Limits of Harmonic Power Recovery by Power Quality Conditioners in Three-Phase Three-Wire Systems Under Non-sinusoidal Conditions

Rodrigo Guzman Iturra and Peter Thiemann(✉)

South Westphalia University of Applied Sciences,
Lübecker Ring 2, 59494 Soest, Germany
{guzmaniturra.rodrigozenon,thiemann.peter}@fh-swf.de
https://www4.fh-swf.de/de/home/

Abstract. Power quality conditioners were originally meant to provide protection to electrical loads connected to the power system against power quality disturbances. Such protection feature brings mostly intangible benefits that are difficult to quantify in monetary terms. This is the reason why some providers of power quality solutions focus more on promote energy savings benefits rather than emphasize the protection feature at the time of advertising their products. Sometimes, the energy savings are exaggerated and inflated with the aim to present a more convincing argument to the customers about why they should acquire a particular solution. This technical paper present presents two formulas that determine the theoretical maximum energy savings that can be achieved when a power quality conditioner targets current harmonics within an industrial facility. In particular, the formulas predict the maximum amount of harmonic active power that can be recovered by power quality conditioners (e.g. harmonic active power filter) in a three-wire three-phase system that contains linear and nonlinear loads. The upper bound of the harmonic active power is the total harmonic apparent power. The upper bound is given in function of the Total Harmonic Distortion of the current and the voltage measured at the point of common coupling and total apparent power of the loads.

Keywords: Active power filter · Energy savings · Harmonic active · Power power quality conditioner

1 Introduction

Due to the huge proliferation of loads of the nonlinear type, nowadays the most common voltage distortions encounter in power systems are related with harmonics, inter-harmonics, notching and noise [1,2]. Power quality solutions as passive harmonic filters and active power filters were originally meant to protect

J. L. Afonso et al. (Eds.): SESC 2020, LNICST 375, pp. 15–29, 2021.
https://doi.org/10.1007/978-3-030-73585-2_2

electrical devices from waveform distortions in the supplied voltage. Economically speaking, due to their protective feature, power quality solutions produce mostly intangible benefits to a particular industrial facility. Those savings are related with the minimization of production down-times and the damage reduction with the consequent lifetime increase of the electrical devices [2]. Indeed, it is very difficult to assign a hard value to such savings and thus they are considered as soft savings. This is the reason why, presently many of the power filter suppliers or representatives choose as sales strategy to promote the energy cost reduction rather than the protection feature [3]. Due to the economical challenges imposed over the current times, it is becoming every single time more and more difficult to get financial resources allocated to maintenance and improvement of existing processes. Therefore, power quality solutions that advertise the return of the investment in some finite amount of time makes a particular power quality conditioner device easier to sell. The former sales strategy offers an easy justification that pursuits to convince controllers or financial officers in charge of the purchase approval to accept the acquisition of a particular power quality conditioner. Consequently, many vendors of harmonic mitigation solutions embellish or inflate the energy savings that can be achieved. Sometimes energy savings on the range of 20% to 30% are claimed, when the true is that the energy savings are much smaller [4].

Many power filters vendors use the kVA instead of kW as method to make claims about significant energy reductions [3]. Equipment that improves or reduce the THD_{U} or THD_{I} can reduce the kVA demanded but have very little effect on the kWs consumed by the facility from the utility point of view. Real energy saving means a reduction on the real kW or kWh that a particular industrial facility demands. The energy required to do actual work (e.g. mechanical, heating, etc.) cannot be eliminated from the electrical costs unless a second energy source is installed locally [4]. Thus, if a device states to save energy on a power system, the only energy that can be saved is the energy wasted in losses through the power system. An important part of such losses are the power line losses. The former arise out due to the power dissipation produced in the line resistance due to the flow of fundamental and harmonics currents from the utility to the customer facility through the feeder and the distribution transformer. In this paper the focus will be solely on the power line losses due to harmonics currents. The power dissipated in the line resistance due to the flow of current harmonics is largely due to the harmonic active power (P_{H}) produced by the nonlinear loads/power converters [12]. The humble goal of this paper is to provide an easy to apply formula to estimate the amount of energy that can be saved if a particular power quality conditioner (e.g. harmonic power filter) processes or transforms the harmonic active power produced by nonlinear loads. In particular, in this paper the formula that determines the upper bound of the harmonic power that can be processed is given in function of the THD_{U}, THD_{I} and the total apparent power S_{e} of the loads measured at the point of common coupling (PCC) of the facility. Moreover, based on some practical experience, a very likely range where the true value of the harmonic power resides is given in function the upper bound.

2 Studied Power System

2.1 Scenario Definition

In order to study the energy savings potential due to the processing of current harmonics, some power definitions and the establishment the electrical quantities that are measured on a particular power system become necessary. Figure 1 shows the sketch of the studied circuit where three-phase symmetrical voltages with harmonic background distortion supply the power system through a distribution transformer [5]. The voltage at the secondary of the transformer is $U_1 = 230\,V$ RMS, 50 Hz with a background distortion at the fifth harmonic U_5 250 Hz and seven harmonic U_7 350 Hz. The impedance of the distribution feeder and transformer referred to the secondary of the transformer is represented by the equivalent components R_S and L_S [2]. The loads are balanced and consist on linear loads with ohmic inductive-behavior represented by the passive electrical elements R_L and L_L (e.g. induction motors) and nonlinear loads (NLV) (e.g. three phase passive diode rectifiers with smoothing DC-Link capacitor). The equivalent circuit can be seen in Fig. 2.

