source nodes and several relay nodes, the optimized power allocation has not yet been studied. This paper aims to fill this research gap. The major contributions and novelties of the paper are summarized as follows:

- 1) An AF system model is proposed, in which several source nodes communication with a destination node with the help of all parallel relay nodes, and no direct path exist between source nodes and destination node. The power of every relay node is equally allocated, as well as the several source nodes, and the total system power of the relay nodes and source nodes is limited.
- 2) The criteria that assess the system performance usually are bit error rate (BER), OP or QoS. Without loss of generality, here the criterion that assesses the system performance is SNR at destination rather than BER, as we know that the maximum SNR can bring the minimum BER.
- 3) The expression of the SNR at destination is derived and transformed into a convex function with respect to power allocation. Then the expressions of maximum SNR at destination and corresponding optimized power allocation are obtained.

The remainder of this paper is organized as follows. In Sect. 2, the multi-relay AF system with multi-source nodes is introduced and the SNR at destination is given, and the expression of the maximum SNR at destination under system

power constraints is derived. The power allocation is optimized and the SNR is maximized in Sect. 3. Calculations are conducted with corresponding parameters and numerical results are analyzed in Sect. 4. Finally, conclusions are given in Sect. 5.

2 Multi-relay AF System with Multi-source Nodes

Figure 1 depicts the proposed AF system, in which no direct link exists between the source S_m and destination D , here $m = 1, 2, \dots, M$. The several sources communicate with destination D with the assistance of L parallel relay nodes $R_i (i = 1, 2, \dots, L)$. The destination collects all signals that amplified and retransmitted by relays with equal weight and recovers original information.

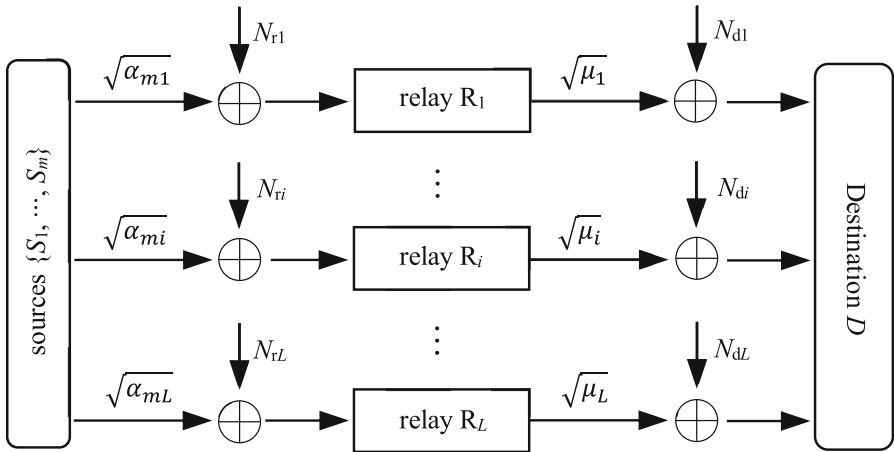


Fig. 1. Multi-relay AF system with several sources.

Assume that the power of the several source nodes are equally distributed, the original signal $X_m(t)$ generated from source S_m follows Gaussian distribution $N(0, P_x)$. The noise in every relay node and destination is independently and identically distributed (i.i.d) additive white Gaussian noise (AWGN) and obeys $N(0, \sigma_n^2)$. Meanwhile, all the wireless channels between source nodes and relay nodes, as well as that between relay nodes and destination node, are all Gaussian channels. It should be mentioned that, here what we considered is a theoretical analysis framework, encoding mechanism and access probability [21] are not taken into account. Furthermore, the numbers of source nodes and relay nodes are all more than one, while the character of the structure of the AF system make it different with the MIMO system [22]. The technologies and their relative theories of MIMO system, such as beam design [23], antenna selection and so on, are not thought about.

