




# Design of Current Controllers for Three Phase Voltage PWM Converters for Different Modulation Methods

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**Abstract.** Grid Tie Three Phase Voltage PWM converters can be conceived as current sources that inject currents into the grid at the point of common coupling (PCC). In order to achieve a good performance, the voltage source inverter (VSI) should be commanded by a current controller to track as accurate as possible a current reference. Pulse width modulation (PWM) and space vector modulation (SVM) are two techniques used in VSIs to generate the time average output voltage demanded by the current controller. By using the average model of the VSI, controlled either by PWM or by SVM, we can use feedback linear control for the analysis and design of the current controller. The main goal of this technical paper is to illustrate an analytical formula to calculate the gains of the proportional-integral current controller. The gains are calculated based on the values of the coupling inductivity, the DC Bus voltage and the switching frequency and given for two PWM modulation methods.

**Keywords:** Current controller · PWM modulation · Space vector modulation · Voltage source inverter

## 1 Introduction

There is large variety of applications that rely on DC/AC converters such as ac motor drives, STATCOMs, active power filters, uninterruptible power supplies and photovoltaic systems [1, 2]. The preferred topology for the DC/AC converters are voltage source inverters (VSIs). Even due converters based on the current source inverter topology (CSI) exist, the switching characteristics of IGBTs and power MOSFETs favor the VSI topology in terms of efficiency in comparison with CSI and made the VSI the predominant topology for DC/AC converters [3]. In many of former mentioned applications, the three phase currents that are generated by the DC/AC converter follow a reference provided by an outer loop. The current references are generated by speed controllers in the case of ac motor drives, a reactive power controller or grid voltage regulator in the

case of STATCOMs and in case of photovoltaic systems they follow a reference generated based on the amount of active power that needs to be injected into the grid. Most of the DC/AC converters are based on a voltage source inverter (VSI) and in order to achieve a good performance, the VSI is commanded by a current controller to track as accurate as possible the three phase currents reference. Pulse with modulation (PWM) and space vector modulation (SVM) are two techniques used in VSIs to generate the time average output voltage demanded by the current controller. By using the average model of the VSI, controlled either by PWM or by SVM, we can use feedback linear control for the analysis and design of the current controller. The main goal of this document is to illustrate an analytical formula to calculate the gain for a proportional-integral current controller based on the values of the coupling inductivity, the DC Bus voltage and the switching frequency of a three phase PWM converter.

This paper is based on the work done by Holmes et al. in [4], however the former paper dealt just with the current controller design for two level converter with conventional PWM as modulation method. The small contribution of this paper is to extend the calculation of the current controller gains for two level converters with SVM modulation and for three level converters with PWM (e.g. Neutral Point Clamped NPC, T Type NPC and others). Moreover, this paper shows a more detailed mathematical derivation of the formulas step by step in contrast with the original on [4]. This paper is organized as follows: Sect. 2 describes the model of the three phase voltage PWM converter connected to the grid. Section 3 presents the derivation of the formula for the calculation of the proportional gain based on the parameters of the three phase voltage PWM converter. Section 4 presents the experimental results obtained by setting up a prototype govern by a digital control system with the current controller proposed in this paper. Finally in Sect. 5, the conclusions are presented.

## 2 System Modeling

Figure 1 shows the structure of the DC/AC converter based on a VSI, which represents the power electronics part. Holmes et al. have shown in [4] that each phase can be controlled independently if the three phase system is balanced. Therefore considering each phase independently as an individual half-bridge, the dynamics of the AC current for the three phases can be described by [5]:

$$L_i \frac{di_A(t)}{dt} + R_i i_A(t) = v_{aI}(t) - v_{GA}(t) \quad (1)$$

$$L_i \frac{di_B(t)}{dt} + R_i i_B(t) = v_{bI}(t) - v_{GB}(t) \quad (2)$$

$$L_i \frac{di_C(t)}{dt} + R_i i_C(t) = v_{cI}(t) - v_{GC}(t) \quad (3)$$

Where  $v_{Gx}$  are the voltage of the grid and  $v_{xI}$  is the time average voltage produced by the inverter. Figure 2 shows the block diagram of the controlled

system considering the time delay introduced by the modulation process. In  $\alpha - \beta$  coordinates the model of the three phase converter is as follows:

$$L_i \frac{di_\alpha(t)}{dt} + R_i i_\alpha(t) = v_{\alpha I}(t) - v_{G\alpha}(t) \quad (4)$$

$$L_i \frac{di_\beta(t)}{dt} + R_i i_\beta(t) = v_{\beta I}(t) - v_{G\beta}(t) \quad (5)$$

