Minimum Class Variance Thresholding Based on Multi-objective Optimization

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Abstract. Variance-based thresholding is one of the most popular methods for image segmentation. The mechanism of variance-based thresholding methods is to minimize the class variance. A novel minimum class variance thresholding method based on multi-objective optimization has been presented, and the ideal threshold is achieved by minimizing the variance of each class and the sum of them, and this will lead to more satisfactory segmentation result. The presented method possesses the merits of restraining the class probability and the class variance effects, and it is more accurate. Firstly, the proposed method is compared quantitatively with other methods on lots of synthetic images with the convenience of obtaining the ideal thresholds precisely and the ground-truth images exactly. The presented method possess better performance at most magnitudes of the noise. At the same time, experiments over real infrared images and visual images also have illustrated the better performance of the presented method.

Keywords: Thresholding · Class variance · Multi-objective optimization

1 Introduction

In digital image processing, thresholding is one of the simple and effective methods for image segmentation. This method is based on the assumption that the intensity is similar within an object and different between different objects, and one appropriate threshold can be found to separate the object from the background [1–3]. And there have been many applications of thresholding for image segmentation, such as document image analysis, CT scan images, nondestructive testing, and so on. A large amount of thresholding methods have been developed due to their efficiency and simplicity, especially for two-class segmentation problems [1–3]. Among these methods, variance-based thresholding is a very popular technology for image segmentation [1–7]. The minimum within-class variance method proposed by Otsu is regarded as one of the classic methods [4]. Nonetheless, the Otsu method tends to shift the threshold towards the component with larger class probability or larger class variance. Hou et al. has explored the underlying mechanism responsible for Otsu method's bias, and proposed the MCVT method, which can overcome the Otsu method's disadvantages [5]. Li has proposed a new statistical thresholding method to avoid the negative influence caused by variance discrepancy between the object and the

H. Jin et al. (Eds.): CROWNCOM 2021/WiCON 2021, LNICST 427, pp. 183–191, 2022.
https://doi.org/10.1007/978-3-030-98002-3_13

background [6], but there is huge bias in the formula, and this method is called Li_1 for convenience in this paper. In addition, Li has proposed another statistical thresholding method for images in which the object and the background have similar statistical distribution [7], and this method is called Li_2 in this paper. Experiments on lots of corresponding images show that the two methods are not as effective as described, which has been analyzed in Sect. 2.

According to the theory of classifying and optimization, we have presented a novel minimum class variance thresholding based on multi-objective optimization (MCVT-MO). The presented method selects the threshold by a compromise solution. It can be applied to many classes of images effectively, such as infrared images, ordinary visual images, and even such images in which the object and background gray levels possess substantially overlapping distributions. Experiments in Sect. 3 show that the presented method possesses the MCVT method's advantages of overcoming the class probability and the class variance effects, but it is more accurate than the MCVT method.

This paper is organized as follows. In Sect. 2, the presented MCVT-MO method is formulated and a generalized MCVT method is defined. In Sect. 3, the performance of the presented method has been tested and compared with other methods on synthetic and real images. Finally, some concluding remarks are presented in Sect. 4.

2 Minimum Class Variance Thresholding Based on Multi-Objective Optimization

2.1 Theory of Multi-objective Optimization

A multi-objective optimization problem is formulated as follows [8]:

$$min\big(f(x) = \big(f_1(x), f_2(x), \cdots f_p(x)\big)\big) \tag{1}$$

Over $x \in X$, X is the set of the non-inferior solutions of the vector optimization problem. This kind of problem is also called a vector optimization.

Generally, an optimal solution that minimizes all the objective functions simultaneous does not exist in multi-objective optimization problems, which are different from the programming problem with a single objective function. In this case, we can take the total balance of objectives into account to find a compromise solution. One of the most popular compromise solutions for the multi-objective optimization problems is the $l_p - norm$, which is defined as follows [8]

$$min\left\{ d_p\left(\boldsymbol{f}(x), \widehat{\boldsymbol{f}}\right) \triangleq \left\{ \sum_{l}^{n} W_j \left| f_j(x) - \widehat{f}_j \right|^p \right\}^{1/p} \right\} \tag{2}$$

Subject to $x \in X$.