When the signal $X_m(t)$ that generated from the m^{th} source reaches the i^{th} relay node R_i , we obtain the following equations [24]

$$Y_{rmi}(t) = \sqrt{\alpha_{mi}}X_m(t) + N_{ri}(t),$$

$$m = 1, 2, \dots, M; i = 1, 2, \dots, L. \quad (1)$$

The power of signal $Y_{rmi}(t)$ can be represented as

$$P_{rmi} = \alpha_{mi}P_x + \sigma_n^2,$$

$$m = 1, 2, \dots, M; i = 1, 2, \dots, L. \quad (2)$$

where $N_{ri}(t)$ denotes the Gaussian noise at the i^{th} relay node, α_{mi} corresponds to the channel attenuation factor between the m^{th} source node and the i^{th} relay node [25].

The i^{th} relay node amplifies the power of the received signal for β times which is defined as

$$\beta = \frac{P_{ri}}{\alpha P_x + \sigma_n^2} \quad (3)$$

where P_{ri} denotes the power of the signal that the i^{th} relay transmitted, then send it to the destination via wireless channel. The signal received by the destination and its power can be presented in the following forms

$$Y_{dmi}(t) = \sqrt{\mu_i}\sqrt{\beta}Y_{rmi}(t) + N_{di}(t)$$

$$= \sqrt{\mu_i}\sqrt{\beta}[\sqrt{\alpha_{mi}}X_m(t) + N_{ri}(t)] + N_{di}(t), \quad (4)$$

$$m = 1, 2, \dots, M; i = 1, 2, \dots, L.$$

$$P_{dmi} = \mu_i\beta P_{rmi} + \sigma_n^2 = \mu_i\beta(\alpha_{mi}P_x + \sigma_n^2) + \sigma_n^2,$$

$$m = 1, 2, \dots, M; i = 1, 2, \dots, L. \quad (5)$$

where μ_i corresponds to the channel attenuation factor between the i^{th} relay node and destination node, $N_{di}(t)$ denotes the Gaussian noise at destination node with mean zero and power σ_n^2 . For the relay nodes are parallel, all $\sqrt{\alpha_{mi}}$ are the same, as well as all $\sqrt{\mu_i}$. For simplicity, we let $\alpha_{mi} = \alpha$, $\mu_i = \mu$. For the destination received signal $Y_{di}(t)$ from the i^{th} relay can be calculated as a coherent summation of the signals transmitted from the sources [26], we can get the following expression

$$Y_{di}(t) = \sum_{m=1}^M \{\sqrt{\mu}\sqrt{\beta}[\sqrt{\alpha_{mi}}X_m(t) + N_{ri}(t)] + N_{di}(t)\}. \quad (6)$$

In this case, we obtain the power of the $Y_{di}(t)$

$$P_{di} = M[\mu\beta(\alpha P_x + \sigma_n^2) + \sigma_n^2] \quad (7)$$

so the SNR of $Y_{di}(t)$ is

$$\gamma_{Ai} = \frac{M\alpha\mu\beta P_x}{M\sigma_n^2 + M\mu\beta\sigma_n^2} = \frac{\alpha\mu\beta P_x}{\sigma_n^2 + \mu\beta\sigma_n^2}. \quad (8)$$

Substituting (3) into (8) gives

$$\gamma_{Ai} = \frac{\alpha\mu P_{ri}P_x}{\sigma_n^2[\alpha P_x + \sigma_n^2 + \mu P_{ri}]} \quad (9)$$

where γ_{Ai} is expressed as a function of the power of every source node and relay node. In the traditional multi-relay AF system, every source-relay-destination sublink is the same. The destination node collects all signals from L relays simultaneously, and SNR of the sum of all signals at the destination node can be presented by

$$\gamma_A = \sum_{i=1}^L \frac{\alpha\mu P_{ri}P_x}{\sigma_n^2[\alpha P_x + \sigma_n^2 + \mu P_{ri}]} \quad (10)$$