The time average voltage produced by the inverter can be controlled directly by the duty cycle ( $m$ ) feed to the PWM modulator. The duty cycle is the output of the current controller. The average model of the VSI is considered as a linear amplifier [5] with a gain  $G_{INV}$  related with the DC bus voltage  $v_{DC}$  and a dead time (transport delay) introduced by the switching process [6]. Thus:

$$v_{\alpha I} = G_{INV} \cdot e^{-sT_d} \cdot m \quad (6)$$

Where for PWM with double sample update the delay can be approximated by  $T_d = \frac{1}{2F_s}$ , being  $F_s$  the switching frequency and  $G_{INV} = \frac{V_{dc}}{2}$  for the case of PWM. For the controller design, many authors approximate the transport delay produced by the switching process by a first order Pade approximation [6]:

$$e^{-sT_d} \approx \frac{1 - s(T_d/4)}{1 + s(T_d/4)} \quad (7)$$

Or even they approximate the transport delay roughly by a first order system of the form [7]:

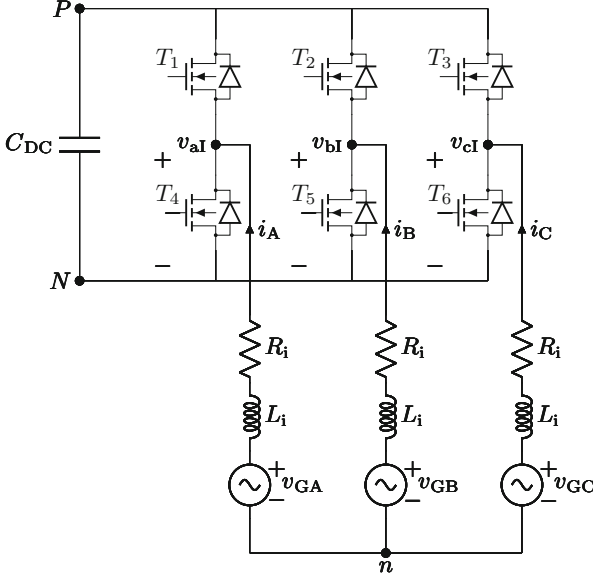
$$e^{-sT_d} \approx \frac{1}{1 + s(3 \cdot T_d/2)} \quad (8)$$

Equations (7) and (8) are a good approximation in case that the switching frequency is at least a factor of 10 times larger than the frequency of the reference signal feed to the PWM modulator. However, there is applications where this condition does not hold, for instance in high power applications when the switching frequency cannot be high due to unacceptable switching losses or in case of active power filters that compensate high order harmonics. In such cases the approximation will lead to a bad performance of the current controller or even to instabilities in the current control loop [8]. Therefore, for such applications is better to design the current controller considering the transport delay directly in the model.

## 2.1 Open Loop Transfer Function

By disregarding the voltage of the grid  $v_{Gx}$  that acts just a disturbance and can be almost entirely canceled by feedforward controller as is explained in [5], the open loop transfer function for the current loop considering the transport delay and a proportional controller is as follows:

$$G(s)_{OL} = K_P \cdot G_{INV} \cdot e^{-sT_d} \cdot \frac{1}{L_i s + R_i} \quad (9)$$



**Fig. 1.** Voltage Source Inverter (VSI) connected to a power grid through a coupling inductor  $L_i$  with an internal resistance  $R_i$ . The grid is represented by the three voltages sources  $v_Gx$ .