Where $1 \leq p \leq \infty$, $\widehat{\boldsymbol{f}}$ is the goal vector, W_j is the weight or priority given to the jth objective. $d(,)$ is the distance between $\boldsymbol{f}(x)$ and $\widehat{\boldsymbol{f}}$. If the goal $\widehat{f}_j$ is unattainable, the distance between $f_j(x)$ and $\widehat{f}_j$ can be considered to represent a measure of regret resulted from un-attainability of $f_j(x)$ to $\widehat{f}_j$. $min\left\{d_p\left(\boldsymbol{f}(x),\widehat{\boldsymbol{f}}\right)\right\}$ represents the minimum combined deviation from the goals $\widehat{f}_1, \widehat{f}_2, \cdots \widehat{f}_n$.

2.2 MCVT Based on Multi-objective Optimization (MCVT-MO)

As is known, the variance of a random variable X is a measure of dispersion or scatter from the mean in the possible values for X. Smaller variance corresponds to smaller dispersion from the center. The mechanism of variance-based thresholding methods is to minimize the class variance [5]. Therefore, the ideal threshold can be achieved by minimizing the variance of each class and the sum of them.

According the theories of variance and classifying, the goals to segment an image with two classes should be as follows

$$\sigma^2_{1min} = min\left\{\sigma^2_1(t), t = 0, 1, 2, \cdots L-1\right\} \tag{3}$$

$$\sigma^2_{2min} = min\left\{\sigma^2_2(t), t = 0, 1, 2, \cdots L-1\right\} \tag{4}$$

$$\sigma^2_{smin} = min\left\{\sigma^2_s(t), t = 0, 1, 2, \cdots L-1\right\} \tag{5}$$

$$\text{with } \sigma^2_s = \sigma^2_1(t) + \sigma^2_2(t), t = 0, 1, 2, \cdots L-1 \tag{6}$$

The above formulas (3–6) mean that when the object and background are well separated, the class variance $\sigma^2_1(t)$, $\sigma^2_2(t)$, and the sum of class variances $\sigma^2_s = \sigma^2_1(t) + \sigma^2_2(t)$ all should be minimized. However, they will not be minimized simultaneous with the same gray value t in most cases.

According to the theory of multi-objective optimization elaborated in Sect. 2.1, we adopt $l_2 - norm$ in this paper, and give three class variances $\sigma^2_1(t)$, $\sigma^2_2(t)$, $\sigma^2_s(t)$ the same weight or priority, $W_j = 1, j = 1, 2, 3$. Then, a compromise solution can be defined to have the minimum combined deviation of class variances from the desired goals $\sigma^2_{1min}, \sigma^2_{2min}$ *and* σ^2_{smin}.

$$J(t) = \left\{\left(\sigma^2_1(t)\right)^2 + \left(\sigma^2_2(t)\right)^2 + \left(\sigma^2_s(t) - \sigma^2_{smin}\right)^2\right\}^{\frac{1}{2}}, t = 0, 1, 2, \cdots, L-1. \tag{7}$$

Then the optimal threshold t^* is determined as follows

$$t^* = Arg \min_{0 \le t \le L-1} J(t) \tag{8}$$

2.3 The Generalized MCVT Method

According to the theory of multi-objective optimization discussed in Sect. 2.1, if we adopt $l_1 - norm$, give the two class variances $\sigma^2_1(t)$ and $\sigma^2_2(t)$ the same weight or priority, $W_j = 1, j = 1, 2,$ and do not consider the sum of class variances $\sigma_s^2 = \sigma_1^2(t) + \sigma_2^2(t)$, we can get another form of discriminating function $J(t)$. A compromise solution can be defined to have the minimum combined deviation from the desired goals σ_{1min}^2 and σ_{2min}^2.

$$J(t) = \sigma_1^2(t) + \sigma_2^2(t), t = 0, 1, 2, \cdots L - 1 \tag{9}$$

Then the optimal threshold t* is determined as follows

$$t^* = Arg \min_{0 \le t \le L-1} J(t) \tag{10}$$

We can find that the MCVT method also can be obtained according to the theory of multi-objective optimization.

In addition, a generalized MCVT method can be defined as follows

$$J(t) = \left\{ \left(\sigma_1^2(t)\right)^p + \left(\sigma_2^2(t)\right)^p \right\}^{\frac{1}{p}}, 1 \le p \le \infty, t = 0, 1, 2, \cdots, L - 1 \tag{11}$$

the optimal threshold t* is determined as follows

$$t^* = Arg \min_{0 \le t \le L-1} J(t) \tag{12}$$

According to aforesaid analysis, we can see that the MCVT method and MCVT-MO method are same in substance, and they just use different $l_p - norm$. Whereas the presented MCVT-MO method takes into account a more complete information including the distance between the sum of class variances $\sigma_s^2(t)$ and ideal σ_{smin}^2, and this will lead to more satisfactory segmentation result. The better performance of the MCVT-MO method has been proved in Sect. 3.

