



Modulation Recognition with Alpha-Stable Noise Over Fading Channels

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Abstract. This paper proposes a method based on kernel density estimation (KDE) and expectation condition maximization (ECM) to realize digital modulation recognition over fading channels with non-Gaussian noise in the cognitive radio networks. A compound hypothesis test model is adopted here. The KDE method is used to estimate the probability density function of non-Gaussian noise, and the improved ECM algorithm is used to estimate the fading channel parameters. Numerical results show that the proposed method is robust to the noise type over fading channels. Moreover, when the GSNR is 10 dB, the correct recognition rate for the digital modulation recognition under non-Gaussian noise is more than 90%. Gaussian noise, and the improved ECM algorithm is used to estimate the fading channel parameters. Numerical results show that the proposed method is robust to the noise type over fading channels. Moreover, when the GSNR is 10 dB, the correct recognition rate for the digital modulation recognition under non-Gaussian noise is more than 90%.

Keywords: Cognitive radio · Modulation recognition · Fading channel · Non-Gaussian noise · Alpha stable distribution

1 Introduction

With the rapid development of communication technology and the increasing tension in spectrum resources, cognitive radio technology has become one of the key technologies to solve these problem [1]. In the cognitive radio network (CRN), spectrum sensing technology and spectrum access technology are particularly important. Among them, spectrum sensing technology not only needs to accurately detect the occurrence of authorized user signals, but also needs to identify the modulation type of authorized user signals [2]. Then we can determine the authorized user information, such as the type of service, the strength of the service, and so on. CRN technology is easily affected by fading channels, inter-user interference and electromagnetic pulse noise. This interference and noise often highlights that the probability density function is a thick tail of non-Gaussian distribution [3]. Therefore, it has practical engineering significance to

study the digital modulation signal identification method under non-Gaussian fading channel in CRN.

Several algorithms have been reported to solve modulation recognition based on non-Gaussian noise. Some scholars have studied the digital modulation recognition under the alpha stable distributed noise model. Some have studied the modulation recognition under the mixed Gaussian noise distribution. In [4], a novel modulation classification method was proposed by using cyclic correntropy spectrum (CCES) and deep residual neural network (ResNet). CCES is introduced to effectively suppress non-Gaussian noise through the designated Gaussian kernel. In [5], T. Dutta et al. proposed a cyclostationary (CS) property based on FB classification technique under non-Gaussian impulsive noise condition. CS features perform well at low signal to noise ratio (SNR). But the performances of the classifiers degrade due to the presence of the impulse noise. In [6], Hu Y H et al. used fractional low-order wavelet packet decomposition and neural network recognition method, but this method has poor recognition performance under low GSNR. In [7], under the selective channel, the Gaussian mixture model is used to model the noise, and the identification problem of the amplitude phase modulation signal is studied. In the actual communication process, the signal is affected by the fading of channel besides a lot of non-Gaussian noise. In [8], D. E. Kebiche et al. investigated the performance of the Rao-test based detector for wideband spectrum sensing under non-Gaussian noise in a multi-carrier transmission framework. It showed that the Rao-test based detector combined with the universal filtered multicarrier (UFMC) outperforms the traditional OFDM based system in a realistic non-Gaussian noise environment. In [9], a signal identification method based on normalized fractional low-order cumulants for alpha-stable distributed noise fading channel was proposed. When the GSNR is low, the classification and recognition performance is poor. In [10], S. Hu, Y. Pei et al. proposed a low-complexity blind data-driven modulation classifier which operates robustly over Rayleigh fading channels under uncertain noise condition modeled using a mixture of three types of noise, namely, white Gaussian noise, white non-Gaussian noise and correlated non-Gaussian noise. The performance of proposed classifier approaches that of maximum likelihood classifiers with perfect channel knowledge.

In view of the above problems, this paper proposes a new method of modulated signal recognition under the alpha stable distributed noise fading channel. Firstly, the kernel density estimation algorithm is used to estimate the probability density function of the alpha stable distribution noise. The improved Expectation Conditional Maximization algorithm is then used to estimate the fading channel parameters. Finally, the modulation type of the signal is identified according to the composite hypothesis test model. The simulation results show that when the characteristic index of alpha stable distribution noise is 1.5 and the GSNR is 10 dB, the recognition rate of the signal in the fading channel is more than 90%. It can be seen that this method is effective and feasible.

