A Sophisticated Anti-jamming Strategy for a Joint Radar and Communication System

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Abstract. In this paper, we consider the problem of determining how a joint (dual) radar and communication system should divide its effort between supporting its radar and communication tasks in the presence of a jammer that wants to obstruct the system's work by means of jamming. The system, besides the basic objective consisting of two tasks (a) to communicate with a receiver and (b) to track a radar target through the reflections witnessed at the system, also has the secondary objective to achieve the basic objective in a manner that is as unpredictable as possible to the jammer. The signal to interference and noise ratio (SINR) of the radar and communication's SINR are considered as the metrics that reflect the radar and communication tasks, respectively. The entropy associated with a system's strategy to switch between two tasks is considered as a metric that reflects unpredictability of its strategy for the jammer. We model this problem by a Bayesian game for a scenario where the system is at a disadvantage to access information about environmental parameters relative to the jammer. The established uniqueness of the equilibrium reflects stability of the designed anti-jamming strategy, even in such a disadvantageous situation for the system.

Keywords: Nash equilibrium · Radar · Communication · Jamming

1 Introduction

There has been recent interest in enabling radar and communication systems to co-exist in the same frequency bands in order to allow spectrum to be utilized more efficiently [17]. This has given rise to a significant amount of research on methods for spectrum sharing between the two systems. One approach to achieve this is to formulate waveform design using Orthogonal Frequency Division Multiplexing (OFDM) signals and then optimally allocate the subcarriers [12,18]. Radar waveform design for controlled interference is considered in [3,4], while the cooperative design of the two systems was explored in [5,14]. In this paper, we consider a dual purpose communication-radar system that employs OFDM style waveforms and explore the *complementary aspect* where the system besides the basic objective consisting of two tasks (a) to communicate with a receiver

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and (b) to track a radar target through the reflections witnessed at the system, also has the secondary objective to achieve the basic objective in a manner that is as unpredictable as possible to the jammer. We model this problem by a non-zero sum (Bayesian) game. The radar's SINR and communication's SINR are considered as the metrics that reflect corresponding tasks of the basic objective. The entropy of the system's strategy for switching between its two tasks of basic objective is considered as the metric that reflects its unpredictability, i.e., its secondary objective. The problem is modeled by a non-zero sum game and the equilibrium is found in closed form. Its uniqueness is proven, which reflects stability of the designed anti-jamming strategy even in the case where the system is at a serious disadvantage when compared with the jammer in terms of the information possessed regarding the jamming fading gains. Specifically, we consider that the system has only statistical information, meanwhile the jammer has complete information about the channel gains.

2 Basic Model of a Dual Purpose Communication-Radar System

Let us consider an operational scenario involving a dual purpose communication-radar system that transmits communication signals only, and uses the received reflections of those signals off targets for target tracking in presence of an jammer (Fig. 1). The system wants to support two different tasks: (a) to communication with a receiver, and (b) to track a radar target through the reflections witnessed at the system. In order to support these two tasks, the system uses a spectrum band that is modeled as consisting of n adjacent sub-channels, which may be associated with n different subcarriers. In this paper we employ a transmission scheme like OFDM, as considered in [11]. With each of these n different subcarriers, two different (fading) channel gains are associated. Specifically, we let $h_{\mathbb{R},i}$, $i \in \mathcal{N} \triangleq \{1, ..., n\}$, correspond to the i-th radar channel gain associated with the round-trip effect of the transmitted signal, reflected off the radar target, and received at the system, while $h_{\mathbb{C},i}$ denotes the i-th channel gain associated with the i-th communication subcarrier at the receiver.

Let $\boldsymbol{P} = (P_1, \ldots, P_n)$ be power-allocation strategy for the system, where P_i, $i \in \mathcal{N}$, is the power assigned for transmitting on subcarrier i, and $\sum_{i \in \mathcal{N}} P_i = \overline{P}$, where $\overline{P}$ is the total power budget.

We assume that to avoid interference at each moment the system focuses only on one of two tasks. We call such focusing on a task by implementing the corresponding *mode*. Thus, at each moment the system can implement one of two modes:

(i) *communication mode*, denoted by $\mathbb{C}$, to focus on the communication's task,
(ii) *radar mode*, denoted by $\mathbb{R}$, to focus on the radar's task.

In order to unify the examination of radar and communication metrics, we note that radar detection/tracking and communication throughput are both closely related to the associated signal-to-interference-plus-noise ratio (SINR)

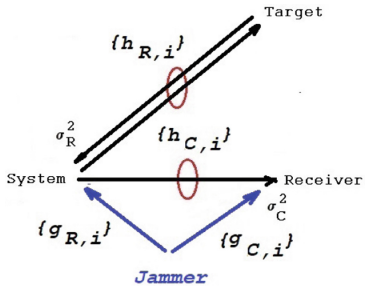

Fig. 1. Dual purpose communication-radar system, target. receiver and jammer

as witnessed at the appropriate recipient. Let the radar SINR and communication SINR be given by $\sum_{i=1}^{n} h_{\mathbb{R},i}P_i/\sigma_{\mathbb{R}}^2$ and $\sum_{i=1}^{n} h_{\mathbb{C},i}P_i/\sigma_{\mathbb{C}}^2$, respectively, with $\sigma_{\mathbb{R}}^2$ and $\sigma_{\mathbb{C}}^2$ are corresponding background noise powers. For the communication objective, the SINR is used as the payoff function for two reasons: first, it is easily linearized; and, second, for a low SINR regime, SINR is an approximation to throughput. For the radar objective, SINR is used as the payoff function since it is closely related to the associated detection metrics [16].