3 Contrastive Experiments and Analysis

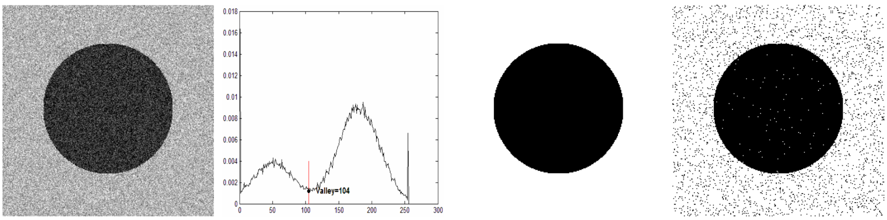
A variety of synthetic of image and real images are chosen to test the five thresholding methods. To put into evidence the different performance features of these methods, we have used the following two performance criteria: misclassification error (ME) and segmentation error (SE). As paper [3], the two performance measures are adjusted so that their scores vary from 0 for a totally correct segmentation to 1 for a totally erroneous case. The arithmetic averaging of the normalized scores obtained from the two criteria is chosen as the indication of segmentation quality. Because the segmentation results of the algorithms are impacted seriously by the noisiness of the image, different performance of the algorithms are tested on the synthetic image added on Gaussian noise with different magnitudes.

3.1 Experiments on Synthetic Images

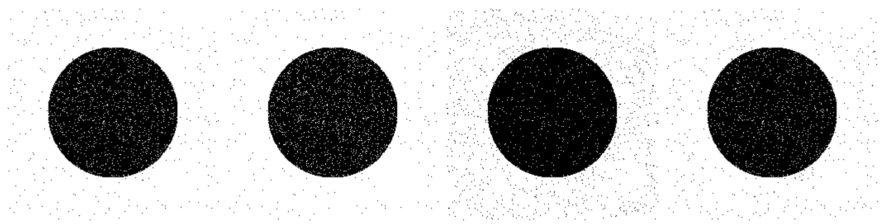
Firstly, the proposed MCVT-MO method is compared quantitatively with other methods on lots of synthetic images with the convenience of obtaining the ideal thresholds precisely and the ground-truth images exactly.

Figure 1(a) is a synthetic image with two intensities (the circle region is 50, the square region is 180), and Gaussian white noise is added to it, whose mean is 0 and variance is 0.015. Figure 1(b) is the histogram of the synthetic image, and the valley is at 104 marked as a dot in the figure. Figure 1(c) is the ground-truth image. Figure 1(d)–(h) are respectively the Otsu result, MCVT result, Li_1 result, Li_2 result, and MCVT-MO result.

From the thresholded images, one can see that the Otsu method almost perfectly segments the smaller circle component, but yields a noisier map in the largest square component. The Li_1 method, MCVT method and the proposed MCVT-MO method all lead to a cleaner segmentation map in the square component. The reason is that the Otsu method tends to shift the threshold towards the component with larger class probability or larger class variance [5], which is verified by the threshold 131 of the Otsu method. Nevertheless, the presented MCVT-MO method and the MCVT method can overcome the class probability and the class variance effects, and detect the valley more accurately than other methods as illustrated in Table 1. The threshold by the MCVT-MO method is exactly the valley, followed by the selected threshold 100 of the MCVT method. The difference between the threshold 131 determined by the Otsu method and the ideal threshold 104 is the biggest, which is followed by the Li_2 method and Li_1 method.



(a)synthetic image (b) the histogram(104) (c) the ground-truth image (d)Otsu method(131)



(e)MCVT method(100) (f)Li_1 method(97) (g)Li_2 method(115) (h)MCVT-MO(104)

Fig. 1. Experiment of different methods on the synthetic image (the number in the bracket denotes the valley of histogram or the threshold by the method)

Table 1. Experiment results of different thresholding methods on the synthetic image.