2.2 Voltages and Currents at the Coupling Point

In presence of nonlinear loads in the power system, the voltage at the PCC (U_N) contains a component at the fundamental u_{x1} with fundamental frequency $w_1 = 2 \cdot \pi \cdot f_1$ and harmonic voltages u_{xh} with frequencies at integers multiples of the fundamental frequency (possible also non-integers when interharmonics are considered) leading to harmonic frequencies $w_h = 2 \cdot \pi \cdot h \cdot f_1$. Therefore, the voltage at the PCC referred to the neutral point N can be mathematically written as in (1) to (3) [6]:

$$u_{\mathrm{nu}} = u_{\mathrm{u1}} + u_{\mathrm{uh}} = \sqrt{2} \cdot U_1 \cdot \sin(\omega_1 t + \alpha_1) + \sum_{h=2}^{\infty} \sqrt{2} \cdot U_{\mathrm{h}} \cdot \sin(h\omega_1 t + \alpha_{\mathrm{h}}) \quad (1)$$

$$u_{\mathrm{nv}} = u_{\mathrm{v1}} + u_{\mathrm{vh}} = \sqrt{2} \cdot U_1 \cdot \sin(\omega_1 t + \alpha_1 - \frac{2\pi}{3}) + \sum_{h=2}^{\infty} \sqrt{2} \cdot U_{\mathrm{h}} \cdot \sin(h\omega_1 t + \alpha_{\mathrm{h}} - h\frac{2\pi}{3}) \quad (2)$$

$$u_{\mathrm{nw}} = u_{\mathrm{w1}} + u_{\mathrm{wh}} = \sqrt{2} \cdot U_1 \cdot \sin(\omega_1 t + \alpha_1 + \frac{2\pi}{3}) + \sum_{h=2}^{\infty} \sqrt{2} \cdot U_{\mathrm{h}} \cdot \sin(h\omega_1 t + \alpha_{\mathrm{h}} + h\frac{2\pi}{3}) \quad (3)$$

Where the terms expressed with capital letters U_h denote the RMS voltage quantity of the individual component. The total RMS voltage U_e of each phase

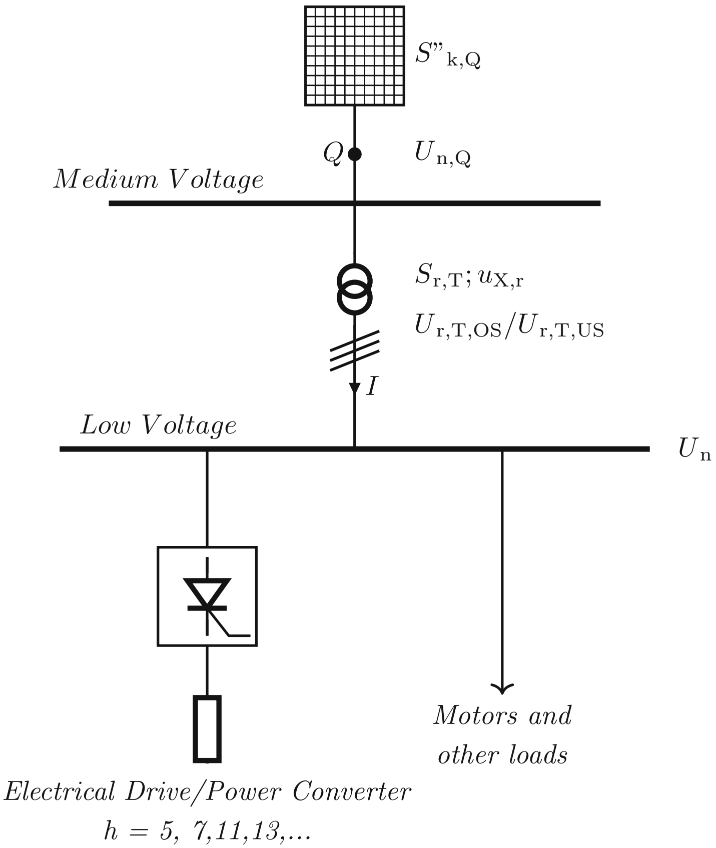

Fig. 1. Industrial power grid with linear and nonlinear loads connected to a utility power system through a power transformer.

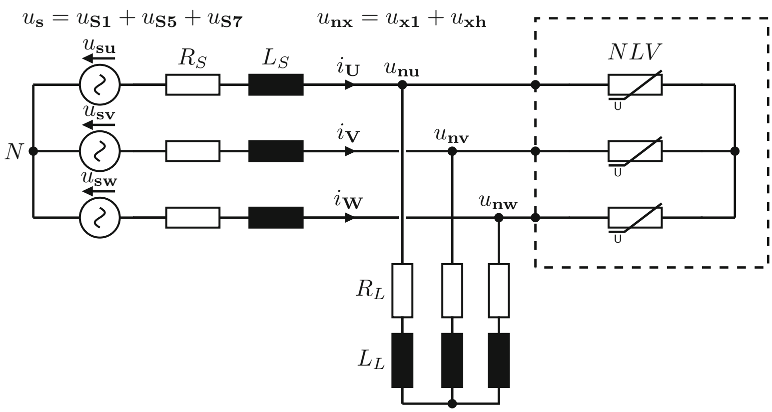

Fig. 2. Equivalent circuit of the power system depicted in Fig. 1.

considering all the components (fundamental and harmonics) was defined by Emanuel in [7] following the approach of Buchholz [8] and can be calculated as:

$$U_{\mathrm{e}} = \sqrt{U_1^2 + U_{\mathrm{H}}^2} = \sqrt{U_1^2 + \sum_{h=2}^{\infty} U_{\mathrm{h}}^2} \tag{4}$$