For some wireless sensor networks, signal source is usually working in harsh conditions, charging and changing battery are too difficult to realize. Here we assume that the total power of all sources and all relays are limited, and the power of every relay node is equally distributed, one can find the following expression

$$MP_x + LP_{ri} = P. \quad (11)$$

Combine the power constraint (11) into (10) to take the place of P_{ri} with independent variable P_x , we can get the expression of SNR at destination

$$\gamma_A = \frac{L\alpha\mu P_x(P - MP_x)}{\sigma_n^2[\mu(P - MP_x) + L(\sigma_n^2 + \alpha P_x)]} \quad (12)$$

Considering that the power of every source node P_x should be lower than the total system power P , the maximum SNR at destination of the AF system can be described as a constrained optimization problem

$$\begin{aligned} & \max : \gamma_A \\ & \text{s.t.} \begin{cases} \gamma_A = \frac{L\alpha\mu P_x(P - MP_x)}{\sigma_n^2[\mu(P - MP_x) + L(\sigma_n^2 + \alpha P_x)]} \\ 0 \leq P_x \leq P \end{cases} \end{aligned} \quad (13)$$

3 Optimizations of Power Allocation and SNR at Destination

Obviously, the value of P_x cannot take 0, for the source nodes have not enough power to transmit signal to relay nodes, nor take P , for the relay nodes will lack enough power to transmit signal to destination node. So γ_A is a convex function of variable P_x . To obtain the expression of the maximum γ_A , here we use a strict mathematical derivation.

If we make γ_A the function with respect to a variable P_x/P , the following expression can be derived from (13)

$$\begin{aligned}
(\gamma_A)_{\max} &= \max \frac{P^2}{P^2} \cdot \frac{L\alpha\mu P_x(P - MP_x)}{\sigma_n^2[\mu(P - MP_x) + L(\sigma_n^2 + \alpha P_x)]} \\
&= \max \frac{L\alpha\mu P^2}{\sigma_n^2} \cdot \frac{P_x(P - MP_x)}{P^2[\mu(P - MP_x) + L(\sigma_n^2 + \alpha P_x)]} \\
&= \frac{L\alpha\mu P^2}{\sigma_n^2} \cdot \max \frac{\frac{P_x}{P}(1 - \frac{MP_x}{P})}{\mu(P - MP_x) + L(\sigma_n^2 + \alpha P_x)}.
\end{aligned} \tag{14}$$

Obviously, the first half part of (14) $\frac{L\alpha\mu P^2}{\sigma_n^2}$ is a constant. When γ_A reaches its maximum, the second part of (14) must get its maximum, too. For further simplification, we will therefore define $\varphi\left(\frac{P_x}{P}\right)$ [27] in the following

$$\varphi\left(\frac{P_x}{P}\right) = \frac{\mu(P - MP_x) + L(\sigma_n^2 + \alpha P_x)}{\frac{P_x}{P}(1 - \frac{MP_x}{P})} \tag{15}$$

The expression given above can further be written as

$$\left[\varphi\left(\frac{P_x}{P}\right)\right]^{-1} \triangleq \frac{\frac{P_x}{P}(1 - \frac{MP_x}{P})}{\mu(P - MP_x) + L(\sigma_n^2 + \alpha P_x)}. \tag{16}$$

Putting (16) into (14), we can obtain

$$(\gamma_A)_{\max} = \max_{\frac{P_x}{P} \in (0,1)} \frac{L\alpha\mu P^2}{\sigma_n^2} \left[\varphi\left(\frac{P_x}{P}\right)\right]^{-1} = \frac{L\alpha\mu P^2}{\sigma_n^2} \left[\min_{\frac{P_x}{P} \in (0,1)} \varphi\left(\frac{P_x}{P}\right)\right]^{-1}. \tag{17}$$