3 Jammer

Let a *jammer* be present, and it is located close to the receiver (Fig. 1). The jammer seeks to introduce hostile interference to disrupt the functionality of the system. Its strategy is a power allocation vector $\boldsymbol{J} = (J_1, \ldots, J_n)$, where J_i, $i \in \mathcal{N}$, is the power assigned for jamming subcarrier i, and $\sum_{i\in\mathcal{N}} J_i = \overline{J}$ where $\overline{J}$ is the total power budget allocated for jamming. We assume that the jammer's signal will not reach the target. Under a jamming attack, the communication's SINR and radar's SINRs are given by:

$$\mathrm{SINR}_{\mathbb{R}}(\boldsymbol{P}, \boldsymbol{J}) = \sum_{i\in\mathcal{N}} \frac{h_{\mathbb{R},i}P_i}{\sigma_{\mathbb{R}}^2 + g_{\mathbb{R},i}J_i} \tag{1}$$

and

$$\mathrm{SINR}_{\mathbb{C}}(\boldsymbol{P}, \boldsymbol{J}) = \sum_{i\in\mathcal{N}} \frac{h_{\mathbb{C},i}P_i}{\sigma_{\mathbb{C}}^2 + g_{\mathbb{C},i}J_i}, \tag{2}$$

where $g_{\mathbb{R},i}$ are fading channels gains between the jammer and the radar, and $g_{\mathbb{C},i}$ are fading gains between the jammer and the communication receiver.

Let the system work in a fixed mode m, $m \in \{\mathbb{C}, \mathbb{R}\}$ and this mode is known to the jammer. Let the system and jammer implement strategies $\boldsymbol{P}$ and $\boldsymbol{J}$, respectively. Then, $\mathrm{SINR}_m(\boldsymbol{P}, \boldsymbol{J})$ is the payoff to system in mode m, and it can be considered as the cost function for the jammer. Thus, for a fixed mode m,

this is a zero-sum game with $\mathrm{SINR}_m(\boldsymbol{P}, \boldsymbol{J})$ as system's payoff. Let us denote this zero-sum game by $\mathfrak{G}_m$.

For each fixed mode $m \in \{\mathbb{C}, \mathbb{R}\}$, we look for a (Nash) equilibrium in this game $\mathfrak{G}_m$. Thus, for such a pair of strategies $(\boldsymbol{P}_m, \boldsymbol{J}_m)$ that the following inequalities hold:

$$\mathrm{SINR}_m(\tilde{\boldsymbol{P}}, \boldsymbol{J}_m) \le \mathrm{SINR}_m(\boldsymbol{P}_m, \boldsymbol{J}_m) \le \mathrm{SINR}_m(\boldsymbol{P}_m, \tilde{\boldsymbol{J}}) \text{ for all } (\tilde{\boldsymbol{P}}, \tilde{\boldsymbol{J}}). \quad (3)$$

The equilibrium strategies $(\boldsymbol{P}_m, \boldsymbol{J}_m)$ in game $\mathfrak{G}_m$ can be found via [1].

4 Mode Selection

Now suppose the system can choose between radar and communication mode, and, then, implement the optimal power allocation strategy for these modes $\boldsymbol{P}_{\mathbb{R}}$ and $\boldsymbol{P}_{\mathbb{C}}$, respectively. The jammer does not know what mode the system implements. Thus, a dilemma for the jammer arises: the best response to each of the system's power allocation strategies $\boldsymbol{P}_{\mathbb{R}}$ or $\boldsymbol{P}_{\mathbb{C}}$ to implement. Specifically, to implement power allocation strategy $\boldsymbol{J}_{\mathbb{R}}$ or $\boldsymbol{J}_{\mathbb{C}}$, respectively. Meanwhile, in its turn, the system does not know which decision will be made by the jammer. Thus, a dilemma also arises for the system: the best response to each of jammer's power allocation strategies $\boldsymbol{P}_{\mathbb{R}}$ or $\boldsymbol{P}_{\mathbb{C}}$ to implement, namely, to apply power allocation strategy $\boldsymbol{J}_{\mathbb{R}}$ or $\boldsymbol{J}_{\mathbb{C}}$, respectively. This scenario leads to the following payoff matrix $\mathcal{M}$, where the rows are the system's strategies and the columns are the jammer's strategies:

$$\mathcal{M} = \begin{array}{c} \\ \mathbb{R} \\ \mathbb{C} \end{array} \begin{array}{c} \begin{array}{cc} \mathbb{R} & \mathbb{C} \end{array} \\ \begin{pmatrix} A & a \\ b & B \end{pmatrix} \end{array} \quad (4)$$

with

$$A \triangleq \mathrm{SINR}_{\mathbb{R}}(\boldsymbol{P}_{\mathbb{R}}, \boldsymbol{J}_{\mathbb{R}}), \quad a \triangleq \mathrm{SINR}_{\mathbb{R}}(\boldsymbol{P}_{\mathbb{R}}, \boldsymbol{J}_{\mathbb{C}}), \quad (5)$$

$$B \triangleq \mathrm{SINR}_{\mathbb{C}}(\boldsymbol{P}_{\mathbb{C}}, \boldsymbol{J}_{\mathbb{C}}) \text{ and } b \triangleq \mathrm{SINR}_{\mathbb{C}}(\boldsymbol{P}_{\mathbb{C}}, \boldsymbol{J}_{\mathbb{R}}). \quad (6)$$

Note that, by (3), without loss of generality we could assume that

$$A < a \text{ and } B < b. \quad (7)$$

Let $\boldsymbol{x} = (x, 1-x)$ be randomized (mixed) strategies [6] for the system, where x and $1-x$ are probabilities that the system is in mode $\mathbb{R}$ and $\mathbb{C}$, respectively. Meanwhile, $\boldsymbol{y} = (y, 1-y)$, be randomized (mixed) strategies for the jammer, where y and $1-y$ are probabilities that the jammer is in mode $\mathbb{R}$ and $\mathbb{C}$, respectively. Then, if the system and the jammer implement strategies $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, the expected system's SINR is given as follows

$$\begin{aligned} \mathrm{SINR}_S(\boldsymbol{x}, \boldsymbol{y}) &= \boldsymbol{x}\mathcal{M}\boldsymbol{y}^T \\ &= Axy + ax(1-y) + b(1-x)y + B(1-x)(1-y). \end{aligned} \quad (8)$$

4.1 Sophisticated System

The (sophisticated) system wants to find a trade-off between two objectives:

(i) *the basic objective*, to maximize the expected system's SINR,
(ii) *the secondary objective*, to achieve this maximum in the most unpredictable way for the jammer.

As a metric for the system to confuse the jammer, we consider the entropy of its strategy, i.e.,

$$H(\boldsymbol{x}) = -x\ln(x) - (1-x)\ln(1-x). \tag{9}$$

The payoff for such (sophisticated) system is taken as a weighted sum of the expected system's SINR and the entropy of its strategy, i.e.,

$$v_S(\boldsymbol{x}, \boldsymbol{y}) = w_S\,\mathrm{SINR}_S(\boldsymbol{x}, \boldsymbol{y}) + w_E H(\boldsymbol{x}), \tag{10}$$

where w_S and w_E are non-negative weighting coefficients.

Note that for any scale positive parameter C, the payoff $C\,v_S(\boldsymbol{x}, \boldsymbol{y})$ achieves its maximum at the same strategy $\boldsymbol{x}$. That is why, without loss of generality we can assume that w_S and w_E are normalized non-negative weighting coefficients. Specifically,

$$w_S \triangleq 1 - w \text{ and } w_E \triangleq w, \text{ where } w \in [0,1]. \tag{11}$$

This and (10) imply that

$$v_S(\boldsymbol{x}, \boldsymbol{y}) = (1-w)\mathrm{SINR}_S(\boldsymbol{x}, \boldsymbol{y}) + wH(\boldsymbol{x}). \tag{12}$$

Note that, in [8] we used entropy metric to design a sophisticated adversary's strategy for illegal sneaking into protected bandwidth, and, in [7,9], to design sophisticated anti-eavesdropping strategies,

Meanwhile, jammer wants to minimize the expected system's SINR. Thus, payoff to the jammer is negative of the expected system's SINR, i.e.,

$$v_J(\boldsymbol{x}, \boldsymbol{y}) = -\mathrm{SINR}_S(\boldsymbol{x}, \boldsymbol{y}). \tag{13}$$

5 Incomplete Information

In this section, we consider a more practical case, in which there is incomplete information about some of the parameters involved. In particular, we assume that the jammer knows all of the parameters perfectly, as this is the most dangerous case for the system. Meanwhile, the system knows all of the parameters except the channel gains for the jammer, for which it has limited statistical information. Specifically, the system only knows that the jammer's channel gains can have values $(g_{\mathbb{R},i,k}, g_{\mathbb{C},i,k}), i \in \mathcal{N}$ with probability α_k, where $k \in \mathcal{K} \triangleq \{1,\ldots,K\}$. In other words, the jammer's channel gains can be in one of K states, and the

system does not know which state occurs (Fig. 2). Note that if the jammer's channel gains are in state k, then the radar's SINR and communication's SINR are give as follows:

$$\mathrm{SINR}_{\mathbb{R},k}(\boldsymbol{P},\boldsymbol{J})=\sum_{i\in\mathcal{N}}\frac{h_{\mathbb{R},i}P_i}{\sigma_{\mathbb{R}}^2+g_{\mathbb{R},i,k}J_i} \tag{14}$$

and

$$\mathrm{SINR}_{\mathbb{C},k}(\boldsymbol{P},\boldsymbol{J})=\sum_{i\in\mathcal{N}}\frac{h_{\mathbb{C},i}P_i}{\sigma_{\mathbb{C}}^2+g_{\mathbb{C},i,k}J_i}. \tag{15}$$