Where the harmonic voltage contribution is:

$$U_{\mathrm{H}} = \sqrt{\sum_{h=2}^{\infty} U_{\mathrm{h}}^2} \tag{5}$$

The currents draw from the power system will have the same frequency components as the voltages at the PCC, thus the three phase currents can be expressed as in (6) to (8):

$$i_{\mathrm{u}} = i_{\mathrm{u1}} + i_{\mathrm{uh}} = \sqrt{2} \cdot I_1 \cdot \sin(\omega_1 t + \beta_1) + \sum_{h=2}^{\infty} \sqrt{2} \cdot I_{\mathrm{h}} \cdot \sin(h\omega_1 t + \beta_{\mathrm{h}}) \tag{6}$$

$$\begin{aligned} i_{\mathrm{v}} = i_{\mathrm{v1}} + i_{\mathrm{vh}} &= \sqrt{2} \cdot I_1 \cdot \sin(\omega_1 t + \beta_1 - \frac{2\pi}{3}) \\ &+ \sum_{h=2}^{\infty} \sqrt{2} \cdot I_{\mathrm{h}} \cdot \sin(h\omega_1 t + \beta_{\mathrm{h}} - h\frac{2\pi}{3}) \end{aligned} \tag{7}$$

$$\begin{aligned} i_{\mathrm{w}} = i_{\mathrm{w1}} + u_{\mathrm{wh}} &= \sqrt{2} \cdot I_1 \cdot \sin(\omega_1 t + \beta_1 + \frac{2\pi}{3}) \\ &+ \sum_{h=2}^{\infty} \sqrt{2} \cdot I_{\mathrm{h}} \cdot \sin(h\omega_1 t + \beta_{\mathrm{h}} + h\frac{2\pi}{3}) \end{aligned} \tag{8}$$

Where capital letters $\mathrm{I_h}$ denote the current RMS quantity of the individual frequencies. The effective current of each phase considering the fundamental and harmonic frequencies can be calculated with an expression similar to (4) leading to:

$$I_{\mathrm{e}} = \sqrt{I_1^2 + I_{\mathrm{H}}^2} = \sqrt{I_1^2 + \sum_{h=2}^{\infty} I_{\mathrm{h}}^2} \tag{9}$$

$$I_{\mathrm{H}} = \sqrt{\sum_{h=2}^{\infty} I_{\mathrm{h}}^2} \tag{10}$$

The total harmonic voltage and current distortion are defined as [9]:

$$THD_{\mathrm{U}} = \frac{U_{\mathrm{H}}}{U_1} = \sqrt{(\frac{U_{\mathrm{e}}}{U_1})^2 - 1} \tag{11}$$

$$THD_{\mathrm{I}} = \frac{I_{\mathrm{H}}}{I_1} = \sqrt{(\frac{I_{\mathrm{e}}}{I_1})^2 - 1} \tag{12}$$

3 Apparent Power Resolution for Nonsinusoidal Three-Wire Three Phase System

Considering that all the loads are connected to the PCC, any potential harmonic power that could processed and recovered later by and active power filters depends necessarily of the total voltage U_e that supplies the loads and the current I_e that such loads draw from the main power system. Therefore, the starting point for the calculation of the harmonic power limit that can be recovered is the total apparent power of the loads.

3.1 Total Apparent Power

Equations (4) and (9) lead to the total apparent power:

$$S_e = 3 \cdot U_e \cdot I_e \tag{13}$$

The apparent power can be squared and expanded as in [10]:

$$\begin{aligned} S_e^2 &= 9 \cdot U_e^2 \cdot I_e^2 = 9 \cdot (U_1^2 + U_H^2) \cdot (I_1^2 + I_H^2) \\ &= (U_1^2 \cdot I_1^2) + (U1^2 \cdot I_H^2) + (U_H^2 \cdot I_1^2) + (U_H^2 \cdot I_H^2) \\ &= (9 \cdot U_1^2 \cdot I_1^2) + (9 \cdot U_1^2 \cdot I_H^2) + (9 \cdot U_H^2 \cdot I_1^2) + (9 \cdot U_H^2 \cdot I_H^2) \\ &= (9 \cdot S_1^2) + (9 \cdot D_I^2) + (9 \cdot D_U^2) + (9 \cdot S_H^2) \\ &= (3 \cdot S_1)^2 + (3 \cdot D_I)^2 + (3 \cdot D_U)^2 + (3 \cdot S_H)^2 \\ &= (S_{e1})^2 + (D_{eI})^2 + (D_{eU})^2 + (S_{eH})^2 \end{aligned} \tag{14}$$

Equation (14) can be written as:

$$S_e^2 = S_{e1}^2 + S_{eN}^2 \qquad [V^2 \cdot A^2] \tag{15}$$

The first term in (15) is the square of the fundamental (50 Hz) apparent power S_{e1}:

$$S_{e1} = 3 \cdot S_1 = 3 \cdot U_1 \cdot I_1 \tag{16}$$

$$S_{e1} = 3 \cdot \sqrt{(P_1)^2 + (Q_1)^2} \tag{17}$$

$$S_{e1} = 3 \cdot \sqrt{\{U_1 \cdot I_1 \cdot cos(\beta_h - \alpha_h)\}^2 + \{U_1 \cdot I_1 \cdot sin(\beta_h - \alpha_h)\}^2} \qquad [V \cdot A] \tag{18}$$

The second term in (15) is the square of the nonfundamental apparent power S_eN, defined as following [11]:

$$S_{eN} = \sqrt{(D_{eI})^2 + (D_{eU})^2 + (S_{eH})^2} \qquad [V \cdot A] \tag{19}$$