The resistance of the coupling inductivity  $R_i$  that is usually smaller compared with the reactance can be neglected, it follows:

$$G(s)_{OL} = K_P \cdot G_{INV} \cdot e^{-sT_d} \cdot \frac{1}{L_i s} \quad (10)$$

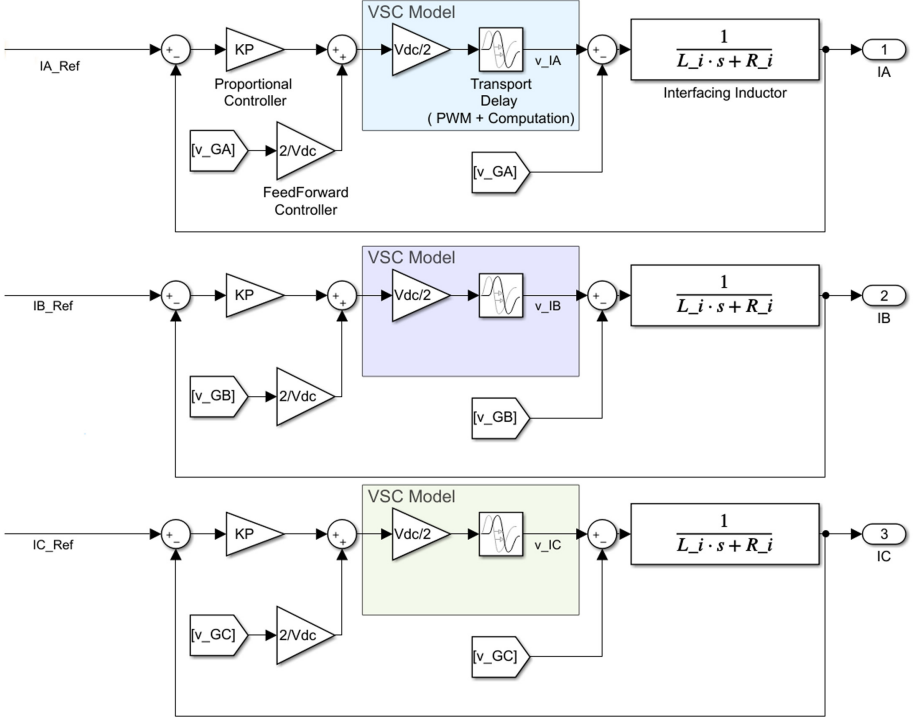
### 3 Derivation of Proportional Gain

The following derivation follows the procedure described on [4], furthermore in this document the derivation is done in a step by step fashion and at the end a final analytical formula is achieved. The phase angle of the frequency dependent expression (10) is equal to:

$$\angle G(s)_{OL} = \angle(K_P \cdot G_{INV}) + \angle(e^{-sT_d}) + \angle\left(\frac{1}{sL_i}\right) \quad (11)$$

$K_P$  and  $G_{INV}$  are just gains and their contribution to the total phase is  $0^\circ$ , the coupling inductivity is always contributing  $-90^\circ$  or  $-\pi/2$  rad to the total phase when  $R_i$  is neglected. The dead time is introducing negative phase in radians that is linearly proportional to the frequency [9], thus:

$$\angle G(s)_{OL} = 0 - (\omega T_d) - \frac{\pi}{2} \quad (12)$$



**Fig. 2.** Block diagram of the current control loop for the three phases. Feedforward and feedback controllers are shown.

The phase of the open loop transfer function can be made function of the desired phase margin (PM) according to:

$$\angle G(s)_{OL} = -\pi + PM \quad (13)$$

$$-\pi + PM = -(\omega T_d) - \frac{\pi}{2} \quad (14)$$

If we choose certain PM, we can use Eq. (14) to calculate which cross over frequency  $\omega_c$  corresponds to the selected PM margin:

$$\omega_c = \frac{\frac{\pi}{2} - PM}{T_d} \quad (15)$$

A PM of  $30^\circ$  or  $\pi/6$  rad, produces a close loop with a damping ratio of approximate 0.4 that should be adequate to track without that much error sinusoidal references and it is judged as the lowest adequate value for the phase margin according to [9]. For a  $PM = 30^\circ$  and  $T_d = \frac{1}{2F_s}$  in (15), it follows:

$$\omega_c = \frac{\pi}{3 \cdot T_d} = \frac{2 \cdot \pi \cdot F_s}{3} \quad (16)$$

The proportional gain  $K_P$  to achieve  $\omega_c$  can be calculated by the gain expression in dB of the open loop transfer function:

$$|G(s)_{OL}|_{dB} = 20\log\left(\frac{K_P \cdot G_{INV}}{L_i}\right) + 20\log(e^{-sT_d}) + 20\log\left(\frac{1}{s}\right) \quad (17)$$