It can be seen that, the value of γ_A will reach the maximum when $\varphi\left(\frac{P_x}{P}\right)$ get its minimum value. Next, we will transform (15) into a sum of two equations.

$$\begin{aligned}
\varphi\left(\frac{P_x}{P}\right) &= \frac{\mu(P - MP_x) + L(\sigma_n^2 + \alpha P_x)}{\frac{P_x}{P}(1 - \frac{MP_x}{P})} \\
&= \frac{P(\mu P - M\mu P_x + L\sigma_n^2 + L\alpha P_x)}{\frac{P_x}{P}(P - MP_x)} \\
&= \frac{\mu P^2 - \mu MPP_x + LP\sigma_n^2 + LP\alpha P_x + (MLP_x\sigma_n^2 - MLP_x\sigma_n^2)}{\frac{P_x}{P}(P - MP_x)} \\
&= \frac{LP_x(M\sigma_n^2 + \alpha P) + (P - MP_x)(L\sigma_n^2 + \mu P)}{\frac{P_x}{P}(P - MP_x)} \\
&= \frac{L\frac{P_x}{P}(M\sigma_n^2 + \alpha P) + (1 - \frac{MP_x}{P})(L\sigma_n^2 + \mu P)}{\frac{P_x}{P}(1 - \frac{MP_x}{P})} \\
&= \frac{L(M\sigma_n^2 + \alpha P)}{1 - \frac{MP_x}{P}} + \frac{L\sigma_n^2 + \mu P}{\frac{MP_x}{P}}.
\end{aligned} \tag{18}$$

For the purpose of simplification, we set

$$u \triangleq \frac{P_x}{P}, \quad (19)$$

$$A = L(M\sigma_n^2 + \alpha P), \quad (20)$$

$$B = L\sigma_n^2 + \mu P. \quad (21)$$

Applying these relationships in connection with (18) leads to

$$\varphi(u) = \frac{A}{1 - Mu} + \frac{B}{u}. \quad (22)$$

Here, we analysis that how we can get the minimum value of $\varphi(u)$. Taking the second derivative with respect to (22) results in the following inequation

$$\varphi''(u) = \frac{M^3 A}{(1 - Mu)^3} + \frac{2B}{u^3} > 0 \quad \text{for } u \in (0, 1), \quad (23)$$

so $\varphi(u)$ is transformed into a convex function with respect to u when $u \in (0, 1)$ and the minimum value of it can be gotten from $u_0 \in (0, 1)$, where the first derivative of $\varphi(u)$ should be zero

$$\varphi'(u_0) = \frac{MA}{(1 - Mu_0)^2} - \frac{B}{u_0^2} = \left(\frac{\sqrt{MA}}{1 - Mu_0} + \frac{\sqrt{B}}{u_0} \right) \left(\frac{\sqrt{MA}}{1 - Mu_0} - \frac{\sqrt{B}}{u_0} \right) = 0. \quad (24)$$

It can be seen that, this result is only valid for

$$\left(\frac{\sqrt{MA}}{1 - Mu_0} - \frac{\sqrt{B}}{u_0} \right) = 0. \quad (25)$$

By doing deformation on (25), we obtain the following expression

$$u_0 = \frac{\sqrt{B}}{\sqrt{MA} + M\sqrt{B}}. \quad (26)$$

Combining the expression of u_0 (26) and (22), it can be found that the minimum value of $\varphi(u)$ can be obtained

$$\begin{aligned} [\varphi(u)]_{\min} &= \varphi(u_0) = \frac{A}{1 - Mu_0} + \frac{B}{u_0} \\ &= \frac{A}{1 - M \frac{\sqrt{B}}{\sqrt{MA} + M\sqrt{B}}} + \frac{B}{\frac{\sqrt{B}}{\sqrt{MA} + M\sqrt{B}}} \\ &= \left(\sqrt{A} + \sqrt{MB} \right)^2. \end{aligned} \quad (27)$$