Where:

$$D_{eI} = 3 \cdot D_I = 3 \cdot U_1 \cdot I_H \qquad [V \cdot A] \tag{20}$$

is the total current distortion power. The distortion power results from the product of the voltage at the fundamental and currents at different harmonic frequencies. The expression contains only oscillatory terms with an average value of zero, therefore D_{eI} produces just reactive power and its units is fundamentally Var. Most of the times, the distortion power D_{eI} makes the largest contribution to the magnitude of S_{eN}. Moreover:

$$D_{\mathrm{eU}} = 3 \cdot D_{\mathrm{U}} = 3 \cdot U_{\mathrm{H}} \cdot I_1 \qquad [V \cdot A] \tag{21}$$

is the total voltage distortion power. The voltage distortion power is the product of harmonic voltages at different frequencies and the current at the fundamental frequencys. It follows that D_{eU}, as D_{eI}, produces only reactive power leading to Var as unit. Finally, the smallest of all the terms in (3.5) is the total harmonic apparent power S_{eH}:

$$S_{\mathrm{eH}} = 3 \cdot S_{\mathrm{H}} = 3 \cdot U_{\mathrm{H}} \cdot I_{\mathrm{H}} \qquad [V \cdot A] \tag{22}$$

The harmonic apparent power can be characterized by two important components [11]:

$$S_{\mathrm{eH}}^2 = P_{\mathrm{H}}^2 + D_{\mathrm{eH}}^2 \qquad [V^2 \cdot A^2] \tag{23}$$

We recall that the active power P_{x} is equal to the average value of the instantaneous power $p_{\mathrm{x}} = v_{\mathrm{x}} \cdot i_{\mathrm{x}}$ over a time window for the averaging equal to one grid voltage period. It is granted that if we consider a three phase system, the contribution of each phase to the active power needs to be considered. With the last arguments on mind, the total harmonic active power P_{H} is formally defined as [12]:

$$P_{\mathrm{H}} = \int_0^{\frac{1}{f_1}} \Big\{ p_{\mathrm{uh}} + p_{\mathrm{vh}} + p_{\mathrm{wh}} \Big\} dt \tag{24}$$

$$P_{\mathrm{H}} = \int_0^{\frac{1}{f_1}} \Big\{ (u_{\mathrm{uh}} \cdot i_{\mathrm{uh}}) + (u_{\mathrm{vh}} \cdot i_{\mathrm{vh}}) + (u_{\mathrm{wh}} \cdot i_{\mathrm{wh}}) \Big\} dt \tag{25}$$

$$P_{\mathrm{H}} = 3 \cdot \sum_{h=2}^{\infty} U_{\mathrm{H}} \cdot I_{\mathrm{H}} \cdot cos(\beta_{\mathrm{h}} - \alpha_{\mathrm{h}}) \qquad [W] \tag{26}$$

Which has an average value different from zero due to the product of currents and voltages at the same frequency. Actually, this one is the only one component of the apparent power than can be processed later to recover energy from the harmonics present in the power system. The term D_{eH} is defined as harmonic distortion power and results from the product of harmonic voltages and currents at different frequencies. The consequence is that D_{eH} contains only oscillatory terms and thus produces only reactive power:

$$D_{\mathrm{eH}} = \sqrt{(S_{\mathrm{eH}})^2 - (P_{\mathrm{H}})^2} \qquad [V \cdot A] \tag{27}$$

3.2 Apparent Power Components as Functions of THD

The different components that form the total apparent power (14) can be expressed as functions of the total harmonic voltage and current distortion THD_U and THD_I [12]. This can be achieved by multiplying and dividing each term by the factor S_{e1}. For instance, take the total current distortion power D_{eI} in equation (20):

$$D_{\mathrm{eI}} = 3 \cdot U_1 \cdot I_{\mathrm{H}} = S_{\mathrm{e1}} \cdot \frac{3 \cdot U_1 \cdot I_{\mathrm{H}}}{3 \cdot U_1 \cdot I_1} = S_{\mathrm{e1}} \cdot \frac{I_{\mathrm{H}}}{I_1} = S_{\mathrm{e1}} \cdot THD_{\mathrm{I}} \tag{28}$$

A similar procedure can be carried out with the total voltage distortion power D_{eU} and the harmonic apparent power S_{eH}:

$$D_{\mathrm{eU}} = 3 \cdot U_{\mathrm{H}} \cdot I_1 = S_{\mathrm{e1}} \cdot \frac{3 \cdot U_{\mathrm{H}} \cdot I_1}{3 \cdot U_1 \cdot I_1} = S_{\mathrm{e1}} \cdot \frac{U_{\mathrm{H}}}{U_1} = S_{\mathrm{e1}} \cdot THD_{\mathrm{U}} \tag{29}$$

$$S_{\mathrm{eH}} = 3 \cdot U_{\mathrm{H}} \cdot I_{\mathrm{H}} = S_{\mathrm{e1}} \cdot \frac{3 \cdot U_{\mathrm{H}} \cdot I_{\mathrm{H}}}{3 \cdot U_1 \cdot I_1} = S_{\mathrm{e1}} \cdot \frac{U_{\mathrm{H}}}{U_1} \cdot \frac{I_{\mathrm{H}}}{I_1} = S_{\mathrm{e1}} \cdot THD_{\mathrm{U}} \cdot THD_{\mathrm{I}} \tag{30}$$

Replacing (28), (29) and (30) in (19) leads to:

$$S_{\mathrm{eN}} = S_{\mathrm{e1}} \cdot \sqrt{(THD_{\mathrm{I}})^2 + (THD_{\mathrm{U}})^2 + (THD_{\mathrm{U}} \cdot THD_{\mathrm{I}})^2} \qquad [V \cdot A] \tag{31}$$