2 System Model

Let B be the number of classes of all possible modulation types in the set of signals to be identified. The transmitted signal of class b can be expressed as:

$$s_b(t) = \sum_{m=-\infty}^{\infty} s_{mb}g(t - nT) \quad (1)$$

where T is the symbol period of the transmitted signal, $g(t)$ is the shaping filter function, and $s_{mb} \in s_b$ is the signal value of a certain modulation method. In CRN, the model of the received signal $r(t)$ can be described as:

$$r(t) = \sum_{l=0}^{L-1} h_l s_b(t - \tau_l) + \omega(t) \quad (2)$$

where L is the number of paths in the fading channel, τ_l is the time delay of each path, h_l is the attenuation range of each path, and $\omega(t)$ is the alpha stable distribution noise. The received signal is sampled at the receiving end by the sampling period T_s . We assume that the time delay τ_l generated by each path is an integer times the sampling period and that the number of sampling points of the time delay after sampling is still represented by τ_l , the sampled received signal can be expressed as:

$$r[k] = \sum_{l=0}^L h_l s_b[kT_s - \tau_l] + \omega[k], k = 1, 2, \dots, k \quad (3)$$

where K is the signal length, $\{h_l\}_{l=0}^{L-1}$ and $\{\tau_l\}_{l=0}^{L-1}$ are unknown channel parameter. Since the alpha stable distribution does not have a uniform closed PDF, $\omega(t)$ is usually described by a feature function [11], which is expressed as:

$$\phi(\theta) = \exp\{j\mu\theta - \gamma|\theta|^\alpha [1 + j\beta \operatorname{sgn}(\theta)\omega(\theta, \alpha)]\} \quad (4)$$

where $\operatorname{sgn}(\theta)$ is a symbolic function,

$$\operatorname{sgn}(\theta) = \begin{cases} 1, & \theta > 0 \\ 0, & \theta = 0 \\ -1, & \theta < 0 \end{cases}, \quad \omega(\theta, \alpha) = \begin{cases} \tan(\alpha\pi/2), & \alpha \neq 1 \\ \frac{2}{\pi} \log|\theta|, & \alpha = 1 \end{cases}$$

$\gamma \geq 0$ is the dispersion coefficient, also known as the scale coefficient, α is called the characteristic index and its value range is $0 < \alpha \leq 2$. When $\alpha = 2$, the alpha stable distribution noise is converted into Gaussian noise. The parameter β ($-1 \leq \beta \leq 1$) is a symmetrical parameter that determines the symmetry of the distribution. When $\beta = 0$, the alpha stable distribution is called the *sas* distribution. The parameter μ is called the positional parameter. For the *sas* distribution, if $0 < \alpha < 1$, μ is the median. If $1 < \alpha \leq 2$, μ is the mean. If $\mu = 0$ and $\gamma = 1$ are satisfied, the alpha stable distribution is called the standard alpha stable distribution.

3 Modulation Identification Under Alpha Stable Noise and Fading Channel

3.1 Probability Density Function Estimation of Alpha Stable Distributed Noise

We assume that each sample in the sample set $X = \{X_1, X_2, \dots, X_N\}$ is independent and obeys the distribution of PDF as $p(X)$. The PDF is estimated using the Kernel Density Estimation (KDE) [12].

First we estimate the probability density in a small area. If a small area R is in the space where the sample is located, the probability that a random variable X is in the area R is

$$P_R = \int_R P(X)dX \quad (5)$$

known by the definition of binomial distribution, the probability that k of the N independent and identically distributed samples of the sample set are in the area R is:

$$P_k = C_N^k P_R^k (1 - P_R)^{N-k} \quad (6)$$

where C_N^k represents the number of combinations of k values randomly taken from N values. Then we get the expectation of k : $E[k] = k = NP_R$. Therefore, the estimate of P_R can be expressed as:

$$P_R = \frac{k}{N} \quad (7)$$

When $p(x)$ is continuous and the volume of R is sufficiently small, it can be considered that $p(x)$ in R is a constant, then Eq. (5) can be expressed as:

$$P_R = \int_R p(x)dx = p(x)V \quad (8)$$

where V is the volume of R . Substituting (7) into (8), we have:

$$\hat{p}(x) = \frac{k}{NV} \quad (9)$$

So we have completed the probability density estimation in a small area. But the probability density function estimated in this way is not continuous. Therefore, this paper uses the KDE method. When the small volume is fixed, the small volume of sliding is used to estimate the probability density of each point.