Combining (19), (20), (21) and (27), we can find

$$\min_{\frac{P_x}{P} \in (0,1)} \varphi \left(\frac{P_x}{P} \right) = \left[\sqrt{L(M\sigma_n^2 + \alpha P)} + \sqrt{M(L\sigma_n^2 + \mu P)} \right]^2. \quad (28)$$

Putting (28) into (17), then $(\gamma_A)_{\max}$ can be represented by

$$(\gamma_A)_{\max} = \frac{L\alpha\mu P^2}{\sigma_n^2 \left[\sqrt{L(M\sigma_n^2 + \alpha P)} + \sqrt{M(L\sigma_n^2 + \mu P)} \right]^2}. \quad (29)$$

Till now, the maximum SNR of the AF system $(\gamma_A)_{\max}$ is written as a function of several variables. The value of $(\gamma_A)_{\max}$ can be easily obtained when the working conditions of the system are determined.

Meanwhile, combining (19) and (26), we can get the corresponding power of every source node allocated

$$P_x = \frac{P\sqrt{L\sigma_n^2 + \mu P}}{\sqrt{ML(M\sigma_n^2 + \alpha P)} + M\sqrt{L\sigma_n^2 + \mu P}} \quad (30)$$

and the power of every relay node

$$P_{ri} = \frac{P - MP_x}{L} = \frac{P\sqrt{ML(M\sigma_n^2 + \alpha P)}}{L[\sqrt{ML(M\sigma_n^2 + \alpha P)} + M\sqrt{L\sigma_n^2 + \mu P}]}. \quad (31)$$

It can be seen that, both P_x and P_{ri} are expressed as the functions of the number of relay node L and that of source node M , as well as the system power P .

4 Numerical Results

From solving (29), (30) and (31), the maximum SNR at destination of the proposed AF system, the corresponding source power and every relay power can be gotten, respectively. As a contrast, we provide the SNR of the scheme that the total power is equally divided up among several source nodes and all relay nodes. For simplicity, we assume that the noise powers in every relay node and destination node are all identical, which is $\sigma_n^2 = 8.28 \times 10^{-14}$ W, here the value is gotten according to internal thermal noise. All signals are transmitted in free space and the channel attenuation factors are obtained from the free space propagation formula, carrier frequency is 2.5×10^3 MHz, the distances between source nodes and relay nodes and that between relay nodes and destination node are all 400m. The following figures are numerical results of SNRs at destination and the power allocations.

In order to demonstrate the effect taken by the number of relay node, the SNRs at destination with respect to the number of relay node are shown in Fig. 2, where the number of source node is 3. From Fig. 2 we can see that, the SNRs at destination of the two schemes grow with the increase of the number of relay node, which are consistent with the previous studies. The reason is that, more relay nodes bring more source-relay-destination sublinks and higher gain in destination. It should be pointed out that, the SNR of the proposed optimized method always outperforms that of the scheme with equal power allocation.

Due to the fact that the total system power determines the performance, the plot of SNRs at destination versus the total system power is given in Fig. 3, where

the number of source node and relay node are 3 and 10, respectively. Obviously, with the increase of the system power, the SNRs at destination are improved. This is because that the noise power is a constant whatever the system power is. For this reason, the bigger the system power, the higher the SNR. Meanwhile, the SNR of the scheme with equal power allocation is similarly with the proposed one while its value is a little lower than it.