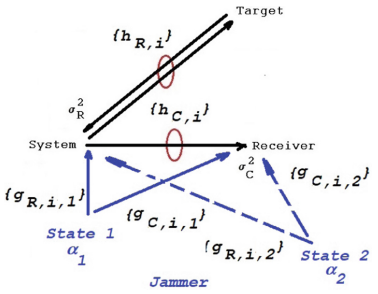

Fig. 2. Dual purpose communication- radar system, target. receiver and jammer, which channel gains can be in one of two states, i.e., $\mathcal{K}=\{1,2\}$.

Similar to (3), for each fixed mode $m\in\{\mathbb{C},\mathbb{R}\}$ and jammer's channel state $k\in\mathcal{K}$ let $\boldsymbol{P}_{m,k}$ and $\boldsymbol{J}_{m,k}$ be equilibrium strategies of the system and jammer, respectively, in zero sum game with $\mathrm{SINR}_{m,k}$ as system's payoff. Further, if channel state k occurs, and this state is known to both players, the system and jammer can choose mode $m\in\{\mathbb{R},\mathbb{C}\}$ and $\tilde{m}\in\{\mathbb{R},\mathbb{C}\}$, respectively, and implement the equilibrium strategies $\boldsymbol{P}_{m,k}$ and $\boldsymbol{J}_{\tilde{m},k}$, associated with their choices independently from each other. This leads to the following payoff matrix $\mathcal{M}_k$ to the system:

$$\mathcal{M}_k=\begin{array}{c}\\ \mathbb{R}\\ \mathbb{C}\end{array}\begin{array}{c}\begin{array}{cc}\mathbb{R} & \mathbb{C}\end{array}\\ \begin{pmatrix}A_k & a_k\\ b_k & B_k\end{pmatrix}\end{array} \tag{16}$$

with

$$A_k\triangleq\mathrm{SINR}_{\mathbb{R},k}(\boldsymbol{P}_{\mathbb{R},k},\boldsymbol{J}_{\mathbb{R},k}),\quad a_k\triangleq\mathrm{SINR}_{\mathbb{R},k}(\boldsymbol{P}_{\mathbb{R},k},\boldsymbol{J}_{\mathbb{C},k}), \tag{17}$$

$$B_k\triangleq\mathrm{SINR}_{\mathbb{C},k}(\boldsymbol{P}_{\mathbb{C},k},\boldsymbol{J}_{\mathbb{C},k})\text{ and }b_k\triangleq\mathrm{SINR}_{\mathbb{C},k}(\boldsymbol{P}_{\mathbb{C},k},\boldsymbol{J}_{\mathbb{R},k}). \tag{18}$$

Then, following (3), we have that

$$A_k<a_k\text{ and }B_k<b_k. \tag{19}$$

Let us denote as a *k-type jammer,* the jammer if its fading gains occur in state k. Also, $\boldsymbol{y}_k = (y_k, 1-y_k)$, be mixed strategies for the k-type jammer, where y_k and $1-y_k$ are probabilities that k-type jammer is in mode $\mathbb{R}$ and $\mathbb{C}$, respectively. Since the system does not know which fading gain state of the jammer occurs, the system strategy is $\boldsymbol{x} = (x, 1-x)$, where x and $1-x$ are probabilities that the system is in mode $\mathbb{R}$ and $\mathbb{C}$, respectively.

Then, the expected system's SINR is given by (20) below if the system and jammer, dependently on its type, implement strategies $\boldsymbol{x}$ and $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_K$, respectively:

$$\mathrm{SINR}_S(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K) = \sum_{k \in \mathcal{K}} \alpha_k \boldsymbol{x} \mathcal{M}_k \boldsymbol{y}_k^T. \tag{20}$$

By (12) and (20), payoffs to sophisticated system k-type jammer are given, respectively as follows:

$$v_S(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K) = (1-w) \sum_{k \in \mathcal{K}} \alpha_k \boldsymbol{x} \mathcal{M}_k \boldsymbol{y}_k^T. + wH(\boldsymbol{x}), \tag{21}$$

$$v_{J,k}(\boldsymbol{x}, \boldsymbol{y}_k) = -\boldsymbol{x} \mathcal{M}_k \boldsymbol{y}_k^T, \quad k \in \mathcal{K}. \tag{22}$$