Substituting (31) in (15) results in:

$$S_{\mathrm{e}} = S_{\mathrm{e1}} \cdot \sqrt{1 + (THD_{\mathrm{I}})^2 + (THD_{\mathrm{U}})^2 + (THD_{\mathrm{U}} \cdot THD_{\mathrm{I}})^2} \qquad [V \cdot A] \tag{32}$$

4 Determination of the Limits or Bounds for $\mathbf{P_H}$

From the discussion at chapter 3, it was concluded that the only single power component from the nonfundamental apparent power that do not produce reactive power is the harmonic active power P_{H}. Therefore, the limit of the power that can be recovered from harmonics is going to be determined by the magnitude of P_{H}. From (26), it can be seen that this quantity depends on the magnitude of each harmonic voltage, current harmonic and the phase angle difference between them. To the best of our knowledge, it is not possible to express the power P_{H} returned by the nonlinear loads to the power system in terms of THD_{U} and THD_{I} at the point of common coupling directly. However, Eq. (23) relates P_{H} with S_{eH}, here rewritten in a different way:

$$S_{\mathrm{eH}} = \sqrt{P_{\mathrm{H}}^2 + D_{\mathrm{eH}}^2} \qquad [V \cdot A] \tag{33}$$

Equation (30) gives a defined value for S_{eH} in terms of THD_{U} and THD_{I}. The question is how to related the limit or upper bound of P_{H} knowing the value of S_{eH}, issue that is not trivial. Let us to define the following terms:

$$a_1 = P_{\mathrm{H}}^2 \tag{34}$$

$$a_2 = D_{\mathrm{eH}}^2 \tag{35}$$

Thus (33) becomes:

$$S_{\mathrm{eH}} = \sqrt{a_1 + a_2} \tag{36}$$

It is straight forward to demonstrate the following inequality [13]:

$$\sqrt{a_1 + a_2} \leq \sqrt{a_1} + \sqrt{a_2} \tag{37}$$

Replacing (34), (35) and (36) in the inequality (37), the following expression is obtained:

$$S_{\mathrm{eH}} \leq \sqrt{P_{\mathrm{H}}^2} + \sqrt{D_{\mathrm{eH}}^2} \tag{38}$$

Moreover, the following mathematical expression holds [14][15]:

$$\sqrt{a^2} = |a| \tag{39}$$

Using (39), Eq. (38) becomes:

$$S_{\mathrm{eH}} \leq |P_{\mathrm{H}}| + |D_{\mathrm{eH}}| \tag{40}$$

It follows:

$$|P_{\mathrm{H}}| \geq S_{\mathrm{eH}} - |D_{\mathrm{eH}}| \tag{41}$$

The value of S_{eH} is by the definition given in Eq. (22) always a positive number [11]. Therefore the maximum absolute value that P_{H} can reach will necessary be equal to S_{eH} when D_{eH} is equal to zero. The former will be the most optimistic scenario of the amount of harmonic power that can be transformed. Indeed having a network with $D_{\mathrm{eH}} = 0\ Var$ is not a practical case, due to the fact that in general $S_{\mathrm{eH}} > D_{\mathrm{eH}} > P_{\mathrm{H}}$, so we are in the safe side if we claim that the upper bound for P_{H} is the S_{eH} magnitude. Therefore, any potential energy saving that can be achieved by transforming harmonic active power is going to be upper bounded by S_{eH}. If we aim to calculate the energy savings potential in percentage that can be achieved by processing or transforming harmonic active power, the ratio $\frac{S_{\mathrm{eH}}}{S_{\mathrm{e1}}}$ can be used. Equation (30) leads to the following $\frac{S_{\mathrm{eH}}}{S_{\mathrm{e1}}}$ ratio expression:

$$\frac{S_{\mathrm{eH}}}{S_{\mathrm{e1}}} = THD_{\mathrm{U}} \cdot THD_{\mathrm{I}} \tag{42}$$

The former equation implies that the harmonic active power recovery limit related to the fundamental apparent power $S_{\mathrm{e1}} = 3 \cdot S_1$ is going to be bounded by the product of $THD_{\mathrm{U}} \cdot THD_{\mathrm{I}}$. For instance if in a particular industrial facility the THD_{U} is 8% (which is already the maximum limit impose by the standard IEC 61000-2-4 for facilities class 2) and the THD_{I} is 60% (which is a typical THD_{I} in many cases [12]), the energy savings potential in percentage related to the fundamental apparent power of the facility is bounded by:

$$\frac{S_{\mathrm{eH}}}{S_1} = \frac{8}{100} \cdot \frac{60}{100} \cdot 100\% = 4.8\% \tag{43}$$

Most of the times the fundamental apparent power $S_{\mathrm{e1}} = 3 \cdot S_1$ cannot be measured without a power analyzer that separates the different frequencies contained in the measured voltages and currents through Fourier Analysis. Most of the times what is available is the TRUE RMS value of currents and voltages that can be acquired easily with typical multimeters. The TRUE RMS value considers several frequency components in a range 45 Hz to ten's of kHz all at once. Instruments measuring the TRUE RMS are for instance the FLUKE 175 [16] or the power meter SIEMENS SENTRONPAC 3200 [17]. If the former is the case, the electrical quantity that is available is the total apparent power S_{e}. In order to related S_{eH} to the total apparent quantity S_{e}, let us square Eq. (30) to obtain a new equation:

$$S_{\mathrm{eH}}^2 = S_{\mathrm{e1}}^2 \cdot (THD_{\mathrm{U}} \cdot THD_{\mathrm{I}})^2 \tag{44}$$