The dead time element is always contributing 0 dB to the overall gain of the system and the integrator produces -20dB per decade thus:

$$|G(s)_{OL}|_{dB} = 20\log\left(\frac{K_P \cdot G_{INV}}{L_i}\right) + 0 - 20\log(\omega) \quad (18)$$

At the cross over frequency  $\omega_c$  the gain of the open loop system is  $|G(s)_{OL}|_{dB} = 0$ , therefore:

$$0 = 20\log\left(\frac{K_P \cdot G_{INV}}{L_i}\right) - 20\log(\omega_c) \quad (19)$$

$$20\log\left(\frac{K_P \cdot G_{INV}}{L_i}\right) = 20\log(\omega_c) \quad (20)$$

Thus:

$$K_P = \omega_c \cdot \frac{L_i}{G_{INV}} = \frac{2 \cdot \pi \cdot F_s}{3} \cdot \frac{L_i}{G_{INV}} \quad (21)$$

### 3.1 Proportional Gain for PWM - 2 Level (2L) and 3L NPC

The static gain for the case of the two level inverter using PWM is  $G_{INV} = \frac{V_{dc}}{2}$ , therefore the proportional gain is finally:

$$K_{PWM} = \frac{4 \cdot \pi \cdot F_s}{3} \cdot \frac{L_i}{V_{dc}} \quad (22)$$

According to Yazdani [5] the average model of the 3 Level NPC topology is the same as the 2 level PWM inverter topology, model that was illustrated in Eqs. (4) and (5). It follows that the gain calculated by the expression (22) is valid for both topologies, 2L PWM and 3L NPC PWM.

### 3.2 Proportional Gain for Space Vector Modulation (SVM) – 2L

Holmes in [11] and Mohan in [10] stated that the maximum amplitude of the output phase voltage (for a waveform measured in reference to the center point of the DC bus) for a sinusoidal reference voltage at one frequency is given by:

$$(v_{alz})_{PWM} = \frac{v_{DC}}{2} \quad (23)$$

In case of SVM, the maximum amplitude of the output phase voltage is limited within the inner circle of the hexagon, in order to do not exceed the carried period, and it is given by [10]:

$$(v_{\text{alz}})_{\text{SVM}} = \frac{v_{\text{DC}}}{\sqrt{3}} \quad (24)$$

Just by inspection and comparison we can conclude that the inverter gain in case of SVM is  $G_{\text{INV}} = \frac{v_{\text{DC}}}{\sqrt{3}}$  and therefore the proportional gain in case of space vector modulation is given by:

$$K_{\text{SVM}} = \frac{2\sqrt{3} \cdot \pi \cdot F_s}{3} \cdot \frac{L_i}{V_{\text{dc}}} \quad (25)$$

### 3.3 Integral Gain

If it is wanted to add an integral term with an integrator in parallel with the proportional controller, the following expression is suggested for the integral gain:

$$K_I = K_P \cdot F_S \cdot \frac{\pi}{3} \cdot \frac{1}{60} \quad (26)$$

The integral gain has been calculated in such a way that the cross over frequency and the phase margin are just determined by the proportional gain in Eq. (21). In other words, the proportional gain calculated by (26) does not introduce any phase lag at the desired cross over frequency.

## 4 Experimental Verification

In order to test the validity of the formulas (21) and (26) a Grid-Tie connected inverter was tested in an experimental setup similar to the scenario depicted in Fig. 1 and the parameters in Table 1. Figure 3 shows a picture of the hardware utilized during the experiment. The Grid-Tie connected converter is operated as STATCOM for injection of reactive power to the power system. The STATCOM is meant with a self-supporting DC-Bus, therefore it only absorbs from the power system the active power at the fundamental frequency necessary to compensate for the internal VSI losses. Invoking the instantaneous power theory [5], the active and reactive power that the power converter exchanges with the grid can be expressed by the following equations:

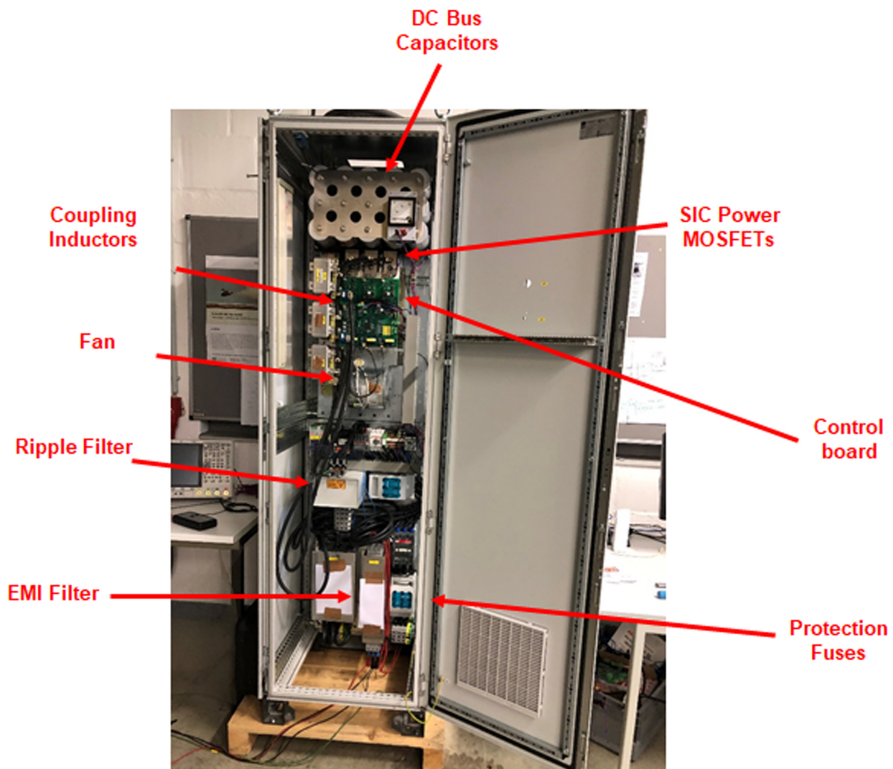
$$P(t) = \frac{3}{2} \cdot (v_{G\alpha}(t) \cdot i_\alpha(t) + v_{G\beta}(t) \cdot i_\beta(t)) \quad (27)$$

$$Q(t) = \frac{3}{2} \cdot (-1 \cdot v_{G\alpha}(t) \cdot i_\beta(t) + v_{G\beta}(t) \cdot i_\alpha(t)) \quad (28)$$

If we set  $P_{\text{Ref}}$  and  $Q_{\text{Ref}}$  as the active and reactive power that the Grid-Tie connected converter should inject into the grid, based on (27) and (28), the current references can be calculated as in (29) and (30).

**Table 1.** Experimental setup parameters

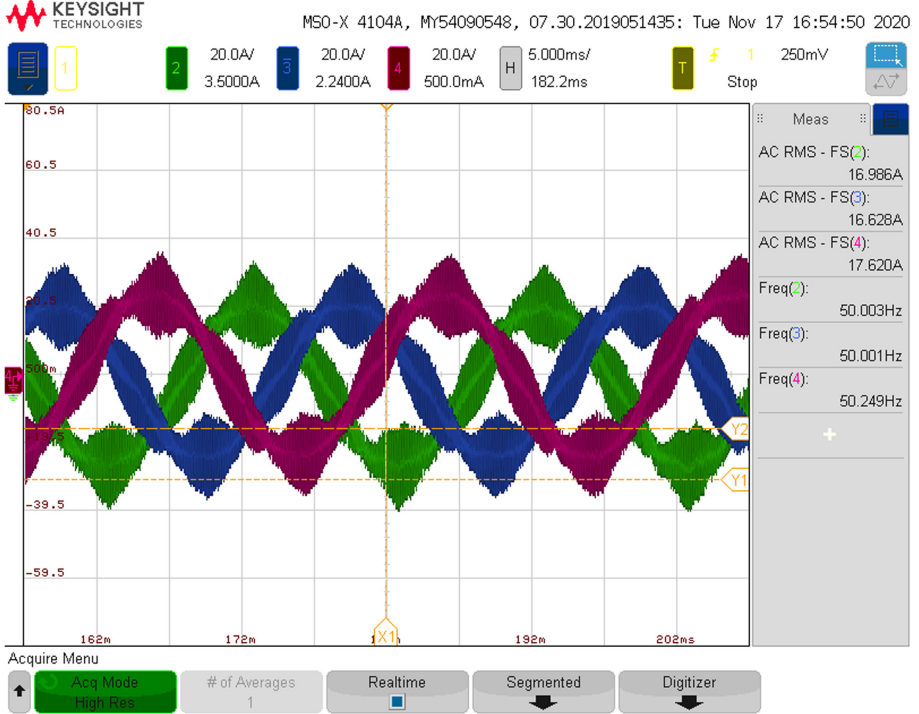
Description	Value
Power System	400 V (L-L), 50 Hz
Power Transformer	12.5 kVA, Imp. 3.3%, X/R = 2
Coupling Inductor	$L_i = 100 \mu\text{H}$ , $R_i = 1.6 \text{ m}\Omega$
Ripple Filter	$C_F = 300 \mu\text{F}$ , $R_F = 45 \text{ m}\Omega$
Modulation Method	PWM
Switching Frequency $F_s$	20 kHz
Current Controller Sampling Frequency	40 kHz
DC bus Voltage/Capacitor	750 V/8 mF