We look for a Nash equilibrium. i.e., for such feasible strategies $(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K)$ that any feasible strategies $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}}_1, \ldots, \tilde{\boldsymbol{y}}_K)$, the following inequalities hold:

$$v_S(\tilde{\boldsymbol{x}}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K) \leq v_S(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K), \tag{23}$$

$$v_{J,k}(\boldsymbol{x}, \tilde{\boldsymbol{y}}_k) \leq v_{J,k}(\boldsymbol{x}, \boldsymbol{y}_k) \text{ for } k \in \mathcal{K}. \tag{24}$$

Denote this non-zero sum (Bayesian) game by Γ_K. Note that in game Γ_K a jammer's type has to be associated per a set of channel gains to reflect that the jammer has complete information about channel gains in contrast to the system, which has access to statistical information about them. Of course, in case if both players have access only to statistical information about channel gains there is no need to introduce a such way associated players' types (see, for example, [2]). Here, as examples of using Bayesian game approach in different communication systems we also refer to [10,13,15].

Without loss of generality throughout the rest part of the paper we will label the system's strategy $\boldsymbol{x} = (x, 1-x)$ by its first component x. Similarly, we will label the k-type jammer's strategy $\boldsymbol{y}_k = (y_k, 1-y_k)$ by its first component y_k.

Proposition 1. *In game Γ_K there exists at least one equilibrium.*

The proof can be found in Appendix A.1.

6 Best Response Strategies

In this section we derive in closed form the best response strategies for the system as well as all jammer types.

Proposition 2. *(a) For fixed $y_k \in [0,1], k \in \mathcal{K}$, the best response of system is given as follows:*

$$BR_S(y_1,\ldots,y_K) \triangleq \begin{cases} \dfrac{1}{1+\exp\left(\left(\sum_{k=1}^{K}\alpha_k D_k y_k - \Theta\right)\delta_w\right)}, & w>0, \\ \begin{cases} 1, & \sum_{k=1}^{K}\alpha_k D_k y_k < \Theta, \\ \in [0,1], & \sum_{k=1}^{K}\alpha_k D_k y_k = \Theta, \\ 0, & \sum_{k=1}^{K}\alpha_k D_k y_k > \Theta, \end{cases} & w=0, \end{cases} \tag{25}$$

where

$$\Theta \triangleq \sum_{k=1}^{K}(a_k - B_k)\alpha_k, \tag{26}$$

$$\delta_w \triangleq \frac{1-w}{w}, \tag{27}$$

$$D_k \triangleq a_k + b_k - A_k - B_k. \tag{28}$$

(b) For a fixed $x \in [0,1]$, the best response $y_k = BR_J(x)$ of the k-type jammer is given as follows:

$$BR_{J,k}(x) \triangleq \begin{cases} 0, & x < X_{0,k}, \\ \in [0,1], & x = X_{0,k}, \\ 1, & x > X_{0,k}, \end{cases} \tag{29}$$

where

$$X_{0,k} \triangleq \frac{b_k - B_k}{D_k}. \tag{30}$$

The proof can be found in Appendix A.2

Remark 1. *Note that, by (19), we have that*

$$D_k > 0, 1 > X_{0,k} > 0 \text{ and } \delta_w > 0. \tag{31}$$

7 Equilibrium

In this section we derive the equilibrium strategies in closed form and prove their uniqueness. To avoid bulkiness in formulas we assume that $X_{0,k} \neq X_{0,\tilde{k}}$ for $k \neq \tilde{k}$. Then, by (31), without loss of generality we can assume that the channel gain states are arranged in an increasing order by $X_{0,k}$, i.e.,

$$X_{0,1} \triangleq 0 < X_{0,1} < X_{0,2} < \ldots < X_{0,K}. \tag{32}$$

Let

$$\varphi_0 \triangleq 0 \text{ and } \varphi_k \triangleq \sum_{i=1}^{k} \alpha_i D_i \text{ for } k \in \mathcal{K} \tag{33}$$

By Remark 1, φ_k is strictly increasing on k.

Theorem 1. *Let $0 < w < 1$. In game Γ_K the equilibrium is an unique. Moreover, this unique equilibrium $(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K)$ is given as follows:*

(a) if

$$\varphi_K - \Theta \leq \ln(1/X_{0,K} - 1)/\delta_w \tag{34}$$

then

$$y_k = 1 \text{ for } k \in \mathcal{K} \tag{35}$$

and

$$x = \frac{1}{1 + \exp\left((\varphi_K - \Theta)\,\delta_w\right)}, \tag{36}$$

(b) if

$$\ln(1/X_{0,1} - 1)/\delta_w \leq \varphi_0 - \Theta \tag{37}$$

then

$$y_k = 0 \text{ for } k \in \mathcal{K} \tag{38}$$

and

$$x = \frac{1}{1 + \exp\left(-\Theta\delta_w\right)}, \tag{39}$$

(c) if

$$\varphi_0 - \Theta < \ln(1/X_{0,1} - 1)/\delta_w \text{ and } \ln(1/X_{0,K} - 1)/\delta_w < \varphi_K - \Theta \tag{40}$$

then there exists an unique k_* such that either (41) or (45) hold which are given below, and