If (44) is divided by the square of (32), it follows automatically that:

$$\frac{S_{\mathrm{eH}}^2}{S_{\mathrm{e}}^2} = \frac{(THD_{\mathrm{U}} \cdot THD_{\mathrm{I}})^2}{1 + (THD_{\mathrm{I}})^2 + (THD_{\mathrm{U}})^2 + (THD_{\mathrm{U}} \cdot THD_{\mathrm{I}})^2} \tag{45}$$

$$\frac{S_{\mathrm{eH}}}{S_{\mathrm{e}}} = \sqrt{\frac{(THD_{\mathrm{U}} \cdot THD_{\mathrm{I}})^2}{1 + (THD_{\mathrm{I}})^2 + (THD_{\mathrm{U}})^2 + (THD_{\mathrm{U}} \cdot THD_{\mathrm{I}})^2}} \tag{46}$$

If the energy savings potential due to harmonic active power is calculated using (46) with $THD_{\mathrm{U}} = 8\%$ and $THD_{\mathrm{I}} = 60\ \%$, as in Eq. (43), the limit of the harmonic active power that can be recovered related to the total apparent power of the facility is given by:

$$\frac{S_{\mathrm{eH}}}{S_{\mathrm{e}}} = \sqrt{\frac{(0.08 \cdot 0.6)^2}{1 + (0.6)^2 + (0.08)^2 + (0.08 \cdot 0.6)^2}} = 4.1\% \tag{47}$$

In the above examples the standard IEC 61000-2-4 was referenced to set the THD_{U} to 8%, where a total voltage distortion of 8% is the maximum value allowed for an industrial facility. Nonetheless, the standard IEC 61000-2-4 do not define a maximum value for the THD_{I}. The value of THD_{I} varies typically between 5% and 120% [12]. Furthermore, if the industrial facility needs to comply with the standard IEEE 519-1992 which impose more severe restrictions on the THD_{U} and THD_{I} than the IEC 61000-2-4 norm, the harmonic power recovery potential is reduced. The IEEE Standard 519-1992 defines a maximum THD_{U} of 5% for low voltage power systems (<69 kV). The IEEE Standard 519-1992 defines the maximum TDD_{I} (TDD_{I} is equal to THD_{I} when the facility operates almost at 100% load) in function of the ratio of the maximum short circuit current at the PCC I_{SC} and the maximum current load at the fundamental frequency I_L [2]. If the maximum allowed TDD_I= 20% is taken, the limit of the harmonic active power that can be recovered according to (46) is reduced to:

$$\frac{S_{eH}}{S_e} = \sqrt{\frac{(0.05 \cdot 0.2)^2}{1 + (0.2)^2 + (0.05)^2 + (0.05 \cdot 0.2)^2}} = 0.97\% \tag{48}$$

5 Simulation Validation

In order to confirm the theoretical derivations stated in Sects. 3 and 4, simulations are carried out. The simulation is performed in Simulink using elements of the Simscape Electrical Specialized Library. The scenario simulated is similar to the one depicted in Fig. 1 with the parameters stated in Table 1. The behavior of the voltages at PCC and the currents drawn from the power system can be seen in Fig. 3 and Fig. 4 respectively.

Table 1. Simulation parameters

Description	Value
Power System Fundamental	$U_1 = 230\,\text{V},\ 50\,\text{Hz}$
Voltage Background Distortion	$U_5 = 4.6\,\text{V},\ 250\,\text{Hz}$ $U_7 = 2.07\,\text{V},\ 350\,\text{Hz}$
Power Transformer	2 MVA, Imp. 5%, X/R = 20
Linear Load (Resistive-Inductive)	$P_1 = 700\,\text{kW};\ Q_1 = 100\,\text{kVAR}$
Nonlinear Load	700 kW, Passive Diode Bridge + Smoothing Capacitor

The components up to the 13th harmonic of the measured voltages u_{xn} at the coupling point and the currents i_x drawn from the power systems, with x $x \in u, v, w$ can be seen in Tables 2 and 3 respectively. The effective value of the voltage and current U_E and I_E are also written in the tables. Based on the results of Tables 2 and 3 and Eqs. (13) to (32), it is possible to calculate each of the components of the apparent power under nonsinusoidal conditions leading to the results in Table 4.

The real absolute harmonic active power that can be recovered or transformed is $P_H = 3712W$ measured in the simulation. Let us test the boundaries determined by the Eq. (42). Taking the $THD_U = 7.92\%$ and $THD_I = 26.73\%$ from Tables 2 and 3, the equation predicts the upper bound of the amount of harmonic active power that can be recovered as:

$$\frac{S_{eH}}{S_{e1}} = \frac{7.92}{100} \cdot \frac{26.93}{100} \cdot 100\% = 2.13\% \tag{49}$$

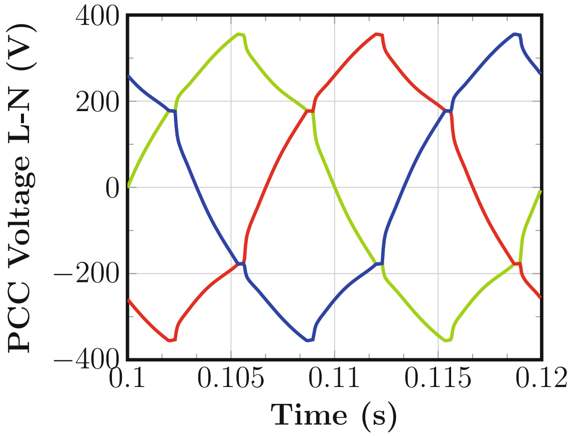

Fig. 3. Voltages at PCC u_{nu}, u_{nv}, u_{nw}.