**Fig. 3.** Measured current of the grid-tie connected converter where the current controller was using the gains suggested in (21) and (26)

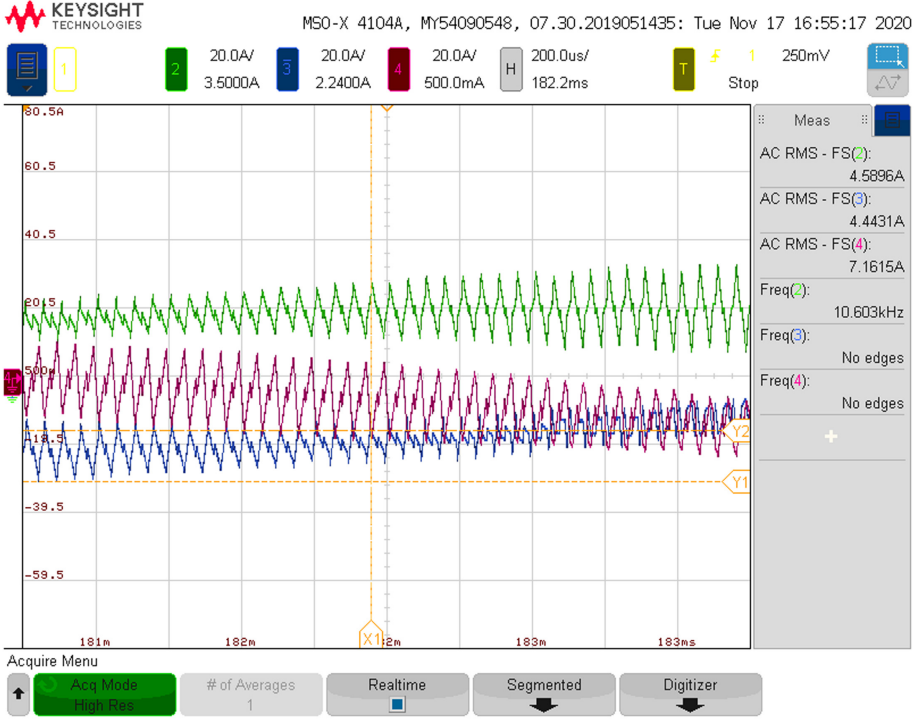
$$i_{\alpha ref}(t) = \frac{2}{3} \cdot \frac{v_{G\alpha}}{v_{G\alpha}^2 + v_{G\beta}^2} \cdot P_{Ref} + \frac{2}{3} \cdot \frac{v_{G\beta}}{v_{G\alpha}^2 + v_{G\beta}^2} \cdot Q_{Ref} \quad (29)$$

$$i_{\beta ref}(t) = \frac{2}{3} \cdot \frac{v_{G\beta}}{v_{G\alpha}^2 + v_{G\beta}^2} \cdot P_{Ref} - \frac{2}{3} \cdot \frac{v_{G\alpha}}{v_{G\alpha}^2 + v_{G\beta}^2} \cdot Q_{Ref} \quad (30)$$