(c-i) if

$$\varphi_{k_*-1} - \Theta < \ln(1/X_{k_*,0} - 1)/\delta_w \leq \varphi_{k_*} - \Theta \text{ with } 2 \leq k_* \leq K \tag{41}$$

then

$$y_i = \begin{cases} 1, & i < k_*, \\ \mathcal{Y}, & i = k_*, \\ 0, & i > k_* \end{cases} \tag{42}$$

and

$$x = X_{0,k_*} \tag{43}$$

where

$$\mathcal{Y} = \frac{\Theta + \ln(1/X_{k_*,0} - 1)/\delta_w - \varphi_{k_*-1}}{\alpha_{k_*} D_{k_*}}, \tag{44}$$

(c-ii) if

$$\ln(1/X_{k_*+1,0} - 1)/\delta_w \le \varphi_{k_*} - \Theta < \ln(1/X_{k_*,0} - 1)/\delta_w \text{ with } 1 \le k_* \le K-1 \tag{45}$$

then

$$y_i = \begin{cases} 1, & i \le k_*, \\ 0, & i > k_* \end{cases} \tag{46}$$

and

$$x = \frac{1}{1 + \exp\left((\varphi_{k_*} - \Theta)\,\delta_w\right)}. \tag{47}$$

The proof can be found in Appendix A.3.

Note that, by (33), $(\varphi_k - \Theta)\delta_w$ is increasing on k. Meanwhile, by (32), $\ln(1/X_{0,k} - 1)$ is decreasing on k. Thus, only one of three conditions (34), (37) or (40) can hold. Also, note that for the boundary case $w = 1$, i.e., if the system is focused on confusing the jammer its equilibrium strategy is $x = 1/2$, where the maximum of entropy is achieved.

8 Discussion of the Results

Let us illustrate the obtained results by an example of the system with $n = 4$ sub-carriers, background noises $\sigma^2 = \sigma_E^2 = 1$, main channel gains $h = (1, 2, 3, 4)$, $h_E = (2, 1, 1, 3)$ and let the total system and jammer power budgets are $\overline{P} = 1$ and $\overline{J} = 5$, respectively. Let the jamming channel gains from the jammer to the communication receiver and radar receiver be $g = (1/d, 1.5/d, 2/d, 0.5/d)$ and $g_E = (2/d, 0.5/d, 1.5/d, 1/d)$, respectively with d being the distance to the receiver from the jammer, where $d \in \{1, 2, 4\}$. Thus, the jamming channel gains can be in $K = 3$ states associated per such a distance. Let $\alpha = (p, (1-p)/2, (1-p)/2)$ with $p \in [0, 1]$ be probabilities that the corresponding channel states occur. Thus, the jamming channel gains might be in state 1 with probability p, meanwhile the rest two states might occur equally probable.

For state 1 we can find power allocation strategies via [1] for the communication and radar modes $\boldsymbol{P}_{\mathbb{C},1} = (0, 0.22, 0.33, 0.44)$, $\boldsymbol{J}_{\mathbb{C},1} = (0, 0.44, 1.66, 2.888)$, and $\boldsymbol{P}_{\mathbb{R},1} = (0.285, 0.142, 0.142, 0.428)$, $\boldsymbol{J}_{\mathbb{R},1} = (1.571, 0.285, 0.285, 2.857)$,