Method	Threshold	ME	SE	Average score (AVE)
Otsu	131	0.04404	0.13900	0.09152
MCVT	100	0.02010	0.04181	0.030955
Li_1	97	0.02235	0.05635	0.03935
Li_2	115	0.01953	0.02661	0.02307
MCVT-MO	104	0.01797	0.02334	0.020655

Table 1 lists the results in terms of threshold, ME, SE and the average scores (AVE), which are obtained by applying these thresholding methods to the synthetic noisy image shown in Fig. 1(a). From Table 1, one can see that the results of ME, SE and the AVE yielded by the proposed MCVT-MO method are the smallest among the five thresholding methods. Comprehensive speaking, the proposed MCVT-MO method should be considered the best among the five methods, followed by the MCVT method.

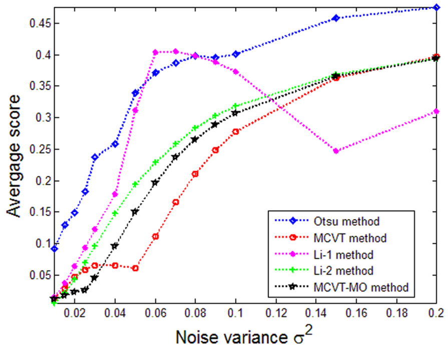


Fig. 2. The relation between the Average scores of different methods and the intensity of Gaussian noise.

Figure 2 shows the relation between the average scores of different methods and the intensity of Gaussian noise. The synthetic image is added with Gaussian white noise with zero mean and gradually increased variance. To be more precise, the AVE of each method is calculated five times at every magnitude of the noise variance, and the mean is chosen as the result.

From Fig. 2, one can see that the segmentation performance of all the methods declines with the increase of noise intensity. The presented MCVT-MO method and MCVT method possess better performance at most magnitudes of the noise. When the magnitude of noise variance is smaller than 0.04, the performance of the presented MCVT-MO method is better than the MCVT method, which is verified in Fig. 1. When the magnitude is bigger than 0.04, the result of comparison is reversed. However, when the magnitude is bigger than 0.1, the two methods' performance tends to be identical. The performance of the proposed MCVT-MO method, MCVT method and the Li_2 method varies gently, and the performance of the Otsu method and the Li_1 method varies sharply, especially the Li_1 method, which indicates that the stability of the method is poor.

3.2 Experiments on Infrared Images

The performance of the proposed MCVT-MO method has been tested and compared with other methods on lots of real infrared images. Owing to the intrinsic characteristics and abominable imaging environment, the infrared images are of poor contrast and faint edges, and the object and background gray levels possess substantially overlapping distributions, even resulting in a unimodal distribution, which can be seen in Fig. 3(a)–(b).

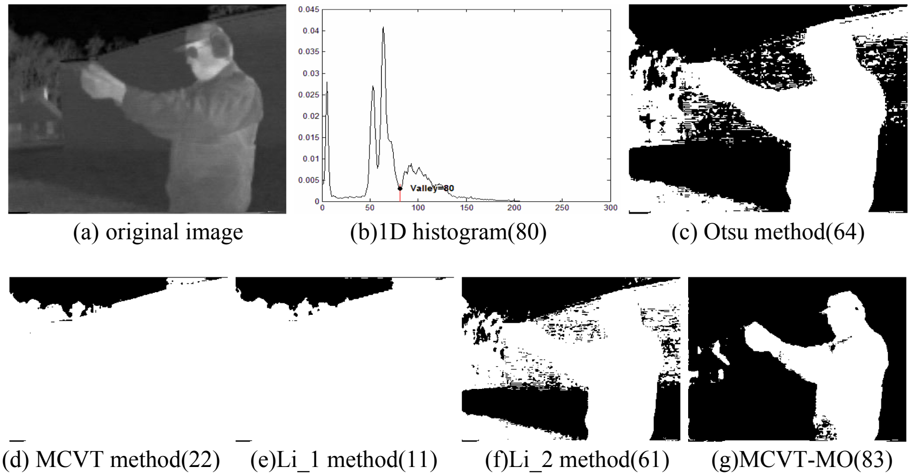


Fig. 3. Experiment of different methods on the real infrared images of irg04 group (the number in the bracket denotes the valley of histogram or the threshold by the method).

Figure 3(a) is infrared images selected from irg04 in Terravic Weapon IR Database of OTCBVS Benchmark Dataset Collection [9], which are video sequences showing muzzle blast and shell discharge. The size of the images is pixels.