We assume that X is a d -dimensional vector, each small volume is a hypercube, each dimension has an edge length of h' and each small volume has a volume of $V = h'^d$. The d -dimensional unit window function is used to calculate the number of samples falling into each small volume:

$$\phi(u_1, u_2, \dots, u_d) \begin{cases} 1, & |u_i| \leq \frac{1}{2}, i = 1, 2, \dots, d \\ 0, & \text{other} \end{cases} \quad (10)$$

At this time, $\phi\left(\frac{x-x_i}{h'}\right)$ can be used to determine whether the sample X_i is in a cube whose center is X and whose edge length is h' . Let the number of observation points be N , then the samples falling in the above cube can be expressed as follows:

$$k_N = \sum_{i=1}^N \phi\left(\frac{x-x_i}{h'}\right) \quad (11)$$

Substituting Eq. (11) into Eq. (9), a PDF estimate at point x is:

$$\hat{P}(x) = \frac{1}{N} \sum_{i=1}^N \frac{1}{V} \phi\left(\frac{x-x_i}{h'}\right) \quad (12)$$

From a nuclear perspective, we can define the kernel function as:

$$K(x, x_i) = \frac{1}{V} \phi\left(\frac{x-x_i}{h}\right) \quad (13)$$

The estimate of the PDF can be regarded as using the kernel function to perform interpolation operation on the sample within the range of values. Its expression is:

$$K(x) = \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{1}{2}x^2\right) \quad (14)$$

In short, a kernel density estimation process needs to move the kernel function to the position of each observation. And then select the global bandwidth to control the smoothness of the probability density function and the expansion of the kernel function. The revised evaluation formula is:

$$\hat{f}(x) = \frac{1}{N} \sum_{i=1}^N \frac{1}{h} K\left(\frac{x-x_i}{h}\right) \quad (15)$$

3.2 Estimation of Fading Channel Parameters

In this paper, based on the principle of ECM algorithm [13], combined with the model of modulation recognition under non-Gaussian fading channel, the parameter estimation steps of the proposed fading channel are as follows:

- 1) E-step: Under the assumption $H_b (b = 1, 2, \dots, B)$ corresponding to each modulated signal, we calculate the conditional expectation of the logarithm likelihood function of the complete data. We have:

$$\begin{aligned} Q\left(\theta, \theta^P | H_b\right) &= E[\log(p(c|\theta, H_b)) | \theta, \theta^P, H_b] \\ &= \sum_h \log(p(c|\theta, H_b)) P(h | \theta, \theta^P, H_b) \end{aligned} \quad (16)$$

where B is the number of modulation modes in the alternative set. Considering that the distribution of $\{r[k]_{k=1}^K\}$ and $\{s_b[k]_{k=1}^K\}$ is different, $\log(p(c|\theta, H_b))$ can be written as:

$$\begin{aligned} & \log(p(c|\theta, H_b)) \\ &= \log\left(\prod_{k=1}^K p(r[k], s_b[k], H_b)\right) \\ &= \sum_{k=1}^K \log\left(\frac{1}{N_b} p(r[k]|s_b[k], \theta)\right) \end{aligned} \tag{17}$$

where:

$$\begin{aligned} & p(r[k]|s_b[k], \theta) \\ &= \frac{1}{N} \sum_{n=1}^N \frac{1}{h\lambda_n} K \left(\frac{r[k] - \sum_{l=0}^{L-1} h_l s_b[kT_s - \tau_l] - \omega_n}{h\lambda_n} \right) \end{aligned} \tag{18}$$