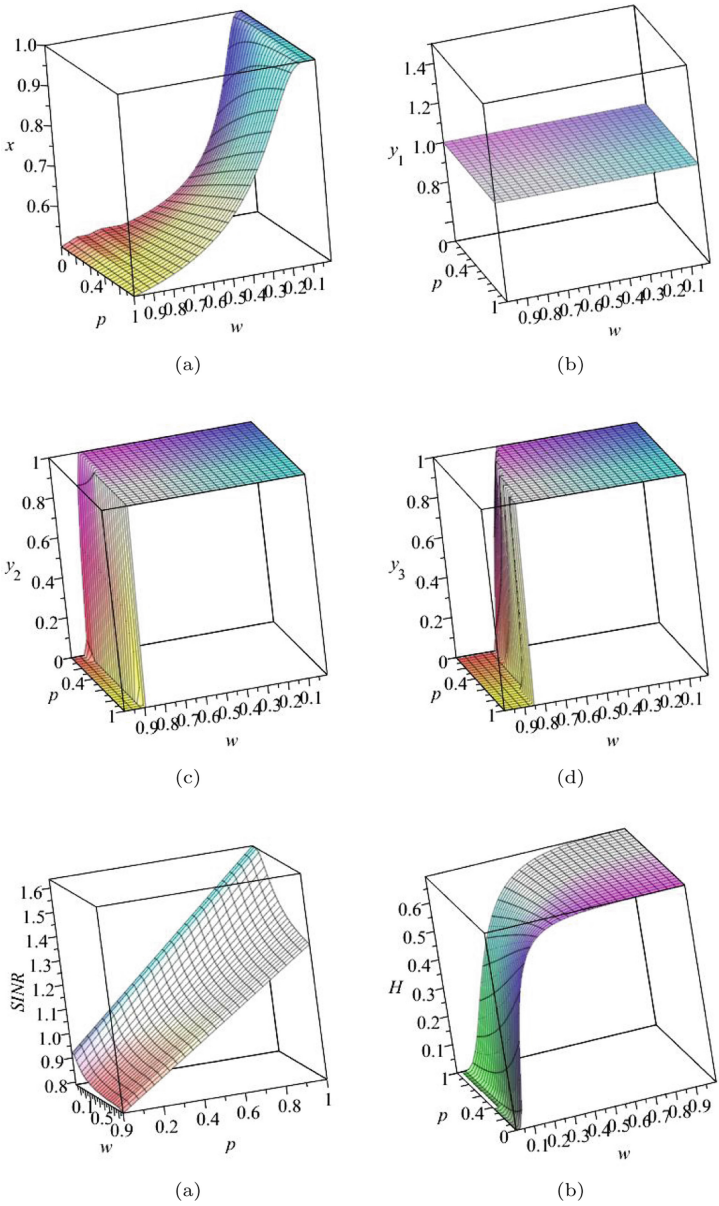

Fig. 3. (a) System strategy x, (b) type-1 jammer strategy y_1, (c) type-2 jammer strategy y_2, (d) type-3 jammer strategy y_3, (e) expected system SINR and (d) entropy $H(x)$ as functions on weighting coefficient w and probability p.

respectively. This leads $A_1 = 1.63$, $a_1 = 1.99$, $B_1 = 0.77$ and $b_1 = 1.05$ as entries of the payoff matrix $\mathcal{M}_1$.

For state 2 we can find power allocation strategies in the communication and radar modes $\boldsymbol{P}_{\mathbb{C},2} = (0, 0.22, 0.33, 0.44)$, $\boldsymbol{J}_{\mathbb{C},2} = (0, 0.44, 1.66, 2.888)$ and $\boldsymbol{P}_{\mathbb{R},2} = (0.285, 0.142, 0.142, 0.428)$, $\boldsymbol{J}_{\mathbb{R},2} = (1.499, 0.499, 0.499, 2.499)$, respectively. This leads to $A_2 = 1.125$, $a_2 = 1.470$, $B_2 = 0.500$ and $b_2 = 0.870$ as entries of the payoff matrix $\mathcal{M}_2$.

Finally, for state 3 we can find power allocation strategies in communication and radar modes $\boldsymbol{P}_{\mathbb{C},3} = (0.10, 0.20, 0.30, 0.40)$, $\boldsymbol{J}_{\mathbb{C},3} = (0.20, 0.90, 1.60, 2.30)$ and and $\boldsymbol{P}_{\mathbb{R},1} = (0.285, 0.142, 0.142, 0.428)$, $\boldsymbol{J}_{\mathbb{R},1} = (1.464, 0.607, 0.607, 2.321)$, respectively. This leads $A_3 = 0.714$, $a_3 = 0.896$, $B_3 = 0.291$ and $b_3 = 0.493$ to entries of the payoff matrix $\mathcal{M}_3$.

Figure 3 illustrates that the system equilibrium strategy x is continuous with respect to the probability p that state 1 occurs and monotonic decreasing with respect to the weighting coefficient w and tends to $x = 1/2$ for w tending to one, which returns the maximum for entropy, i,e., maximum of unpredictability for the jammer and the minimum for the expected system's SINR. The jammer's type strategies implement boundary values (0 or 1) except maybe the only jammer's type. This reflects the advantage that the jammer has since it has complete information about its channel states in contrast to the system, which has only statistical information. Specifically, this disadvantage makes the system to be more flexible in tuning its strategy in contrast to the jammer since such a task to tune strategy arises for the jammer in the only state.

9 Conclusions

In this paper, in framework of Bayesian game approach, we have modeled a dual purpose communication-radar system that besides the basic objective consisting of two tasks (a) to communicate with a receiver and (b) to track a radar target through the reflections witnessed at the system, also has the secondary objective to achieve the basic objective in a manner that is as unpredictable as possible to the jammer. The entropy associated with the system's strategy to switch between its two tasks of the basic objective has been considered as the metric that reflects the unpredictability of its strategy for the jammer. The uniqueness of the equilibrium is proven, and this reflects the stability of the designed anti-jamming protocol even in the case where the system has less information about the environment than the jammer, as is reflected by knowledge of the underlying jamming fading gains.