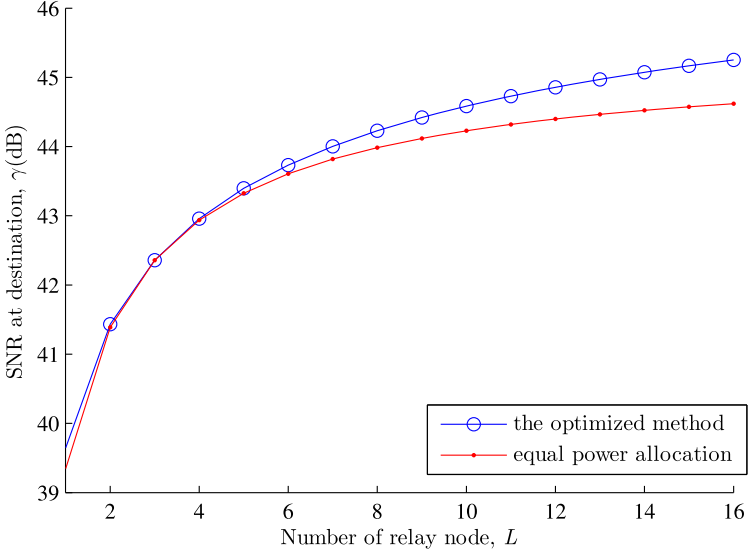


Fig. 2. SNRs at the destination of AF system versus the number of relay node ($M = 3$).

To illustrate how the distance influences the performance, the SNRs at destination with respect to the Distance are shown in Fig. 4, here the distances between the source nodes and the destination node are no longer 400 m, they range from 200 m to 1200 m and the relay nodes lie in the middle position, and the number of source node and relay node are 3 and 10, respectively. It apparently shows that both of the two values of SNR at destination decrease with the increase of the distance, which is consistent with experience and fact. For the farther distance between the source node and destination node is, the weaker the received signal is, wherever for the relay node or the destination node, so the lower the SNR at destination is. Similarly, the SNR of the optimized method outperforms that of the scheme with equal power allocation. In order to learn the power allocation among source nodes and relay nodes in the proposed AF system, and that of every node of the scheme with equal power allocation, the power allocation versus the number of relay node is given in Fig. 5, where the number of source node is 3. Obviously, for the proposed method, the power of every relay node and source node are different. When the number of relay node is more than 2, the power of every source node is bigger than that of every relay

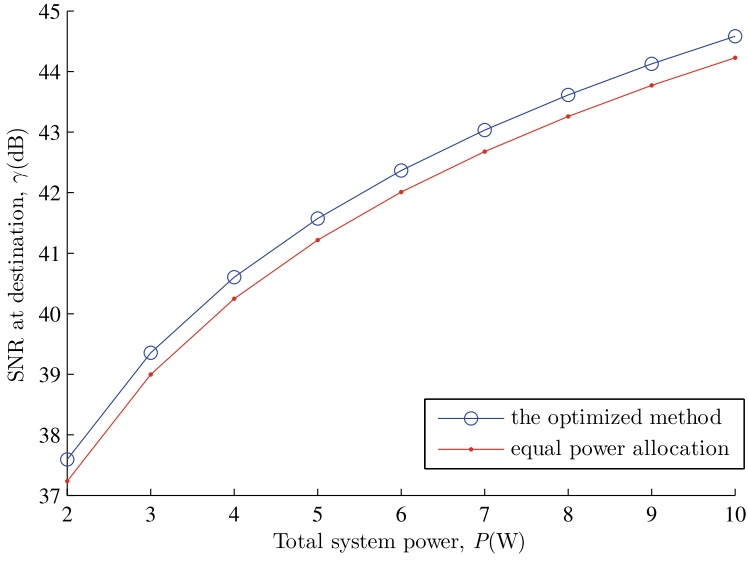


Fig. 3. SNRs at the destination of AF system versus the total system power ($M = 3$, $L = 10$).