By Bayesian theory:

$$\begin{aligned} & p(h|o, \theta^p, H_b) \\ &= \prod_{i=1}^K P(s_b[k]|r[k], \theta^p, H_b) \\ &= \prod_{i=1}^K \frac{1}{N_b} \frac{p(r[k]|s_b[k], \theta^p)}{P(r[k]|\theta^p, H_b)} \end{aligned} \tag{19}$$

where:

$$\begin{aligned} & p(r[k]|s_b[k], \theta^p) \\ &= \frac{1}{N} \sum_{n=1}^N \frac{1}{h\lambda_n} K \left(\frac{r[k] - \sum_{l=0}^{L-1} h_l^p s_b[kT_s - \tau_l^p] - \omega_n}{h\lambda_n} \right) \\ & p(r[k]|\theta^p, H_b) \\ &= \frac{1}{N_b} \sum_{s_b[k]} \left[\frac{1}{N} \sum_{n=1}^N \frac{1}{h\lambda_n} K \left(\frac{r[k] - \sum_{l=0}^{L-1} h_l^p s_b[kT_s - \tau_l^p] - \omega_n}{h\lambda_n} \right) \right] \end{aligned} \tag{20}$$

Finally, $Q(\theta, \theta^p)$ can be obtained:

$$Q(\theta, \theta^p|H_b)$$

$$= \sum_{k=1}^K \sum_{s_b[k]} \left[\log \left(\frac{1}{N_b} p(r[k]|s_b[k], \theta) \right) \frac{1}{N_b} \frac{p(r[k]|s_b[k], \theta^p)}{p(r[k]|\theta^p, H_b)} \right] \quad (21)$$

- 2) M-step: This process maximizes the $Q(\theta, \theta^p)$ in the E-step to find a new estimate of the unknown parameter and then substitutes it as the known quantity into the next iteration.

$$\hat{\theta} = \arg \max_{\theta} Q(\theta, \theta^p | H_b) \quad (22)$$

The E-step of the ECM algorithm is the same as the EM algorithm, and each M-step is replaced with several simpler conditions to maximize the CM-step. For a certain hypothesis, the unknown parameter vector $\theta = \{h_1, \dots, h_L, \tau_1, \dots, \tau_L\}$ is divided into $\theta_1 = \{h_1, \dots, h_L\}$ and $\theta_2 = \{\tau_1, \dots, \tau_L\}$. That is, the unknown channel parameter vector is $\theta = \{\theta_1, \theta_2\}$, $s = 2$. Each unknown parameters is given an initial value before the iterative process begins. In the iteration, keeping θ_2^p of the current iteration unchanged, we derivate $Q(\theta, \theta^p | H_b)$ to θ_1 and let its derivative be 0. We get the estimation θ_1^{p+1} of θ_1 in the next iteration. Then keeping the value of θ_1^{p+1} unchanged, we derivate $Q(\theta, \theta^p | H_b)$ to θ_2 and let its derivative be 0. We get the estimate θ_2^{p+1} of θ_2 in the next iteration. When the set convergence condition is satisfied, the iteration stops. The obtained parameter estimation value is the final unknown parameter estimation value of the iteration under the $p + 1$ th assumption.

3.3 Modulation Recognition Based on Compound Hypothesis Test

The recognition method based on the likelihood function describes the recognition process as a compound hypothesis test process. The modulation mode corresponding to the maximum hypothesis in the (logarithmic) likelihood function of the received signal is determined as the modulation mode of the signal [14].

We use the KDE method to estimate the PDF of alpha stable noise distribution. Combing

$$\begin{aligned} & p(r[1], \dots, r[k] | H_b) \\ &= \prod_{k=1}^K \sum_{i=1}^{N_b} p(r[k] | s_{bi}[k]) P(s_{bi}[k] | H_b) \end{aligned} \quad (23)$$

$$P(s_{bi}[k] | H_b) = 1 / N_b \quad (24)$$

we obtain:

$$\begin{aligned} & p(r[1], \dots, r[K] | H_b) \\ &= \prod_{k=1}^K \frac{1}{N_b} \sum_{i=1}^{N_b} p(r[k] | s_{bi}[k]) \end{aligned} \quad (25)$$

where N_b represents the possible number of values of the amplitude of the b th modulation signal. It can be seen that the probability density distribution of the received signal is equal to the probability density distribution of the noise when the transmitted signal is known. This knowledge can be obtained by the estimation method of the fading channel parameters. Therefore, the process of modulation recognition under non-Gaussian fading channels is expressed as:

$$\hat{H} = \arg \max_{H_b} \log(r[1], \dots, r[K]|H_b) \tag{26}$$

where $H_b (b = 1, 2, \dots, B)$ indicates the modulation type of the received signal S_b .