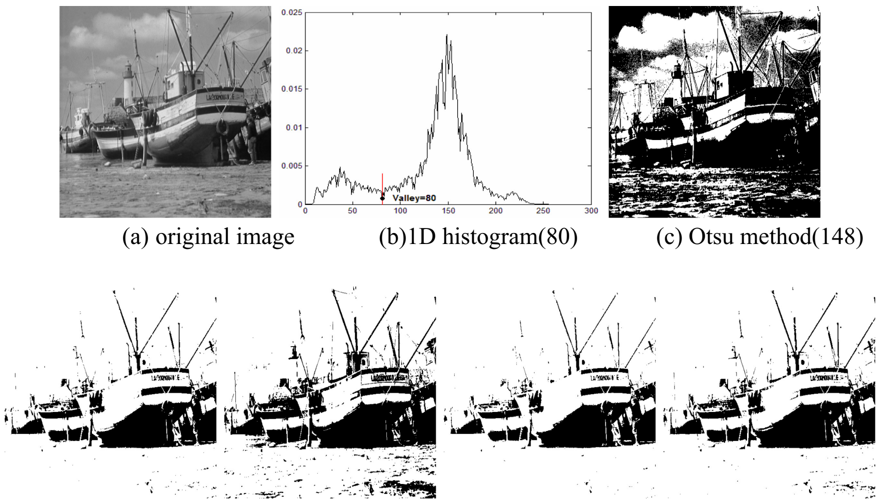
From the histogram in Fig. 3(b), it can be seen that the image consists of three components. The darkest component mainly corresponds to the sky. The middle component is composed of the building and its shadow, the ground and trees. The brightest component is largely contributed by the shooter who is diffusing more heat. The valley between the middle and the brightest component is at 80. The threshold by the presented MCVT-MO method is 83, which is close to the valley. The shooter has been separated completely in the infrared image by the presented method. The threshold by the Otsu method is 64, which shifts towards the middle component with larger class probability. Portions of the building, the ground and the trees are misclassified as the object, and the segmented result is noisier by the Otsu method as illustrated in Fig. 3(c). The threshold by the Li_2 method is 61, and the result is worse. The thresholds by the MCVT method and Li_1 method are respectively 22 and 11. The building and its shadow, the ground and trees are all misclassified as the object, the shooter cannot be identified absolutely in Fig. 3(d)–(e).

3.3 Experiments on Other Images

The proposed MCVT-MO method has also been tested on lots of visible images, follows are two examples.

Figure 4(a) is a gray image chosen from the miscellaneous volume of USC-SIPI image database, which is maintained primarily to support research in image processing, image analysis, and machine vision [10]. The size of the image is pixels. The objects in Fig. 4(a) are three fishing boats. The darkest component mainly corresponds to the bilge, the top of the shipboard and the masts. The largest component is composed of the sky, the beach and most part of the hull.

As illustrated in the histogram in Fig. 4(b), the valley is at 80. The thresholds by the MCVT method and the proposed MCVT-MO method are respectively 75 and 85, which are close to the valley. Most features of the fishing boats can be discriminated clearly in their results. The Li_1 result is noisy with the threshold 96, which misclassifies some part of the beach as the bilge. The Li_2 result is under-segmented with the threshold 68, and some details of the object have been lost, such as the masts of the two smaller fishing boat. The Otsu result is over-segmented with the threshold 148 biasing towards the larger component, and it is very noisy perceptually.



(a) original image (b)1D histogram(80) (c) Otsu method(148)

(d) MCVT method (75) (e)Li_1 method (96) (f)Li_2 method(68) (g)MCVT-MO(85)

Fig. 4. Experiment of different methods on the fishing boat image (the number in the bracket denotes the valley of histogram or the threshold by the method)

4 Conclusions

The mechanism of variance-based thresholding methods is to minimize the class variance. Therefore, the ideal threshold can be achieved by minimizing the variance of each class and the sum of them. According to the theory of multi-objective optimization, we have defined a compromise solution to have the minimum combined deviation from the desired goals.

In fact, the MCVT method can also be obtained through this theory as elaborated in Sect. 2.3. The presented MCVT-MO method possesses the MCVT method's merits of restraining the class probability and the class variance effects, and takes into account a more complete information including the distance between the sum of class variances and its ideal, which results in more reasonable thresholds. Experiments over synthetic images, real infrared images and visual images have illustrated that the performance of the presented method is better than the Otsu method, the MCVT method, Li_1 method, and Li_2 method.

Acknowledgements. This work was supported by Science Foundation of Hebei Normal University (Grant No. L2021B31). Our gratitude is extended to the anonymous reviewers for their valuable comments and professional contributions to their improvement of this paper.

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