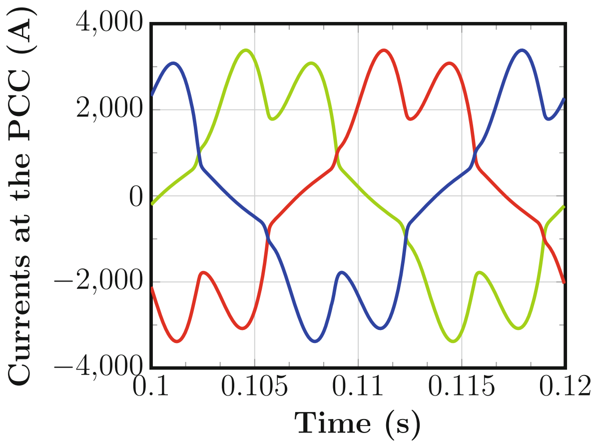

Fig. 4. Currents drawn from the power system i_{u}, i_{v}, i_{w}.

Table 2. Measured voltages [V] (Magnitudes in RMS)

h	u	v	w
1	227.6 ∠ 177.7°	227.6 ∠ 57.7°	227.6 ∠ −62.3°
5	14.82 ∠ 191.8°	14.82 ∠ −48.2°	14.82 ∠ 71.8°
7	7.48 ∠ −73.5°	7.48 ∠ 166.5°	7.48 ∠ 46.5°
11	3.8 ∠ −33.8°	3.8 ∠ 86.2°	3.8 ∠ 206.2°
13	2.84 ∠ 41.7°	2.84 ∠ −78.3°	2.84 ∠ 161.7°
U_{e}	228.3	228.3	228.32
THD_{U}	7.92%	7.92%	7.92%

Table 3. Measured currents [A] (Magnitudes in RMS)

h	u	v	w
1	2002 ∠ 167.3°	2002 ∠ 47.3°	2002 ∠ -72.7°
5	481.54 ∠ 268.1°	481.54 ∠ −28.1°	481.54 ∠ 148.1°
7	211.34 ∠ -33°	211.34 ∠ −87°	211.34 ∠ 153°
11	72.11 ∠ -56.5°	72.11 ∠ 176.5°	72.11 ∠ −63.5°
13	45.6 ∠ 131.9°	45.6 ∠ −11.9°	45.6 ∠ -63.5°
I_{e}	2072	2072	2072
THD_{I}	26.73%	26.73%	26.73%

Table 4. Components of the apparent power calculated based on Tables 2 and 3

Quantity	Unit	Value
S_{e}	VA	1419100
S_1	VA	1367000
S_{eN}	VA	381160
$P_{\mathrm{e1}} = 3 \cdot P_1$	W	1344500
$Q_{\mathrm{e1}} = 3 \cdot Q_1$	Var	246760
D_{eI}	Var	365390
D_{eV}	Var	108260
S_{H}	VA	28939
P_{H}	W	3712
D_{eH}	Var	28700

If S_{eH} and S_{e1} are read directly from the simulation results in Table 4, the former ratio can be directly calculated as:

$$\frac{S_{\mathrm{eH}}}{S_{\mathrm{e1}}} = \frac{28939VA}{1367000VA} \cdot 100\% = 2.12\% \tag{50}$$

It is possible to observe the very good agreement between the results of (49) and (50). Moreover, Eq. (49) suggest that the upper bound (O) of harmonic active power that can be recovered ($D_{\mathrm{eH}} = \ 0\ Var$) is:

$$]O(P_{\mathrm{H}}) = S_{\mathrm{e1}} \cdot cos(0) \cdot THD_{\mathrm{U}} \cdot THD_{\mathrm{I}} = 28939\,\mathrm{W} \tag{51}$$

The bound predicted by Eq. (51) is clearly above the real $P_{\mathrm{H}} = 3712W$ measured in simulation proving that Eq. (42) is correct. It is possible to infer that the real amount of harmonic active power that can be recovered will be in the range $0 < P_{\mathrm{H}} < O(P_{\mathrm{H}})$. Probably a most realistic range for the amount of harmonic active power that can be recovered is between $0.1 \cdot O(P_{\mathrm{H}}) < P_{\mathrm{H}} < 0.5 \cdot O(P_{\mathrm{H}})$ due to the fact that in most cases D_{eH} is different from 0. On the other hand, Eq. (46) based on the total apparent power leads to the same results:

$$\frac{S_{\mathrm{eH}}}{S_{\mathrm{e}}} = \sqrt{\frac{(0.0792 \cdot 0.26)^2}{1 + (0.26)^2 + (0.0792)^2 + (0.0792 \cdot 0.26)^2}} = 2.03\% \tag{52}$$

From the simulation results in Table 4, it is possible to calculate:

$$\frac{S_{\mathrm{eH}}}{S_{\mathrm{e}}} = \frac{28939VA}{1419100VA} \cdot 100\% = 2.04\% \tag{53}$$

Equation (46) agrees very well with the simulation results as is demonstrated by the same output of equations (52) and (53). The amount of harmonic active power that can be recovered related to the total apparent power is:

$$O(P_{\mathrm{H}}) = S_{\mathrm{e}} \cdot cos(0) \cdot \sqrt{\frac{(THD_{\mathrm{U}} \cdot THD_{\mathrm{I}})^2}{1 + (THD_{\mathrm{I}})^2 + (THD_{\mathrm{U}})^2 + (THD_{\mathrm{U}} \cdot THD_{\mathrm{I}})^2}} \tag{54}$$

$$O(P_{\mathrm{H}}) = \; 1419100\,\mathrm{VA} \cdot cos(0) \cdot 2.03\% = 28935\,\mathrm{W} \tag{55}$$

Leading to the same result as Eq. (51) and proving the correctness of (46).