4 Simulation Results and Analysis

In order to verify the effectiveness of the method, simulation experiments are carried out by MATLAB simulation software. The simulation parameters are set as follows: the signal set to be identified is BPSK, QPSK, 16QAM and 64QAM, which are four commonly used digital modulated signals. The noise is additive standard τ_l distributed noise. The carrier frequency of the modulated signal is 10 kHz. The code element rate is 1200 baud. The roll-off factor of the shaping filter is 0.35. The sampling frequency is 40 kHz. The number of signal sampling points is 3000 and the multipath channel is ITU_V_B channel of Rec.ITU-RM.225. The number of Monte Carlo simulations is 2000.

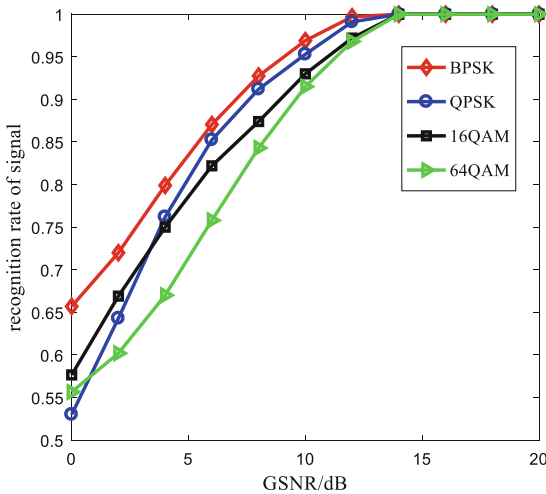


Fig. 1. Identification performance of signals under different GSNR

When $\alpha = 1.5, h_{opt} = 0.7$, the recognition performance of the signal at different GSNR is shown in Fig. 1. It can be seen from Fig. 1 that the recognition performance of different signals under the fading channel increases as the GSNR increases. When

the GSNR is 10 dB, the recognition accuracy of different signals is more than 90%. It can be seen that the proposed identification method has good recognition performance under the fading channel. It is effective and feasible.

When $h_{opt} = 0.7$, $GSNR = 10$ dB, the recognition performance of the signals under different characteristic indices is shown in Fig. 2. It can be seen from Fig. 2 that the recognition performance of different signals under the fading channel increases as the characteristic indices increases. When α is greater than 1, the correct recognition rate of different signal recognition is above 85%. When $\alpha = 2$, the correct recognition rate of different signals under the fading channel is 100%. It is shown that the proposed method is not only suitable for non-Gaussian noise fading channels, but also has good recognition performance under the environment of Gaussian noise fading channels. Therefore, the proposed method is robust to different noise types.

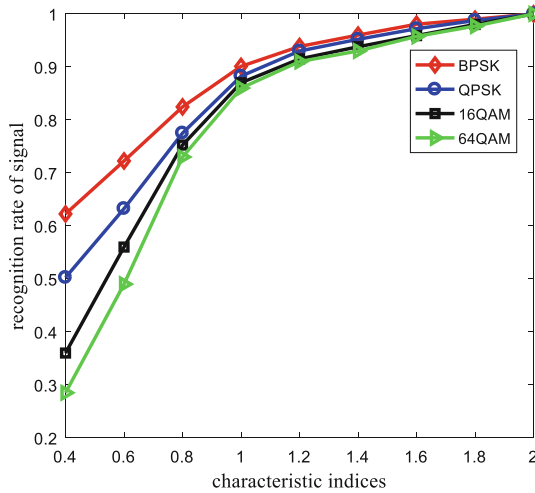


Fig. 2. Identification performance of signals under different characteristic indices

5 Conclusion

Aiming at the problem of modulation recognition in non-Gaussian fading channel in cognitive networks, this paper proposes a modulation recognition model based on compound hypothesis test suitable for this scenario. Under this model, the probability density function of non-Gaussian noise is estimated by the kernel density estimation method. The parameters of the fading channel are estimated using an improved expectation condition maximization algorithm. The simulation results show that the proposed identification method is effective and feasible under non-Gaussian noise fading channels and robust to different noise types.

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