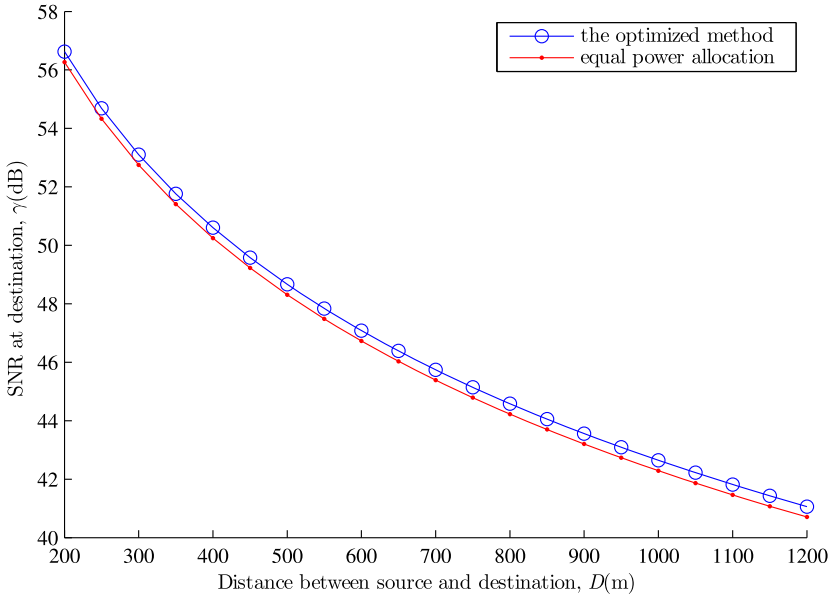


Fig. 4. SNRs at the destination of AF system versus the Distance ($M = 3$, $L = 10$).

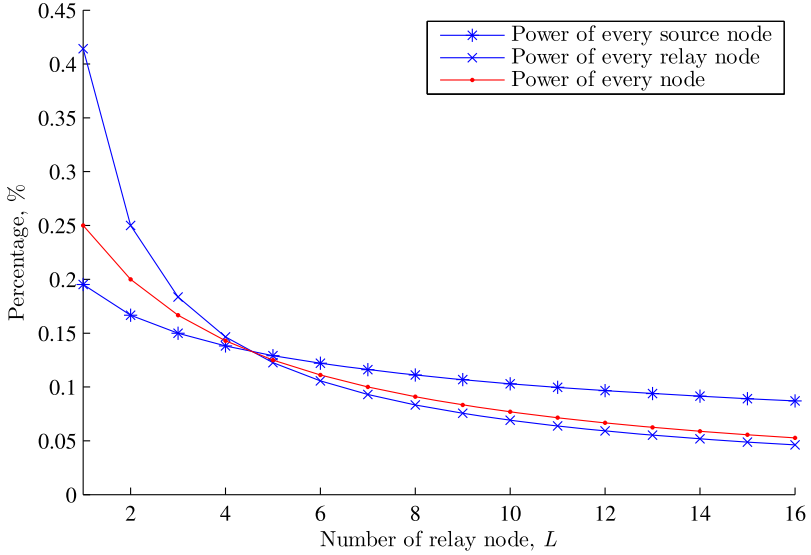


Fig. 5. Percentage of power allocation of AF system versus the number of relay node ($M = 3$).

node. This is because that, the power that the destination node received is coming from all relay nodes, while every relay node received power is only coming from several source nodes. When the relay nodes lie between the source node and the destination node, the scheme will bring higher SNR than the scheme of the latter one, in which the power of every node allocated is similar and the percentage is just the inverse of 1.

5 Conclusion

This paper has proposed an optimized transmitting scheme for the AF system which includes several source nodes and several parallel relay nodes. By an optimization procedure, we have derived the expression of the SNR and obtained the maximum SNR at the destination. Based on the expression and corresponding parameters, validations by the simulation method have been conducted. The simulation and analyze results have showed that the SNR at destination of the proposed scheme outperforms the one in which the power is equally allocated among all source and relay nodes.

Acknowledgments. This work was supported in part by the National Key Scientific Instrument and Equipment Development Project under Grant No. 61827801, and in part by the National Natural Science Foundation of China under Grant No. 61471192.

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