6 Conclusions

This paper has shown the derivation of two formulas (42) and (46) that determine the maximum amount of harmonic active power (P_{H}) that can be recovered in a three-wire three-phase system that contains linear and nonlinear loads. The harmonic active power recovery could be performed by a power conditioner device such as harmonic active power filter. The upper bound (O) of the harmonic active power $O(P_{\mathrm{H}})$ is S_{eH}. The upper bound is given in function of the Total Harmonic Distortion of the currents (THD_{I}) and the voltage (THD_{U}) measured at the point of common coupling (PCC), the fundamental apparent power S_{e1} and total apparent power S_{e} of the loads. Probably the realistic range of the real amount of P_{H} that can be recovered is $0.1 \cdot O(P_{\mathrm{H}}) < P_{\mathrm{H}} < 0.5 \cdot O(P_{\mathrm{H}})$. For example, if in a particular industrial facility, the THD_{U} is 8% (which is already at the maximum limit imposed by the standard IEC 61000-2-4 for facilities class 2) and the THD_{I} is 60% which is typical for many cases, the maximum saving potential in percentage by processing harmonic active power will be bounded by 4.10%. Where it is in highly probable that most realistic value will be between the 0.4% and 2.05%.

References

1. Abu-Rub, H., Malinowski, M., Al-Haddad, K.: Power Electronics for Renewable Energy Systems, Transportation and Industrial Applications, pp. 534–535. Wiley, Chichester (2014)
2. Singh, B., Chandra, A., Al-Haddad, K.: Power Quality: Problems and Mitigation Techniques, pp. 356–358. Wiley, Hoboken (2014)

3. Carnovale, D.J., Barchowsky, A., Groden, B.: Energy savings-realistic expectations for commercial facilities (2015). https://www.eaton.com/FR/FTC/buildings/KnowledgeCenter/WhitePaper2/index.htm. Accessed 15 Oct 2020
4. Carnovale, D.J., Hronek, T.: Power quality solutions and energy savings–what is real? Energy Eng. **106**, 26–50 (2009). https://doi.org/10.1080/01998590909509179. Fluke 175 True-RMS Digital Multimeter—Fluke. https://www.fluke.com/en-us/product/electrical-testing/digital-multimeters/fluke-175. Accessed 17 Sept 2020
5. Schlabbach J., Mombauer W.: Power Quality: Entstehung und Bewertung von Netzrückwirkungen, Netzanschluss erneuerbarer Energiequellen, Theorie, Normung und Anwendung von DIN EN 61000-3-2 (VDE 0838-2), DIN EN 61000-3-12 (VDE 0838-12), DIN EN 61000-3-3 (VDE 0838-3), DIN EN 61000-3-11 (VDE 0838-11), DIN EN 61000-2-2 (VDE 0839-2-2), DIN EN 61000-2-4 (VDE 0839-2-4), DIN EN 61000-4-7 (VDE 0847-4-7), DIN EN 61000-4-15 (VDE 0847-4-15), DIN EN 50160, DIN EN 61000-4-30 (VDE 0847-4-30), VDN-Technische Regeln zur Beurteilung von Netzrückwirkung: VDE, pp. 247–248 (2008)
6. Santoso, S.: Fundamentals of Electric Power Quality, 2009th edn., pp. 207–208. Published by Surya Santoso Through CreateSpace, Austin (2009)
7. Emanuel, A.E.: Summary of IEEE standard 1459: definitions for the measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions. IEEE Trans. Ind. Appl. **40**(3), 869–876 (2004). https://doi.org/10.1109/TIA.2004.827452
8. Buchholz, F.: Die drehstrom-scheinleistung bei ungleichmassiger belastung der drei zweige. Licht und Kraft **2**, 9–11 (1922)
9. IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions, pp. 8–9 (2010)
10. Mohamadian, S., Shoulaie, A.: Comprehensive definitions for evaluating harmonic distortion and unbalanced conditions in three- and four-wire three-phase systems based on IEEE standard 1459. IEEE Trans. Power Deliv. **26**(3), 1774–1782 (2011). https://doi.org/10.1109/TPWRD.2011.2126609
11. Schulz, D.: Netzrückwirkungen-Theorie, Simulation, Messung und Bewertung. In: VDE-Schriftenreihe-Normen verständlich, vol. 115 (2004)
12. Emanuel, A.E.: Power Definitions and the Physical Mechanism of Power Flow, pp. 110–189. Wiley, Hoboken (2011)
13. Mathematics Stack Exchange, real analysis - square root of a sum? Bound? - Mathematics Stack Exchange. https://math.stackexchange.com/questions/318649/square-root-of-a-sum-bound/444640. Accessed 17 Sept 2020
14. Stewart, J.: Calculus: Concepts and Contexts. Cengage, Boston (2019)
15. Polianin, A.D., Manzhirov, A.V.: Handbook of Mathematics for Engineers and Scientists. Chapman & Hall/CRC, Boca Raton (2007)
16. Fluke 175 True-RMS Digital Multimeter—Fluke. https://www.fluke.com/en-us/product/electrical-testing/digital-multimeters/fluke-175. Accessed 17 Sept 2020
17. 7KM2112-0BA00-3AA0 - Industry Support Siemens. https://support.industry.siemens.com/cs/pd/107189?pdti=td&dl=en&lc=en-CN. Accessed 17 Sept 2020