**Fig. 4.** Measured current of the grid-tie connected converter where the current controller was using the gains suggested in (21) and (26)

The complete system is controlled by a Texas Instrument microcontroller TMS320F28379DZWTT. The current controller is implemented in the microcontroller using the gains proposed in Sect. 3 with the parameters on Table 1. The digital implementation is carried out by transforming the continuous time domain current controller into a discrete time domain current controller using the bilinear transformation. The reference for the DC Bus voltage is set to 750 V, and the  $Q_{Ref}$  is set to 12 kVar. The measured currents during the experiment

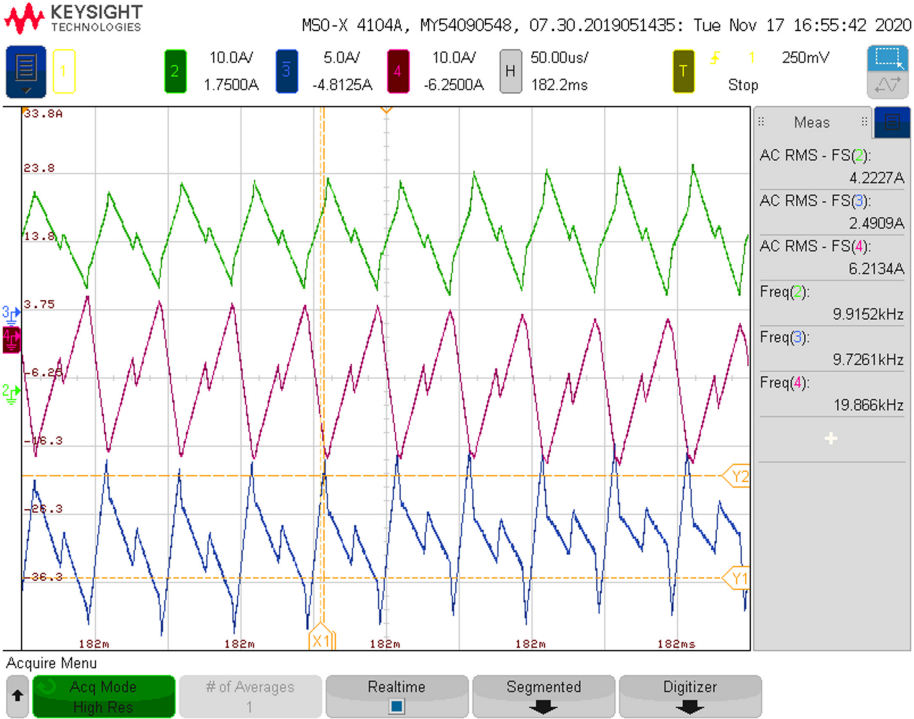


**Fig. 5.** Measured current of the grid-tie connected converter where the current controller was using the gains suggested in (21) and (26)

are depicted in Fig. 4. The RMS value of the currents measured are in the order of 17 A, leading to a total apparent power of:

$$S = 3 \cdot V \cdot I = 3 \cdot 230V \cdot 17A = 11.7kVA. \quad (31)$$

Due to the fact that the Grid-Tie connected converter uses SiC power MOS-FETs, the VSI internal losses are very small and the total apparent power injected by the converter is very close to the reference of 12 kVar set for  $Q_{Ref}$ . The current is zoomed in Fig. 5 and Fig. 6 in order to appreciate the current ripple of the injected currents. The current ripple is in the range of 20 A peak to peak.



**Fig. 6.** Measured current of the grid-tie connected converter where the current controller was using the gains suggested in (21) and (26)

## 5 Conclusion

This paper has shown the derivation of an analytical formula to calculate the proportional gain of a current controller for a three PWM converter. The analytical formula is easy to follow; it only depends on the parameters of the three phase PWM converter. The idea behind the formula is to guarantee always a gain that maintains the inner current loop stable with a controller that provides a fast response. Furthermore, the formula can be used on an adaptive current controller when a nonlinear coupling inductivity is used. When the last is the case, based on the current reference at certain moment and a loop-up table with the L-I characteristic of the coupling inductivity, the proportional gain can be recalculated at all times for new current references feed to the current controller.

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