A Appendix

A.1 Proof of Proposition 1

By (21), we have that

$$\frac{\partial v_S^2(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K)}{\partial x^2} = -\frac{w}{x(1-x)} < 0. \tag{48}$$

Thus, $v_S(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K)$ is concave in $\boldsymbol{x}$. By (22), we have that $v_{J,k}(\boldsymbol{x}, \boldsymbol{y}_k)$ is linear on $\boldsymbol{y}_k$, and the result follows from Nash theorem. □

A.2 Proof of Proposition 2

By (21), we have that

$$\frac{\partial v_S(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K)}{\partial x} = (1-w)\left(\sum_{k=1}^{K} \alpha_k (a_k - B_k) - \sum_{k=1}^{K} \alpha_k D_k y_k\right) + w \ln\left(\frac{1-x}{x}\right) \tag{49}$$

with D_k given by (28). Thus, for a fixed $w \in (0,1)$ function $\partial v_S(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K)/\partial x$ is decreasing on x from infinity for $x \downarrow 0$ to negative infinity for $x \uparrow 1$. Thus, for a fixed $y \in [0,1]$, the best response x is given as the unique root of the following equation:

$$(1-w)(\Theta - \sum_{k=1}^{K} \alpha_k D_k y_k) + w \ln\left(\frac{1-x}{x}\right) = 0. \tag{50}$$

Solving this equation by x implies the first row of (25).

For $w = 0$, by (49), we have that

$$v_S(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K) = (\Theta - \sum_{k=1}^{K} \alpha_k D_k y_k)x + \sum_{k=1}^{K} ((b_k - B_k)y_k + B_k). \tag{51}$$

Thus, $v_S(\boldsymbol{x}, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K)$ is linear in x, and this implies that for a fixed $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_K$ the best response x is given by the second row of (25)

By (22), we have that

$$v_{J,k}(\boldsymbol{x}, \boldsymbol{y}_k) = (B_k - b_k + D_k x)y_k + (B_k - a_k)x - B_k. \tag{52}$$

Thus, $v_{J,k}(\boldsymbol{x}, \boldsymbol{y}_k)$ is linear in y_k, and this implies that for a fixed x the best response y_k is given by (29). □

A.3 Proof of Theorem 1

Let $(x, \boldsymbol{y}_1, \ldots, \boldsymbol{y}_K)$ be an equilibrium. Then, by (29) and (32), we have that there is a t such that

$$y_i = \begin{cases} 1, & i < t, \\ \in [0,1], & i = t, \\ 0, & i > t. \end{cases} \tag{53}$$

Let us consider separately three cases for $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_K$: (a) (35) holds, (b) (38) holds and (c) neither (35) nor (38) hold.

Let (35) hold. This corresponds $t = K+1$ in (53). Substituting (35) into (25) implies (36). Then, substituting (36) into (29) implies (34), and (a) follows.

Let (38) hold. This corresponds $t = 0$ in (53). Substituting (38) into (25) implies (39). Then, substituting (36) into (29) implies (37), and (b) follows.

(c) Let neither (35) nor (38) hold. Then, by (a) and (b), (40) hold, and, by (29) and (53), two cases arise to consider: (i) $0 < y_t < 1$ and (ii) $y_t = 1$

(i) Let $0 < y_t < 1$. Then, by (29)) and (53), we have that

$$x = X_{0,t}. \tag{54}$$

Substituting (53) into (25), and, then, such obtained x substituting into (54) implies the following equation for y_t:

$$\frac{1}{1+\exp\left(\left(\sum_{k=1}^{t-1}\alpha_k D_k + \alpha_t D_t y_t - \Theta\right)\delta_w\right)} = X_{t,0}, \tag{55}$$

which, by (33), is equivalent to

$$\varphi_{t-1} + \alpha_t D_t y_t - \Theta = \ln(1/X_{t,0} - 1)/\delta_w. \tag{56}$$

Note that for a fixed t the left side of this equation is increasing on y_t Thus,(33), this equation has the root y_t in $[0, 1)$ if and only if the following relation holds

$$(\varphi_{t-1} - \Theta)\delta_w \le \ln(1/X_{t,0} - 1) < (\varphi_t - \Theta)\delta_w. \tag{57}$$

By (33), $(\varphi_t - \Theta)\delta_w$ is increasing on t. Meanwhile, by (32), $\ln(1/X_{0,t} - 1)$ is decreasing on t. Then, since (40) holds, inequalities (57) has the unique solution which we denote by k_*. Finally, solving linear equation (56) on y_t with $t = k_*$ implies (44), and (c-i) follows.

(ii) Let $y_t = 1$. Substituting (53) with such y_t into (25) implies that

$$x = \frac{1}{1+\exp\left(\left(\varphi_t - \Theta\right)\delta_w\right)}. \tag{58}$$

Then, by (25) and (53) with $y_t = 1$, we have that

$$X_{0,t} \le \frac{1}{1+\exp\left(\left(\varphi_t - \Theta\right)\delta_w\right)} < X_{0,t+1}. \tag{59}$$

This inequality is equivalent to

$$\ln(1/X_{t+1,0} - 1) < (\varphi_t - \Theta)\delta_w \le \ln(1/X_{t,0} - 1). \tag{60}$$

This implies (45), and (c-ii) follows. Finally, existence of the unique k_* given by (41) and (45) follow from the fact that $\ln(1/X_{t,0}-1)/\delta_w$ is decreasing, meanwhile $\varphi_t - \Theta$ is increasing on t. □

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